Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed - to - be - University under section 3 of UGC Act 1956)

COURSE FILE CONTENTS

NAME OF THE FACULTY	**Ms. J. Arthy**	**FACULTY DEPT**	**Mathematics**
COURSE	**ENGINEERING MATHEMATICS - I**	**COURSE CODE**	**U18BSMA101**
YEAR	**2019 – 2020**	**SEMESTER**	**I**
DEG & BRANCH	**B.TECH (All Branches)**	**DURATION**	**60 HRS**

SL.NO	**DETAILS IN COURSE FILE**	**REMARKS**
1.	LEARNING OUTCOMES	
2.	LESSON PLAN WITH CO MAPPING	
3.	INDIVIDUAL TIME TABLE	
4.	SYLLABUS WITH COURSE OUTCOMES	
5.	LECTURE NOTES	
6.	INTERNAL ASSESSMENT TEST I - QUESTION PAPER	
7.	INTERNAL ASSESSMENT TEST I – ANSWER KEY	
8.	INTERNAL ASSESSMENT TEST I – SAMPLE ANSWER SHEETS	
9.	INTERNAL ASSESSMENT TEST II - QUESTION PAPER	
10.	INTERNAL ASSESSMENT TEST II – ANSWER KEY	
11.	INTERNAL ASSESSMENT TEST II- SAMPLE ANSWER SHEETS	
12.	ASSIGNMENT QUESTION PAPER	
13.	SAMPLE ASSIGNMENTS	
14.	END SEMESTER EXAM QUESTION PAPER	
15.	END SEMESTER EXAM-ANSWER KEY	
16.	TEXT BOOK & REFERENCE BOOK FOLLOWED	
17.	PREVIOUS QUESTION PAPERS	
18.	QUESTION BANK	
19.	STUDENT PERFORMANCE RECORD	
20	STUDENT ATTENDANCE RECORD	
21.	COURSE END SURVEY	
22.	CO ATTAINMENT	

Arthy
STAFF

K. M
HOD

Bhararh Institute of Higher Education and Research
School of Basic Science

U18BSMA 101 Engineering Mathematics I

LEARNING OUTCOMES

1. Analyze the optimum solution of various engineering problems involving single variables.
2. Know the basic concepts of integration and evaluating the problems which involves Beta and Gamma functions.
3. Solve the differential functions and optimizes the problems with two variables functions.
4. Apply multiple integrals to compute area and volume over curves, surface and domain in two dimensional and three-dimensional spaces.
5. Evaluate Eigen value and eigen vector problems from practical areas using transformations.
6. Construct the eigen vector for the problem in engineering field and Diagonalizing the matrix

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BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY

No.173, Agharam Road, Selaiyur, Chennai - 600 073

B.Tech First year 2019-2020(ODD SEM)
LESSION PLAN

SUBJECT CODE / NAME : **U18BSMA101 / ENGINEERING MATHEMATICS - I**

COURSE : **Common to B.Tech**

SEMESTER / BRANCH : **I / Mech, Mect, Auto, Aero, EEE, ECE, CSE, IT, Civil & BME**

Sl No	Topic Name	Reference Book	No. of. Periods	Cumulative No. of. Periods	Teaching Aids (LCD / OHB / BB)
	UNIT-V :MATRICES				
1.	Characteristic Equations	R1 Chapter 1 (Pages 1.1 to 1.80)	1		BB
2.	Cayley Hamilton Theorem		1	2	BB
3.	Eigen Values and Eigen Vectors		2	4	BB
4.	Diagonalization of Matrices		1	5	BB
5.	Quadratic form to Canonical Form		1	6	BB
6	Nature of Quadratic form		1	7	BB
7.	Properties Eigen Values and Eigen Vectors		2	9	BB
8.	Tutorial		3	**12**	
	UNIT-IDIFFERENTIAL CALCULUS(One Variable)				
9	Representation of Functions& Limit of a Function.	**R3** Chapter 4.1 to 4.9 (Pages 267 to306)	1	13	BB
10	Derivatives (Implicit Function, Logarithmic Diff., Parametric Form, Trigonometric Function & One function with respect to another function)		4	17	BB
11	Maxima and Minima (One Variable)		1	18	BB
12	Rolle's and Mean Value Theorem		1	19	BB
13	Maclaurin's Theorem		1	20	BB
14	Taylor's Theorem		1	21	BB
15	Tutorial		3	**24**	
	UNIT-IIIDIFFERENTIAL CALCULUS(Several Variable)				
16	Partial Derivatives	**R3** Chapter 5.6 to 5.17 (Pages 354 to385)	2	26	BB
17	Euler's Theorem on Homogeneous Function		1	27	BB
18	Total Derivatives		2	29	BB
19	Directional Derivatives		1	30	BB
20	Jacobian		2	32	BB

29:7:2019

6. Ms. J. Arthy, Asst. Prof.

Days\ Period	1	2	3	4	5	6	7
Mon			I yr CSE A	I yr CSE B		I yr CSE M	
Tue	I yr CSE A		I yr CSE F			I yr CSE M	
Wed		I yr CSE A	I yr CSE M		I yr CSE M		
Thur	I yr CSE F		I yr CSE B		I yr CSE F		
Fri	I yr CSE M	I yr CSE B		I yr CSE A		I yr CSE F	

		L	T	P	C
U18BSMA101	**Engineering Mathematics – I (Common to B.Tech - Mech, Mechatronics, Automobile, Aero, EEE, EIE, ECE, CSE, IT, Civil & Bio Medical admitted from July 2018)**				
	Total Contact Hours – 60	3	1	0	4
	Prerequisite Course– School Level Mathematics				
	Course Coordinator Name& Department – Ms.J.Aiswarya& Dr.K.Manimekala Department of Mathematics				
COURSE OBJECTIVES	➢ The objective of this course is to familiarize the prospective engineers with techniques in calculus, multivariate integration analysis and linear algebra. ➢ It aims to equip the students with standard concepts and tools at an intermediate to advanced level that will serve them well towards tackling more advanced level of mathematics and applications that they would find useful in their disciplines.				

COURSE OUTCOMES (COs)	
CO1	Analyze the optimum solution of various engineering problems involving single variables.
CO2	Know the basic concepts of integration and evaluating the problems which involves Beta and Gamma functions.
CO3	Solve the differential functions and optimizes the problems with two variables functions.
CO4	Apply multiple integrals to compute area and volume over curves, surface and domain in two dimensional and three-dimensional spaces.
CO5	Evaluate Eigen value and eigen vector problems from practical areas using transformations;
CO6	Construct the eigen vector for the problem in engineering field and Diagonalizing the matrix

Mapping of Course Outcomes with Program outcomes(POs)

H –High, M-Medium, L-Low

1	COs/Pos	1	2	3	4	5	6	7	8	9	10	11	12	PSO1	PSO2
2	CO1	H	H										L		
	CO2	H	H			M							M		
	CO3	H	H	M		M							M		
	CO4	H	H			M							M		
	CO5	H	H	M									L		
	CO6	H	H										L		
3	Category	Basic Science (BS)													
4	Approval	47th Academic Council Meeting held in Aug, 2018													

Bharath Institute of Higher Education and Research

UNIT I DIFFERENTIAL CALCULUS - One Variable **(9+3) Hrs**
Representation of functions – limit of a function – continuity – Derivatives – Differentiation rule – Maxima and minima of functions of one variable – Rolle's Theorem – Mean Value Theorem – Taylor's and Maclaurin's Theorem with remainders.

UNIT II INTEGRAL CALCULUS - One Variable **(9+3) Hrs**
Definite integrals – Substitution rule – Techniques of integration – Integration by parts – Trigonometric integrals – Trigonometric substitutions – Integrations of rational functions by partial fractions – Integrations of irrational functions- Integration of improper functions - Beta. Gamma functions and their properties.

UNIT III DIFFERENTIAL CALCULUS - Several Variables **(9+3) Hrs**
Partial derivatives –Euler's theorem on Homogeneous functions - directional derivatives – total derivative – Jacobian – Maxima and minima of two variables.

UNIT IV MULTIPLE INTEGRALS - Several Variables **(9+3) Hrs**
Double integrals in Cartesian co-ordinates – Change of order of integrations – Area as a double integral – Triple integrals in Cartesian co-ordinates –Volume as triple integrals – Double integrals in polar co-ordinates – simple problems.

UNIT V MATRICES **(9+3) Hrs**
Characteristic Equations – Eigen value and Eigenvectors of the real matrix– Properties— Cayley-Hamilton Theorem – Diagonalization of matrices – Reduction of quadratic form to canonical form by orthogonal transformation – Nature of Quadratic form.

TEXTBOOKS

1. Grewal B. S, Higher Engineering Mathematics, Khanna Publisher, Delhi – 2014.
2. Kreyszig. E, Advanced Engineering Mathematics, 10th edition, John Wiley & Sons, Singapore, 2012.

REFERENCE BOOKS

1. Veerarajan T, Engineering Mathematics, II edition, Tata McGraw Hill Publishers, 2008.
2. Kandasamy P &co., Engineering Mathematics, 9th edition, S. Chand & co Pub., 2010.
3. N.P.Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications, Reprint, 2010.
4. Narayanan S., Manicavachagam Pillai T.K., Ramanaiah G., Advanced Mathematics for Engineering students, Volume I (2nd edition), S.Viswanathan Printers and Publishers.
5. George B. Thomas Jr .Maurice D.Weir, Joel Hass., Thomas' Calculus .Twelfth Edition Addison-Wesley, Pearson.

Course Coordinator	**HOD**
Ms. J. Aiswarya	Dr.K.Manimekalai
Asst.Professor	Prof. & Head
Department of Mathematics	Department of Mathematics

Unit - 1
Multiple Integrals.

Part - A

① Evaluate $\int_0^1\int_0^1 xy\,dy\,dx$.

Solu:-

$$\int_0^1\int_0^1 xy\,dy\,dx = \int_0^1 x\left[\frac{y^2}{2}\right]_0^1 dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}.$$

② Evaluate $\int_0^3\int_0^1 (x^2+3y^2)\,dy\,dx$

Solu:-

$$\int_0^3\int_0^1 (x^2+3y^2)\,dy\,dx = \int_0^3 \left(x^2y+3\frac{y^3}{3}\right)_0^1 dx$$

$$= \int_0^3 (x^2+1)\,dx = \left(\frac{x^3}{3}+x\right)_0^3$$

$$= \left(\frac{27}{3}+3\right) = 9+3 = 12.$$

③ Evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}} dy\,dx$.

Solu:-

$$\int_0^a\int_0^{\sqrt{a^2-x^2}} dy\,dx = \int_0^a (y)_0^{\sqrt{a^2-x^2}}\,dx$$

$$= \int_0^a \sqrt{a^2-x^2}\,dx \quad [\text{formula}]$$

$$= \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \left[\left(0+\frac{a^2}{2}\sin^{-1}1\right) - (0+0)\right]$$

$$= \frac{a^2}{2}\left(\frac{\pi}{2}\right) = \frac{\pi a^2}{4}.$$

4) Evaluate $\int_0^1\int_0^y\int_0^{x+y} dx\,dy\,dz$.

Soln.

Let $\int_0^1\int_0^y\int_0^{x+y} dz\,dx\,dy = \int_0^1\int_0^y (z)_0^{x+y}\,dx\,dy$

$= \int_0^1\int_0^y (x+y)\,dx\,dy$

$= \int_0^1 \left(\frac{x^2}{2}+xy\right)_0^y dy$

$= \int_0^1 \left(\frac{y^2}{2}+y^2\right) dy = \left(\frac{y^3}{6}+\frac{y^3}{3}\right)_0^1$

$= \frac{1}{6}+\frac{1}{3} = \frac{1+2}{6} = \frac{3}{6} = \frac{1}{2}.$

5) What is the formula to find the volume V of a three dimensional region interms of triple integration?

Soln:

Volume of a three dimensional region V is

$$V = \iiint_V dz\,dy\,dx.$$

6) Evaluate $\int_C x^2dy + y^2dx$ where c is the path $y=x$ from $(0,0)$ to $(1,1)$

Soln:

Given that $y=x$, $dy=dx$ and the limits of x is from $x=0$ to $x=1$. Hence

$\int_C x^2dy + y^2dx = \int_0^1 x^2dx + x^2dx$

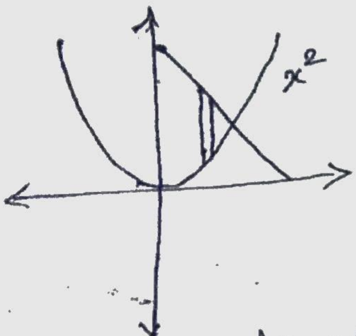
$= 2\int_0^1 x^2dx = 2\left(\frac{x^3}{3}\right)_0^1$

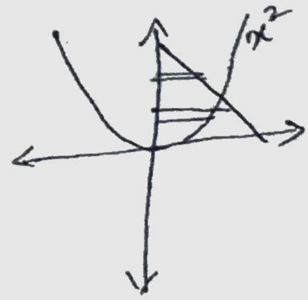
$= 2/3.$

(7) change of the order of integration in

$$I = \int_0^1 \int_{x^2}^{2-x} f(x,y)\,dx\,dy$$

Soln:-





The given limits are

$y = x^2 \qquad y = 2-x.$

After changing the order of integration the region of integration can be divided into two parts namely I_1 and I_2 and the limits are given by.

$y=0 \quad y=1$ and $y=1 \quad y=2.$

$x=0 \quad x=\sqrt{y}$ and $x=0 \quad x=2-y.$

Hence we get.

$$\int_0^1 \int_{x^2}^{2-x} f(x,y)\,dy\,dx = \int_0^1 \int_0^{\sqrt{y}} f(x,y)\,dx\,dy + \int_1^2 \int_0^{2-y} f(x,y)\,dx\,dy$$

5) Evaluate $\iint_R dx\,dy$ over the region bound by $x \geq 0$, $y \geq 0$, $x+y=1$

Sol:-

Given $\iint_R dx\,dy$ where R is the region bounded by $x \geq 0, y \geq 0, x+y \leq 1$.

Hence the limits are

$$x=0 \qquad x=1-y$$
$$y=0 \qquad y=1.$$

$$\iint_R dx\,dy = \int_0^1 \int_0^{1-y} dx\,dy = \int_0^1 (x)_0^{1-y}\,dy.$$

$$= \int_0^1 (1-y)\,dy = \left(y - \frac{y^2}{2}\right)_0^1$$

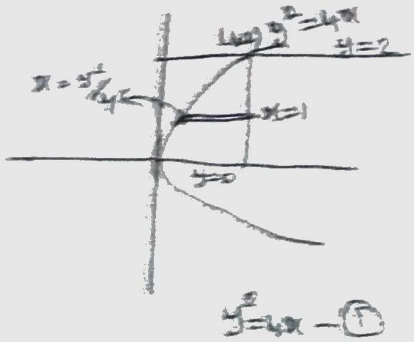
$$= 1 - \frac{1}{2} = 1/2.$$

6) Find the limits of integration in the double integral $\iint_R f(x,y)\,dx\,dy$ where R is the first quadrant and bounded by $x=1$, $y=0$ and $y^2=4x$

Sol:-

Given that $\iint_R f(x,y)\,dx\,dy$ where R is in the first quadrant and bounded by

$$x=1, y=0, y^2=4x$$



So the limits are

$$x=\frac{y^2}{4}, \quad x=1$$
$$y=0 \qquad y=2$$

$y^2=4x$ —(1)

$x=1$ —(2)

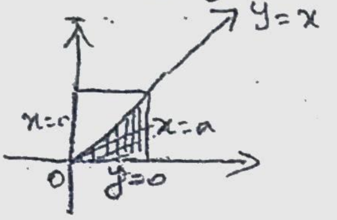
$\Rightarrow y^2=4$

$\Rightarrow y=2$

⑩ Sketch roughly the region of integration for $\int_0^a \int_0^x f(x,y)\,dy\,dx$.

Soln:-

Given $I = \int_0^a \int_0^x f(x,y)\,dy\,dx$.

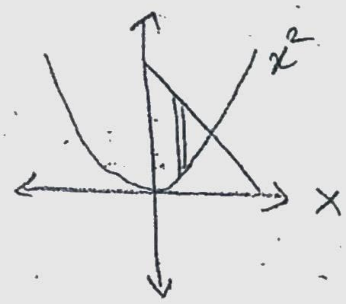


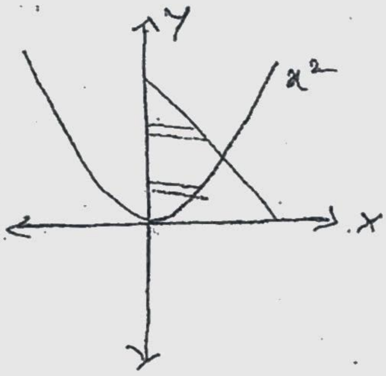
$y=0$
$y=x$
$x=0$
$x=a$

Part - B

① Change the order of Integration & hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

Soln:-





The given limits are

$x=0$ $\quad$ $x=1$
$y=x^2$ $\quad$ $y=2-x$

After changing the order of integration the region of integration can be divided into two parts namely I_1 and I_2 and the limits are given by

$y=0$ $\quad$ $y=1$ $\quad$ and $y=1$ $\quad$ $y=2$
$x=0$ $\quad$ $x=\sqrt{y}$ $\quad$ and $x=0$ $\quad$ $x=2-y$.

Hence we get,

$$\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx = \int_0^1 \int_0^{\sqrt{y}} xy\,dx\,dy + \int_1^2 \int_0^{2-y} xy\,dx\,dy$$

$$= \int_0^1 y\left(\frac{x^2}{2}\right)_0^{\sqrt{y}} dy + \int_1^2 y\left(\frac{x^2}{2}\right)_0^{2-y} dy.$$

$$= \frac{1}{2}\int_0^1 y(y)\,dy + \int_1^2 y\left(\frac{(2-y)^2}{2}\right) dy.$$

$$= \frac{1}{2}\int_0^1 y^2\,dy + \frac{1}{2}\int_1^2 (4y-4y^2+y^3)\,dy$$

$$= \frac{1}{2}\left(\frac{y^3}{3}\right)_0^1 + \frac{1}{2}\left[4\frac{y^2}{2} - 4\frac{y^3}{3} + \frac{y^4}{4}\right]_1^2$$

$$= \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left[\left(4\cdot\frac{4}{2} - 4\cdot\frac{8}{3} + \frac{16}{4}\right) - \left(\frac{4}{2} - \frac{4}{3} + \frac{1}{4}\right)\right]$$

$$= \frac{1}{6} + \frac{1}{2}\left[\left(8 - \frac{32}{3} + 4\right) - \left(2 - \frac{4}{3} + \frac{1}{4}\right)\right]$$

$$= \frac{1}{6} + \frac{1}{2}\left\{\left(\frac{24-32+12}{3}\right) - \left(\frac{24-16+3}{12}\right)\right\}$$

$$= \frac{1}{6} + \frac{1}{2}\left(\frac{4}{3} - \frac{11}{12}\right) = \frac{1}{6} + \frac{1}{2}\left(\frac{16-11}{12}\right)$$

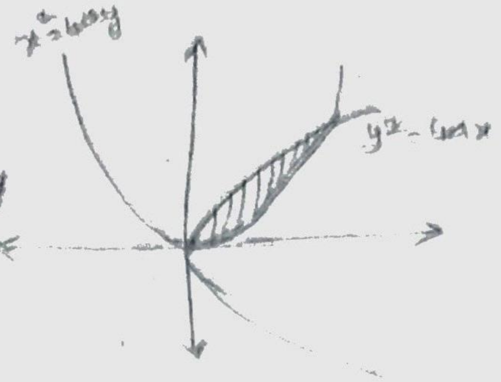
$$= \frac{1}{6} + \frac{1}{2}\left(\frac{5}{12}\right) = \frac{1}{6} + \frac{5}{24}$$

$$= \frac{4+5}{24} = \frac{9}{24} = 3/8.$$

(5)

(2) Find by double integration, the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$

Soln:



Given that $y^2 = 4ax$ and $x^2 = 4ay$

Solving these two curves we get

$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$

$$x^4 = 64a^3x$$

$$x^4 - 64a^3x = 0$$

$$x(x^3 - 64a^3) = 0$$

$$x = 0 \quad x = 4a.$$

Now the limits are

$$x = 0 \quad x = 4a$$

$$y = \frac{x^2}{4a} \quad y = 2\sqrt{ax}$$

$$\text{Area} = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$$

$$= \int_0^{4a} (y)_{x^2/4a}^{2\sqrt{ax}} dx.$$

$$= \int_0^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= \int_0^{4a} \left(2\sqrt{a}(x)^{1/2} - \frac{1}{4a} \cdot x^2\right) dx$$

$$= \left(2\sqrt{a} \cdot \frac{2}{3} x^{3/2} - \frac{1}{4a} \cdot \frac{x^3}{3}\right)_0^{4a}$$

$$= \left(2\sqrt{a}\cdot\frac{2}{3}(4a)^{3/2} - \frac{1}{4a}\cdot\frac{(4a)^3}{3}\right)$$

$$= 2\sqrt{a}\cdot\frac{2}{3}(8a\sqrt{a}) - \frac{1}{4a}\cdot\frac{64a^3}{3}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3} = \frac{16a^2}{3}$$

(3) Using triple integration, find the volume of the sphere $x^2+y^2+z^2=a^2$.

Soln:-

Volume = 8 x volume in the I octant

$$= 8\iiint dz\,dy\,dx.$$

$x^2+y^2+z^2=a^2$

$z=\sqrt{a^2-x^2-y^2}$ $\therefore$ z varies 0 to $\sqrt{a^2-x^2-y^2}$

y varies from 0 to $\sqrt{a^2-x^2}$

x varies from 0 to a.

$$V = 8\int_0^a\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} dz\,dy\,dx.$$

$$= 8\int_0^a\int_0^{\sqrt{a^2-x^2}} (z)_0^{\sqrt{a^2-x^2-y^2}}\,dy\,dx.$$

$$= 8\int_0^a\int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2}\,dy\,dx.$$

$$= 8\int_0^a\left[\frac{y}{2}\sqrt{a^2-x^2-y^2} + \frac{(a^2-x^2)}{2}\sin^{-1}\frac{y}{\sqrt{a^2-x^2}}\right]_0^{\sqrt{a^2-x^2}}$$

(5)

$$= 8\int_0^a \left[0 + \frac{a^2-x^2}{2}\sin^{-1}(1)\right] dx$$

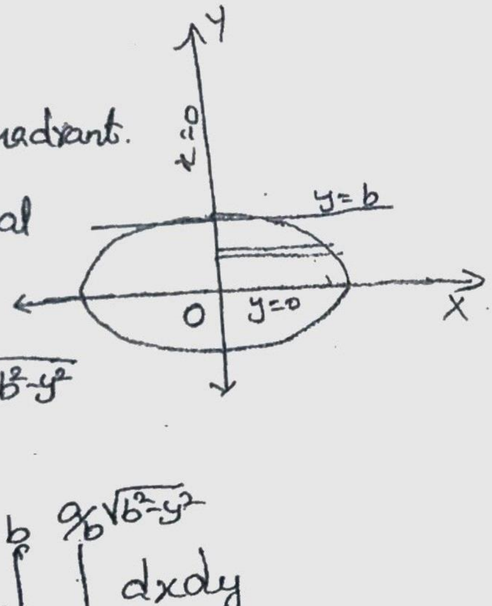
$$= \overset{2}{\cancel{8}}\cdot\frac{\pi}{\cancel{4}}\int_0^a (a^2-x^2)\,dx = 2\pi\left(a^2x - \frac{x^3}{3}\right)_0^a$$

$$= 2\pi\left(a^3 - \frac{a^3}{3}\right) = 2\pi\left(\frac{2a^3}{3}\right)$$

$$= \frac{4\pi a^3}{3}.$$

(4) Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Soln:-



Area of ellipse $= 4\times$ area of quadrant.

Divide the area into horizontal strips of width δy.

x varies from $x=0$ to $x=\frac{a}{b}\sqrt{b^2-y^2}$

y varies from $y=0$ to $y=b$.

$$\therefore \text{The required area} = 4\int_0^b\int_0^{\frac{a}{b}\sqrt{b^2-y^2}} dx\,dy$$

$$= 4\int_0^b [x]_0^{\frac{a}{b}\sqrt{b^2-y^2}}\,dy.$$

$$= 4\int_0^b \left(\frac{a}{b}\sqrt{b^2-y^2}\right)dy.$$

$$= \frac{4a}{b}\int_0^b \sqrt{b^2-y^2}\,dy.$$

$$= \frac{4a}{b}\left[\frac{b^2}{2}\sin^{-1}\frac{y}{b} + \frac{y}{2}\sqrt{b^2-y^2}\right]_0^b$$

$$= \frac{4a}{b}\left[\left(\frac{b^2}{2}\sin^{-1}(1) + 0\right) - (0+0)\right]$$

$$= \frac{4a}{b}\cdot\frac{b^2}{2}\cdot\frac{\pi}{2} = ab\pi.$$

(5) Find the volume bounded by the cylinder $x^2+y^2=4$ & the planes $y+z=4$ & $z=0$.

Solu:-

Here z varies from $z=0$ to $z=4-y$

x varies from $x=-2$ to $x=2$

y varies from $y=-\sqrt{4-x^2}$ to $y=\sqrt{4-x^2}$.

$$I = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{4-y} dz\,dy\,dx = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}[z]_0^{4-y}\,dy\,dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}(4-y-0)\,dy\,dx$$

$$= \int_{-2}^{2}\left[4y-\frac{y^2}{2}\right]_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}dx$$

$$\int_{-2}^{2}\left[\left(4\sqrt{4-x^2}-\frac{4-x^2}{2}\right)-\left(-4\sqrt{4-x^2}-\frac{4-x^2}{2}\right)\right]dx$$

$$= \int_{-2}^{2}\left[4\sqrt{4-x^2}-\cancel{\frac{1}{2}(4-x^2)}+4\sqrt{4-x^2}+\cancel{\frac{1}{2}(4-x^2)}\right]dx$$

$$= \int_{-2}^{2} 8\sqrt{4-x^2}\,dx = 8\int_{-2}^{2}\sqrt{4-x^2}\,dx = 16\int_{0}^{2}\sqrt{4-x^2}\,dx.$$

$$= 16\left[\frac{4}{2}\sin^{-1}\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2}\right]_0^2$$

$$= 16\left[\left(\frac{4}{2}\sin^{-1}(1) + 0\right) - (0-0)\right]$$

$$= 16\left(\frac{4}{2}\cdot\frac{\pi}{2}\right) = 16\pi.$$

Part B

UNIT I MATRICES

1 If $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ find the eigen values of A^4

Soln. The characteristic equ is

$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$ ------ (1)

where a_1 = Sum of main diagonal elements

$= 2 + 3 + 2 = 7$

a_2 = Sum of minors of main diagonal elements

$= \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 6 - 2 + 4 - 1 + 6 - 2 = 11$

$a_3 = |A| = 2(6-2) - 2(2-1) + 1(2-3) = 5$

Substitute $a_1 = 7,\ a_2 = 11,\ a_3 = 5$ in (1)

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -7 & 11 & -5 \\ & 0 & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & \boxed{0} \end{array}$$

$\lambda = 1$ is a root.

The other roots are given by $\lambda^2 - 6\lambda + 5 = 0$

$(\lambda - 1)(\lambda - 5) = 0$

$\lambda = 1,\ \lambda = 5.$

The eigen values of A are 1, 1, 5

The eigen values of A^4 are $1^4, 1^4, 5^4 = 1, 1, 625$

2) Find the eigen values and eigen vectors of $\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$.

Solu:-

The characteristic eqn is

$$\begin{vmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = 0.$$

$$-1-\lambda+\lambda+\lambda^2-3=0$$

$$\lambda^2-4=0$$

$$(\lambda-2)(\lambda+2)=0. \quad \therefore \lambda = 2, -2.$$

Eigen values are 2 and -2

Eigen vectors are given by

$$\begin{pmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0.$$

$$\left.\begin{aligned} (1-\lambda)x_1+3x_2=0. \\ x_1+(-1-\lambda)x_2=0 \end{aligned}\right\} \quad \text{——— (I)}$$

(i) When $\lambda=2$.

$$\left.\begin{aligned} -x_1+3x_2=0 \\ x_1-3x_2=0 \end{aligned}\right\} \Rightarrow \begin{aligned} x_1-3x_2=0 \\ x_1=3x_2. \end{aligned}$$

If $x_2=1$ then x_1 is 3

$\therefore$ Eigen vector for $\lambda=2$ is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(ii) When $\lambda=-2$

$$\left.\begin{aligned} 3x_1+3x_2=0 \\ x_1+x_2=0 \end{aligned}\right\} \Rightarrow \begin{aligned} x_1+x_2=0 \\ \therefore x_1=-x_2 \end{aligned}$$

If $x_1=1$, $x_2=-1$

$\therefore$ Eigen vector for $\lambda=-2$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

3) Verify cayley-Hamilton theorem for $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$

<u>Solu:-</u>

The characteristic equ is

$$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0 \quad \text{——} \ (1)$$

$$a_1 = 1+2+2 = 5$$

$$a_2 = \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}$$

$$= 4-0+2+0+2+0 = 8.$$

$$a_3 = |A| = 1(4-0) - 0(4-0) - 2(0-0)$$

$$= 4.$$

Sub $a_1 = 5$, $a_2 = 8$, $a_3 = 4$ in (1)

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0.$$

To verify cayley-Hamilton theorem, we have to verify $A^3 - 5A^2 + 8A - 4I = 0$ —— (2)

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 0+0+0 & -2+0-4 \\ 2+4+0 & 0+4+0 & -4+8+8 \\ 0+0+0 & 0+0+0 & 0+0+4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 0+0+0 & -2+0-12 \\ 6+8+0 & 0+8+0 & -12+16+24 \\ 0+0+0 & 0+0+0 & 0+0+8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{pmatrix}$$

$$8A = 8\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 & -16 \\ 16 & 16 & 32 \\ 0 & 0 & 16 \end{pmatrix}$$

(2) becomes

$$A^3 - 5A^2 + 8A - 4I = \begin{pmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{pmatrix} - \begin{pmatrix} 5 & 0 & -30 \\ 30 & 20 & 60 \\ 0 & 0 & 20 \end{pmatrix} + \begin{pmatrix} 8 & 0 & -16 \\ 16 & 16 & 32 \\ 0 & 0 & 16 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0.$$

Hence cayley - Hamilton theorem is verified.

4) Using cayley - Hamilton theorem find the inverse of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

Soln:

The characteristic equ is

$$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0 \quad \text{—— (1)}$$

$a_1 = 1+1+1 = 3$

$$a_2 = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$= 1-0+1-0+1-0 = 3.$

$a_3 = |A| = 1(1-0) - 2(0-0) + 3(0-0)$

$= 1.$

Sub $a_1 = 3$, $a_2 = 3$, $a_3 = 1$ in (1)

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0.$$

By cayley - Hamilton theorem, every square matrix satisfies its characteristic equ,

$\therefore A^3 - 3A^2 + 3A - I = 0$

$\therefore I = A^3 - 3A^2 + 3A$

$$A^{-1}I = A^{-1}A^3 - 3A^{-1}A^2 + 3A^{-1}A$$

$$A^{-1} = A^2 - 3A + 3I \quad \text{--- (2)}$$

$$A^2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 2+2+0 & 3+4+3 \\ 0+0+0 & 0+1+0 & 0+2+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ 0 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

5. Discuss the nature of the quadratic form $6x^2+2y^2+3z^2-4xy+8xz$

Solu:-

The matrix of quadratic form is

$$A = \begin{pmatrix} 6 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & 3 \end{pmatrix}$$

$a_{12} = a_{21} = \frac{1}{2}(-4) = -2$

$a_{23} = a_{32} = \frac{1}{2}(0) = 0$

$a_{31} = a_{13} = \frac{1}{2}(8) = 4$

$a_{11} = 6, a_{22} = 2, a_{33} = 3$

$D_1 = |6| = 6 > 0$, +ve

$D_2 = \begin{vmatrix} 6 & -2 \\ -2 & 2 \end{vmatrix} = 12 - 4 = 8 > 0$, +ve

$D_3 = \begin{vmatrix} 6 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = 6(6-0) + 2(-6-0) + 4(0-8)$

$= 36 - 12 - 32 = -8 < 0$, −ve

$\therefore$ The nature of the Q.F is indefinite

UNIT-II

Three Dimensional Analytical Geometry:

1. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point $(1,2,3)$

Soln:

Equation of the sphere passes through the given circle is

$$S+kP=0.$$

$$(x^2+y^2+z^2-9)+k(2x+3y+4z-5)=0 \quad \text{—— (1)}$$

It passes through the point $(1,2,3)$

$$\therefore (1+4+9-9)+k(2+6+12-5)=0.$$

$$5+15k=0, \quad 15k=-5$$

$$k=\frac{-5}{15}=-\frac{1}{3}.$$

Put $k=-\frac{1}{3}$ in (1)

$$\therefore (x^2+y^2+z^2-9)-\frac{1}{3}(2x+3y+4z-5)=0.$$

$$3x^2+3y^2+3z^2-27-2x-3y-4z+5=0.$$

$$\Rightarrow 3x^2+3y^2+3z^2-2x-3y-4z-22=0.$$

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2) Find the centre and radius of the circle $x^2+y^2+z^2=9$, $x+y+z=3$

Soln:

The centre of the sphere is $C(0,0,0)$ and is radius $CP=3$.

Let Q be the centre of the circle.

Then Q is the foot of the $\perp^r$ from the centre $C(0,0,0)$ to the plane $x+y+z=3$.

The direction ratio of CQ are $x-0, y-0, z-0$.

The direction ratio of the normal to the plane $x+y+z=3$ are $1,1,1$.

$$A^{-1}I = A^{-1}A^{3} - 3A^{-1}A^{2} + 3A^{-1}A$$

$$A^{-1} = A^{2} - 3A + 3I \quad \cdots (2)$$

$$A^{2} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+0 & 2+2+0 & 3+4+3 \\ 0+0+0 & 0+1+0 & 0+2+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ 0 & 0 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

→ Discuss the nature of the quadratic form $6x^2+2y^2+3z^2-4xy+8xz$.

Soln:-

The matrix of quadratic form is

$$A = \begin{pmatrix} 6 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & 3 \end{pmatrix}$$

$a_{12} = a_{21} = \frac{1}{2}(-4) = -2$

$a_{23} = a_{32} = \frac{1}{2}(0) = 0$

$a_{31} = a_{13} = \frac{1}{2}(8) = 4$

$a_{11} = 6, a_{22} = 2, a_{33} = 3$

$D_1 = |6| = 6 > 0$, +ve

$D_2 = \begin{vmatrix} 6 & -2 \\ -2 & 2 \end{vmatrix} = 12 - 4 = 8 > 0$, +ve

$D_3 = \begin{vmatrix} 6 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = 6(6-0) + 2(-6-0) + 4(0-8)$

$= 36 - 12 - 32 = -8 < 0$, −ve.

$\therefore$ The nature of the Q.F is indefinite

Since CQ is parallel to the normal to the plane, the equation of CQ is $\frac{x}{1}=\frac{y}{1}=\frac{z}{1}=k$.

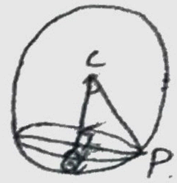
Any point on this line is (k,k,k)

If this point lies on the plane $x+y+z=3$.

We have $k+k+k=3 \Rightarrow 3k=3, \quad \therefore k=1$.

$\therefore$ The centre Q is $(1,1,1)$

$$CQ=\sqrt{(1-0)^2+(1-0)^2+(1-0)^2}=\sqrt{3}.$$



Radius of the circle $QP=\sqrt{CP^2-CQ^2}$

$$=\sqrt{3^2-(\sqrt{3})^2}=\sqrt{9-3}=\sqrt{6}$$

——x——x——

3) Find the equation of the sphere passing through the points $(0,0,0),(0,1,-1),(-1,2,0)$ and $(1,2,3)$

Soln:

Let $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ —— ① be the equation of the sphere

Put $x=0, y=0, z=0$.

$0+0+0+\cdots+d=0 \Rightarrow d=0$.

Put $x=0, y=1, z=-1$.

$0+1+1+0+2v-2w+d=0$

$\Rightarrow 2v-2w+0=-2$

$\therefore v-w=-1$ —— ②

Put $x=-1, y=2, z=0$.

$1+4+0-2u+4v+0+0=0$

$-2u+4v=-5$ —— ③

Put $x=1, y=2, z=3$.

$$1+4+9+2u+4v+6w+0=0$$

$$2u+4v+6w=-14.$$

$$u+2v+3w=-7 \quad \text{—— ④}$$

To solve ②, ③, ④

$$③\times 1 \Rightarrow -2u+4v+0=-5$$

$$④\times 2 \Rightarrow 2u+4v+6w=-14$$

Add $8v+6w=-19$ —— ⑤

$$②\times 6 \Rightarrow 6v-6w=-6$$

Add $14v=-25$ $\therefore \boxed{v=\frac{-25}{14}}$

Put $v=\frac{-25}{14}$ in ②

$$\frac{-25}{14}-w=-1$$

$$w=\frac{-25}{14}+1=\frac{-11}{14}$$

Put $v=\frac{-25}{14}$ in ③

$$-2u+\cancel{4}^{2}\left(\frac{-25}{\cancel{14}_{7}}\right)=-5$$

$$-2u-\frac{50}{7}=-5$$

$$-2u=-5+\frac{50}{7}$$

$$-2u=\frac{-35+50}{7}=\frac{15}{7}$$

$$\therefore u=\frac{-15}{14}$$

sub $u=\frac{-15}{14}$, $v=\frac{-25}{14}$, $w=\frac{-11}{14}$ and $d=0$ in ①

$$x^2+y^2+z^2+2\left(\frac{-15}{14}\right)x+2\left(\frac{-25}{14}\right)y+2\left(\frac{-11}{14}\right)z+0=0$$

$$x^2+y^2+z^2-\frac{15}{7}x-\frac{25}{7}y-\frac{11}{7}z=0.$$

$$7(x^2+y^2+z^2)-15x-25y-11z=0.$$

—— x —— x ——

4) Find the equation of the cone with vertex at the origin and which passes through the curves $x^2+y^2+z^2-x-1=0$, $x^2+y^2+z^2+y-2=0$.

Soln:-

The equation of the guiding curves are

$$x^2+y^2+z^2-x-1=0 \quad \text{---} (1)$$

$$\& \quad x^2+y^2+z^2+y-2=0 \quad \text{---} (2)$$

$(1)-(2) \Rightarrow \quad -x-y+1=0 \Rightarrow x+y=1 \quad \text{---} (3)$

Equation (1) & (3) represent the same curve as (1) & (2)

$\therefore$ making (1) as homegeneons using (3)

$$x^2+y^2+z^2-x(1)-(1)^2=0 \qquad \boxed{\text{Put } x+y=1}$$

$$x^2+y^2+z^2-x(x+y)-(x+y)^2=0.$$

$$\cancel{x^2}+\cancel{y^2}+z^2-\cancel{x^2}-xy-x^2-\cancel{y^2}-2xy=0$$

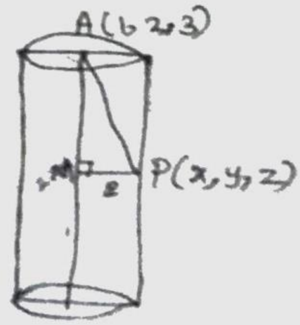
$$-x^2-3xy+z^2=0.$$

$x^2+3xy-z^2=0$ which is the equation of the cone.

---x---x---

5) Find the equation of right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines Proportional to 2,-3,6.

Soln:-



Here point A is (1,2,3)

The direction cosines are

$$\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$$

$$\boxed{\sqrt{2^2+(-3)^2+(6)^2}=\sqrt{4+9+36} =\sqrt{49}=7}$$

Let P(x,y,z) be any point on the cylinder.

From the right-angled triangle AMP,

$$AP^2-AM^2=PM^2$$

AM is the projection of AP on the axis

$$[(x-1)^2+(y-2)^2+(z-3)^2]$$

[illegible]

Simplifying this we get,

[illegible]

UNIT III DIFFERENTIAL CALCULUS

1) Find the radius of curvature of the parabola $x = at^2$, $y = 2at$ at t.

Soln:

$$\frac{dx}{dt} = 2at, \quad \frac{dy}{dt} = 2a$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{t}\right) = \frac{d}{dt}\left(\frac{1}{t}\right)\cdot\frac{dt}{dx}$$

$$= \left(\frac{-1}{t^2}\right)\left(\frac{1}{2at}\right) = \frac{-1}{2at^3}$$

Radius of curvature is

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\left(1+\frac{1}{t^2}\right)^{3/2}}{\frac{-1}{2at^3}} = \left(\frac{t^2+1}{t^2}\right)^{3/2}(-2at^3)$$

$$= \frac{(1+t^2)^{3/2}}{t^3}(-2at^3) = -2a(1+t^2)^{3/2}$$

$$|\rho| = 2a(1+t^2)^{3/2} = 2a(1+t^2)\sqrt{1+t^2}$$

2) Find the radius of curvature for the curve $xy = 30$ at $(3, 10)$

Soln:

$xy = 30$

Diff with respect to x

$$x\frac{dy}{dx} + y(1) = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

At $(3, 10)$

$$y_1 = \frac{dy}{dx} = \frac{-10}{3}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right]$$

4) Find the equation of the cone with vertex at the origin and which passes through the curves $x^2+y^2+z^2-x-1=0,\ x^2+y^2+z^2+y-2=0$.

Soln:-

The equation of the guiding curves are

$$x^2+y^2+z^2-x-1=0 \quad \text{——} (1)$$

$$\& \quad x^2+y^2+z^2+y-2=0 \quad \text{——} (2)$$

$$(1)-(2) \Rightarrow \quad -x-y+1=0 \Rightarrow x+y=1 \quad \text{——} (3)$$

Equation (1) & (3) represent the same curve as (1) & (2)

$\therefore$ making (1) as homegeneous using (3)

$\boxed{\text{Put } x+y=1}$

$$x^2+y^2+z^2-x(1)-(1)^2=0$$

$$x^2+y^2+z^2-x(x+y)-(x+y)^2=0.$$

$$\cancel{x^2}+\cancel{y^2}+z^2-\cancel{x^2}-xy-x^2-\cancel{y^2}-2xy=0$$

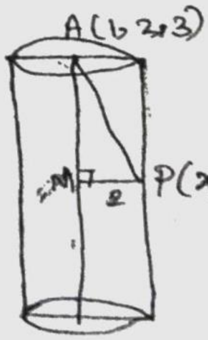
$$-x^2-3xy+z^2=0.$$

$$x^2+3xy-z^2=0 \text{ which is the equation of the cone.}$$

———x———x———

5) Find the equation of right circular cylinder of radius 2 whose axis passes through (1,2,3) and has direction cosines proportional to 2,−3,6.

Soln:-



Here point A is (1,2,3)

The direction cosines are

$$\frac{2}{7},\ \frac{-3}{7},\ \frac{6}{7}$$

$$\sqrt{2^2+(-3)^2+(6)^2}=\sqrt{4+9+36}$$
$$=\sqrt{49}=7$$

Let P(x,y,z) be any point on the cylinder.

From the right-angled triangle AMP,

AM is the projection of AP on the axis

$$AP^2-AM^2=PM^2$$

$$\left[(x-1)^2+(y-2)^2+(z-3)^2\right]$$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right]$$

At $(3, 10)$

$$-\left[\frac{3\left(\frac{-10}{3}\right) - 10}{3^2}\right]$$

$$y_2 = \frac{20}{9}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\left[1+\left(\frac{-10}{3}\right)^2\right]^{3/2}}{\frac{20}{9}} = \left(1+\frac{100}{9}\right)^{3/2} \frac{9}{20}$$

$$= \left(\frac{109}{9}\right)^{3/2} \cdot \left(\frac{9}{20}\right) = \frac{109\sqrt{109}}{9\sqrt{9}} \cdot \frac{9}{20} = \frac{109\sqrt{109}}{60}.$$

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3) Find the radius of curvature of the curve $x = a\cos\theta$, $y = a\sin\theta$ at $\theta = \frac{\pi}{4}$.

Soln:-

$$\frac{dx}{d\theta} = -a\sin\theta, \quad \frac{dy}{d\theta} = a\cos\theta$$

$$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-\cot\theta) = \frac{d}{d\theta}(-\cot\theta)\frac{d\theta}{dx}$$

$$= \operatorname{cosec}^2\theta\left(\frac{1}{-a\sin\theta}\right) = \frac{-1}{a\sin^3\theta}$$

At $\theta = \frac{\pi}{4}$,

$$y_1 = -\cot\left(\frac{\pi}{4}\right) = -1$$

$$y_2 = \frac{-1}{a\sin^3(\pi/4)} = \frac{-1}{a\left(\frac{1}{\sqrt{2}}\right)^3} = \frac{-1}{a\left(\frac{1}{2\sqrt{2}}\right)} = -\frac{2\sqrt{2}}{a}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\left[1+(-1)^2\right]^{3/2}}{-\frac{2\sqrt{2}}{a}} = \frac{(2)^{3/2}}{\left(-\frac{2\sqrt{2}}{a}\right)} = 2\sqrt{2} \cdot \frac{-a}{2\sqrt{2}}$$

$$\boxed{\rho = -a}$$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right]$$

At (3, 10)

$$= -\left[\frac{\cancel{3}\left(\frac{-10}{\cancel{3}}\right) - 10}{+3^2}\right]$$

$$y_2 = \frac{20}{9}$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\left[1+\left(\frac{-10}{3}\right)^2\right]^{3/2}}{\frac{20}{9}} = \left(1+\frac{100}{9}\right)^{3/2} \cdot \frac{9}{20}.$$

$$= \left(\frac{109}{9}\right)^{3/2} \cdot \left(\frac{9}{20}\right) = \frac{109\sqrt{109}}{\cancel{9}\sqrt{9}} \cdot \frac{\cancel{9}}{20} = \frac{109\sqrt{109}}{60}.$$

---

3) Find the radius of curvature of the curve $x = a\cos\theta, y = a\sin\theta$ at $\theta = \pi/4$.

Solu:-

$$\frac{dx}{d\theta} = -a\sin\theta, \quad \frac{dy}{d\theta} = a\cos\theta$$

$$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{-a\sin\theta} = -\cot\theta$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-\cot\theta) = \frac{d}{d\theta}(-\cot\theta)\frac{d\theta}{dx}$$

$$= \operatorname{cosec}^2\theta\left(\frac{1}{-a\sin\theta}\right) = \frac{-1}{a\sin^3\theta}.$$

At $\theta = \pi/4$,

$$y_1 = -\cot(\pi/4) = -1.$$

$$y_2 = \frac{-1}{a\sin^3(\pi/4)} = \frac{-1}{a\left(\frac{1}{\sqrt{2}}\right)^3} = \frac{-1}{a\left(\frac{1}{2\sqrt{2}}\right)} = -\frac{2\sqrt{2}}{a}.$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{[1+(-1)^2]^{3/2}}{-\frac{2\sqrt{2}}{a}} = \frac{(2)^{3/2}}{\left(-\frac{2\sqrt{2}}{a}\right)} = 2\sqrt{2} \cdot \frac{-a}{2\sqrt{2}}$$

$$\boxed{\rho = -a}$$

---

4) Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2+b^2}$ where m is the parameter.

Solu:

Given $y = mx + \sqrt{a^2m^2+b^2}$

$$y - mx = \sqrt{a^2m^2+b^2}.$$

squaring on both sides

$$(y-mx)^2 = a^2m^2+b^2.$$

$$y^2 - 2mxy + m^2x^2 - a^2m^2 - b^2 = 0.$$

$$(x^2-a^2)m^2 - 2xym + (y^2-b^2) = 0.$$

This is a quadratic equation in m.

Here $A = x^2-a^2$, $B = -2xy$, $C = y^2-b^2$.

$\therefore$ Envelope is $B^2 = 4AC$.

$$(-2xy)^2 = 4(x^2-a^2)(y^2-b^2)$$

$$4x^2y^2 = 4[x^2y^2 - b^2x^2 - a^2y^2 + a^2b^2] \quad [\div \text{ by } 4]$$

$$\cancel{x^2y^2} = \cancel{x^2y^2} - b^2x^2 - a^2y^2 + a^2b^2$$

$$b^2x^2 + a^2y^2 = a^2b^2 \quad [\div \text{ by } a^2b^2]$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

5) Find the centre of curvature of the curve $y = 3x^3 + 2x^2 - 3$ at $(0, -3)$.

Solu:

$$\frac{dy}{dx} = y_1 = 9x^2 + 4x, \quad \frac{d^2y}{dx^2} = y_2 = 18x + 4.$$

At $(0,-3)$.

$$y_1 = 9(0)^2 + 4(0) = 0.$$

$$y_2 = 18(0) + 4 = 4.$$

The centre of curvature is $(\bar{x}, \bar{y})$

where $\bar{x} = x - \frac{y_1}{y_2}(1 + y_1^2)$

$$= 0 - \frac{0}{4}(1+0) = 0.$$

$\therefore 1 + \frac{1}{4}(1+1+0)$ [illegible]

$\therefore \frac{1}{4} = \frac{1+11}{4} = \frac{11}{4}$ [illegible]

The centre of curvature is $(11, \frac{11}{4})$ [illegible]

UNIT IV

Functions of several variables

1. If $u = \log(x^2+y^2+z^2)$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2+y^2+z^2}$.

Soln.

$$\frac{\partial u}{\partial x} = \frac{1}{(x^2+y^2+z^2)}(2x+0+0) = \frac{2x}{x^2+y^2+z^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2+z^2)(2) - 2x(2x+0+0+0)}{(x^2+y^2+z^2)^2}$$

$$= \frac{-2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^2}$$

Illy

$$\frac{\partial^2 u}{\partial y^2} = \frac{2x^2-2y^2+2z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{2x^2+2y^2-2z^2}{(x^2+y^2+z^2)^2}$$

$$\therefore \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{-2x^2+2y^2+2z^2+2x^2-2y^2+2z^2+2x^2+2y^2-2z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^2} = \frac{2\cancel{(x^2+y^2+z^2)}}{(x^2+y^2+z^2)^{\cancel{2}}}$$

$$\therefore \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2+y^2+z^2}$$

2) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

Solu:

Given $u = \sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$

$\therefore \sin u = \frac{x^2+y^2}{x-y}$

Let $f(x,y) = \sin u = \frac{x^2+y^2}{x-y}$

Put $x = tx,\ y = ty$.

$$\therefore f(tx, ty) = \frac{(tx)^2+(ty)^2}{tx - ty} = \frac{t^2(x^2+y^2)}{t(x-y)}$$

$$= t\left(\frac{x^2+y^2}{x-y}\right) = t^1 f(x,y)$$

$\therefore f(x,y)$ is a homogeneons function of degree 1.

$\therefore$ By Euler's theorem of f

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = 1.\sin u.$$

$$x.\cos u.\frac{\partial u}{\partial x} + y.\cos u\frac{\partial u}{\partial y} = \sin u$$

$$\cos u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = \sin u$$

$$\therefore x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin u}{\cos u} = \tan u.$$

---

3. Expand e^{xy} at $(1,1)$ upto second degree terms.

Solu:

Here $a=1, b=1$.

$f(x,y) = e^{xy}$

$\therefore f(1,1) = e^1 = e$

$f_x(x,y) = ye^{xy}$

$f_x(1,1) = 1e^1 = e$

$f_y(x,y) = xe^{xy}$

$f_y(1,1) = 1e^1 = e$

$f_{xx}(x,y) = y^2 e^{xy}$ $\therefore f_{xx}(1,1) = 1e^1 = e$

$f_{xy}(x,y) = xye^{xy}$ $\therefore f_{xy}(1,1) = 1e^1 = e.$

$f_{yy}(x,y) = x^2 e^{xy}$ $\therefore f_{yy}(1,1) = 1e^1 = e.$

Taylor series of $f(x,y)$ at (a,b) is

$$f(x,y) = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b)$$
$$+ \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right]$$
$$+ \cdots$$

$$\therefore f(x,y) = e + (x-1)e + (y-1)e + \frac{1}{2!}\left[(x-1)^2 e + 2(x-1)(y-1)e + (y-1)^2 e\right]$$
$$+ \cdots$$

$$= e\left[1 + (x-1) + (y-1) + \frac{1}{2}\left\{(x-1)^2 + 2(x-1)(y-1) + (y-1)^2\right\} + \cdots\right]$$

x ——— x

4) Find the maximum or minimum values of $3x^2 - y^2 + x^3$.

Solu:- Let $f(x,y) = 3x^2 - y^2 + x^3$.

$p = \frac{\partial f}{\partial x} = 6x + 3x^2$, $q = \frac{\partial f}{\partial y} = -2y$, $r = \frac{\partial^2 f}{\partial x^2} = 6 + 6x$.

$s = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(-2y) = 0$, $t = \frac{\partial^2 f}{\partial y^2} = -2$.

Put $p = 0, q = 0$:

$6x + 3x^2 = 0$ $\quad -2y = 0.$

$3x(2+x) = 0.$ $\quad y = 0.$

$x = 0, x = -2$

$\therefore$ The points are $(0,0)$ and $(-2,0)$

At $(0,0)$

$rt - s^2 = (6+0)(-2) - 0^2 = -12 < 0$

$\therefore f$ has neither maximum nor minimum at $(0,0)$.

At $(-2,0)$

$rt - s^2 = [6+6(-2)](-2) - 0^2 = (-6)(-2) = 12 > 0.$

Also $r = -6 < 0$.

$\therefore f$ has maximum value at $(-2,0)$.

Maximum Value,

put $x=-2,\ y=0$ in $f(x,y) = 3x^2 - y^2 + x^3$

$= 3(-2)^2 - (0)^2 + (-2)^3$

$= 12 - 8 = 4.$

———x———x———

Q) If $y_1 = 1-x_1$, $y_2 = x_1(1-x_2)$, $y_3 = x_1x_2(1-x_3)$, find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3.

Soln:

$\frac{\partial y_1}{\partial x_1} = -1,\quad \frac{\partial y_1}{\partial x_2} = 0,\quad \frac{\partial y_1}{\partial x_3} = 0.$

$\frac{\partial y_2}{\partial x_1} = 1-x_2,\quad \frac{\partial y_2}{\partial x_2} = x_1(-1) = -x_1,\quad \frac{\partial y_2}{\partial x_3} = 0.$

$\frac{\partial y_3}{\partial x_1} = x_2(1-x_3),\quad \frac{\partial y_3}{\partial x_2} = x_1(1-x_3),\quad \frac{\partial y_3}{\partial x_3} = x_1x_2(-1) = -x_1x_2.$

$$\therefore \frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)} = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} \end{vmatrix}$$

$$= -1\left[x_1^2 x_2 - 0\right] - 0(\quad) + 0(\quad)$$

$$= -x_1^2 x_2.$$

UNIT-V

MULTIPLE INTEGRALS.

* Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2}$.

Solu:-

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\,dx}{1+x^2+y^2} = \int_0^1 \left[\int_0^{\sqrt{1+x^2}} \frac{dy}{(\sqrt{1+x^2})^2+y^2}\right] dx$$

$$\boxed{\int \frac{dx}{a^2+x^2} = \frac{1}{a}}$$

$$= \int_0^1 \left[\frac{1}{\sqrt{(1+x^2)}} \tan^{-1}\left(\frac{y}{\sqrt{1+x^2}}\right)\right]_0^{\sqrt{1+x^2}} dx$$

$$= \int_0^1 \left[\frac{1}{\sqrt{1+x^2}}\left(\tan^{-1}\left(\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}}\right) - \tan^{-1} 0\right)\right] dx.$$

$$= \int_0^1 \frac{1}{\sqrt{1+x^2}}\left[\tan^{-1}(1) - 0\right] dx.$$

$$= \int_0^1 \frac{1}{\sqrt{1+x^2}}\left(\frac{\pi}{4}\right) dx.$$

$$= \pi/4. \int_0^1 \frac{dx}{\sqrt{1+x^2}}$$

$$= \pi/4\left[\log(x+\sqrt{1+x^2})\right]_0^1$$

$$= \pi/4\left[\log(1+\sqrt{1+1}) - \log(0+\sqrt{1+0})\right]$$

$$= \pi/4\left[\log(1+\sqrt{2}) - \log 1\right] \qquad [\because \log 1 = 0]$$

$$= \pi/4 \log(1+\sqrt{2}).$$

——— α ——— α ———

2) Evaluate $\int_0^2 \int_x^{x^2} e^{\frac{y}{x}}\, dy\, dx$.

Solu:-

$$\int_0^2 \int_x^{x^2} e^{\frac{y}{x}}\, dy\, dx = \int_0^2 \left[\int_x^{x^2} e^{\frac{y}{x}}\, dy\right] dx.$$

$\int u\,dv$

$\frac{1}{x} = a$

$\int e^{ax} = \frac{e^{ax}}{a}$

$$= \int_0^2 \left[\frac{e^{y/x}}{1/x}\right]_x^{x^2} dx.$$

$\boxed{\int u\,dv = uv - u'v_1 \qquad \int x e^x dx = x e^x - e^x}$

$$= \int_0^2 \left[\frac{e^{\frac{x^2}{x}} - e^{x/x}}{1/x}\right] dx$$

$\int uv = u\int v - \int u' \int v$

$$= \int_0^2 (e^x - e^1)(x)\, dx$$

$u = x$

$v = e^x$

$$= \int_0^2 (x e^x - e^1 x)\, dx = \left[x e^x - e^x - e\frac{x^2}{2}\right]_0^2$$

$$= \left[2e^2 - e^2 - e\frac{(2)^2}{2}\right] - \left[0e^0 - e^0 - e\frac{(0)^2}{2}\right]$$

$$= [2e^2 - e^2 - 2e] - [0 - 1 - 0]$$

$$= e^2 - 2e + 1 = (1-e)^2.$$

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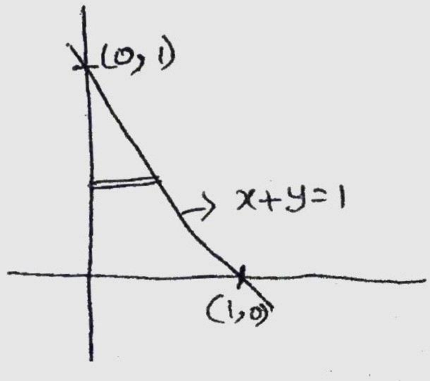
③ Evaluate $\iint_R (x^2+y^2)\,dx\,dy$, where R is the region in the positive quadrant for which $x+y \leq 1$.

Solu:



Let $x+y=1 \Rightarrow x = 1-y$.

$\therefore$ x changes from 0 to $1-y$

y changes from 0 to 1.

$$\therefore \iint_R (x^2+y^2)\,dx\,dy = \int_0^1 \left[\int_0^{1-y} (x^2+y^2)\,dx\right] dy.$$

$$= \int_0^1 \left(\frac{x^3}{3} + xy^2\right)_0^{1-y} dy$$

$$= \int_0^1 \left[\frac{(1-y)^3}{3} + (1-y)y^2\right] dy$$

$$= \int_0^1 \left[\frac{1}{3}(1-y)^3 + y^2 - y^3\right] dy$$

$$= \left[\frac{1}{3}\frac{(1-y)^4}{4(-1)} + \frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$

$$= \left[0 + \frac{1}{3} - \frac{1}{4}\right] - \left[\frac{1}{3}\left(\frac{-1}{4}\right) + 0 - 0\right]$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{12} = \frac{4-3+1}{12} = \frac{2}{12} = \frac{1}{6}$$

4) Evaluate $\int_0^a \int_0^b \int_0^c (x+y+z)\,dz\,dy\,dx$

Soln:

Let $\int_0^a \int_0^b \int_0^c (x+y+z)\,dz\,dy\,dx = \int_0^a \int_0^b \left(xz + yz + \frac{z^2}{2}\right)_0^c dy\,dx$

$$= \int_0^a \int_0^b \left(xc + yc + \frac{c^2}{2}\right) dy\,dx$$

$$= \int_0^a \left(cxy + \frac{cy^2}{2} + \frac{c^2}{2}y\right)_0^b dx$$

$$= \int_0^a \left(bcx + \frac{b^2c}{2} + \frac{bc^2}{2}\right) dx$$

$$= \left(bc\frac{x^2}{2} + \frac{b^2c}{2}x + \frac{bc^2}{2}x\right)_0^a$$

$$= \left(\frac{a^2bc}{2} + \frac{ab^2c}{2} + \frac{abc^2}{2}\right) = \frac{abc}{2}(a+b+c)$$

⑤ evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dz\,dy\,dx$

Solu:-

$$\cancel{\text{Let} \int_0^1 \int_0^{1-x} \int_0^{x+y} (e^z)_0^{x+y} \, dy\,dx}$$

$$\text{Let} \int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dz\,dy\,dx = \int_0^1 \int_0^{1-x} (e^z)_0^{x+y} \, dy\,dx.$$

$$= \int_0^1 \int_0^{1-x} (e^{x+y} - e^0) \, dy\,dx$$

$$= \int_0^1 \int_0^{1-x} (e^{x+y} - 1) \, dy\,dx.$$

$$= \int_0^1 (e^{x+y} - y)_0^{1-x} \, dx.$$

$$= \int_0^1 \left[\left\{ e^{x+1-x} - (1-x) \right\} - \left\{ e^{x+0} - 0 \right\} \right] dx.$$

$$= \int_0^1 (e - 1 + x - e^x) \, dx$$

$$= \left[e(x) - x + \frac{x^2}{2} - e^x \right]_0^1$$

$$= \left[e(1) - 1 + \frac{1}{2} - e^1 \right] - \left[e(0) - 0 + 0 - e^0 \right]$$

$$= \left[\cancel{e} - 1 + \frac{1}{2} - \cancel{e} \right] - (0 - 0 + 0 - 1)$$

$$= -\frac{1}{2} + 1 = \frac{1}{2}.$$

——x——x——

Bharath University

Mathematics - I ( )

Question Bank (Batch 2019)

Unit - I (PART - A) (2 marks)

1. State any two properties of eigen values.

Ans: (i) If $\lambda_1, \lambda_2, \dots \lambda_n$ are the eigen values of A, then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots \frac{1}{\lambda_n}$ are the eigen values of A^{-1}.

(ii) If $\lambda_1, \lambda_2, \dots \lambda_n$ are the eigen values of A, then $\lambda_1^m, \lambda_2^m, \dots \lambda_n^m$ are the eigen values of A^m.

2. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

Sol:- The characteristic equation of A is $|A - \lambda I| = 0$

(i.e.) $\begin{vmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(3-\lambda) - 8 = 0$

$\Rightarrow \lambda^2 - 4\lambda - 5 = 0$

$\Rightarrow \lambda = -1, \lambda = 5$

$\therefore$ The eigen values are $\lambda = -1, 5$

3. Find the eigen values of $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

Sol:- The eigen values are 2, 2, 2. ($\because$ A is upper triangular).

4. State Cauchy'-Hamilton Theorem.

Ans: Every square matrix satisfies its own characteristic equation.

5. Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

Sol:- The matrix of the given quadratic form is

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$D_1 = |2| = +ve$

$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5 = +ve$

$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2(6) - 1(2)$

$= 12 - 2$

$\therefore D_1, D_2, D_3$ are +ve.

$\therefore$ The nature of the quadratic form is positive definite.

6. Find the nature of the quadratic form $2x_1^2 + 2x_1x_2 + 3x_2^2$.

Sol:- The matrix form of the given quadratic is

$$\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$$

$D_1 = |2| = +ve$

$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6-1 = 5 = +ve$

$\therefore D_1, D_2$ are positive.

$\therefore$ The nature of the quadratic is positive Definite.

7. Write the matrix of the quadratic form $2x^2+y^2+z^2+2xy-4zy-6zx$.

Sol:-

$$\begin{pmatrix} 2 & 1 & -3 \\ 1 & 1 & -2 \\ -3 & -2 & 1 \end{pmatrix}$$

8. Write the quadratic form corresponding to the following symmetric matrix $\begin{pmatrix} 2 & 2 & 5 \\ 2 & 3 & -1 \\ 5 & -1 & 8 \end{pmatrix}$.

Sol

$2x^2 + 3y^2 + 8z^2 + 4xy + 10xz - 2yz$.

9. Write the quadratic form corresponding to the following matrix

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Ans: $x_1^2 + x_2^2$

10. Find the sum and product of the eigen values of $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

Ans:- Eigen values are $3, 2, 5$

Sum of Eigen values $= 3+2+5 = 10$

Product of Eigen values $= 3\times2\times5 = 30$.

Unit - II

1. Write the formula for radius of curvature.

Ans: Radius of curvature $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$

2. Define Evolute.

Ans The locus of centre of curvature is called an Evolute.

3. If the value of $\frac{dy}{dx}$ becomes infinite, what is the formula for radius of curvature.

Ans:
$$\rho = \frac{\left[1+\left(\frac{dx}{dy}\right)^2\right]^{3/2}}{d^2x/dy^2}$$

4. Write the equation of circle of curvature.

Ans. $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

5. Define Envelope.

Ans: A curve touches all the curves of a family is called the envelope of the family.

6. Evaluate $\int_0^1 \int_y^1 dx\,dy$.

Sol: $\int_0^1 \int_y^1 dx\,dy = \int_0^1 \left[\int_y^1 dx\right] dy = \int_0^1 [x]_y^1\, dy = \int_0^1 (1-y)\,dy$

$= \left[y - \frac{y^2}{2}\right]_0^1 = (1-\frac{1}{2}) = \frac{1}{2}$

7. Evaluate $\int_1^2 \int_0^{4-x} dy\,dx$

Sol: $\int_1^2 \left[\int_0^{4-x} dy\right] dx = \int_1^2 [y]_0^{4-x}\, dx = \int_1^2 (4-x)\,dx = \left[4x - \frac{x^2}{2}\right]_1^2$

$= \left[\left(4(2) - \frac{2^2}{2}\right) - \left(4(1) - \frac{1}{2}\right)\right]$

$= \left[(8-2) - (4-\frac{1}{2})\right] = \frac{5}{2}$

8. Evaluate $\int_0^1 \int_0^2 xy^2\,dy\,dx$

Sol: $\int_0^1 \left[\int_0^2 xy^2\,dy\right] dx = \int_0^1 x\left[\frac{y^3}{3}\right]_0^2 dy = \frac{8}{3}\int_0^1 x\,dx.$

$= \frac{8}{3}\left[\frac{x^2}{2}\right]_0^1 = \frac{8}{3}\left(\frac{1}{2}\right) = \frac{8}{6}.$

9. Evaluate $\int_0^1 \int_0^x dy\,dx$;

Sol: $\int_0^1 \left[\int_0^x dy\right] dx = \int_0^1 [y]_0^x\, dx = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1$

$= \frac{1}{2}$

10. Write down the formula for centre of curvature.

Ans: Centre of curvature C is $(\bar{x}, \bar{y})$.

where $\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$, $\bar{y} = y + \frac{1}{y_2}(1+y_1^2)$

Unit - III :-

① Solve $(D^2 - 7D + 12)y = 0$

Sol:- The Auxiliary equation is $m^2 - 7m + 12 = 0$

$(m-4)(m-3) = 0$

$\therefore m = 4, 3$

12
-4 -3
-7

The roots are different.

$C.F = c_1 e^{4x} + c_2 e^{3x}$

$\therefore$ The General solution is $y = c_1 e^{4x} + c_2 e^{3x}$.

② Find the particular integral of $(D^2 + D + 1)y = \cancel{\sin} e^{2x}$.

Sol $P.I = \frac{1}{(D^2+D+1)} \cancel{\sin} e^{2x}$

$= \frac{1}{4+2+1} e^{2x} = \frac{e^{2x}}{7}$.

③ Write down the formula for wronskian of y_1 & y_2.

Ans: $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

4) Solve $(D^2 + 9)y = 0$

Sol:- The A.E is $m^2 + 9 = 0$ $\Rightarrow m^2 = -9$

$\Rightarrow m = \pm 3i$

The roots are complex.

$\therefore C.F = c_1 \cos 3x + c_2 \sin 3x$.

$\therefore$ The General solution is $y = c_1 \cos 3x + c_2 \sin 3x$.

5) Solve $(D^2 - 10D + 25)y = 0$

Sol:- The A.E is $m^2 - 10m + 25 = 0$

$(m-5)(m-5) = 0$

$m = 5, 5$

25
-5 -5
-10

$\therefore$ The roots equal.

$\therefore C.F = (c_1 + c_2 x)e^{5x}$

6) Find the particular integral of $(D^2+4D+4)y = e^{-2x}$

Sol:- P.I $= \frac{1}{(D^2+4D+4)} e^{-2x} = \frac{e^{-2x}}{4+4(-2)+4} = \frac{e^{-2x}}{0}$

$\therefore = \frac{x e^{-2x}}{2D+4} = \frac{x e^{-2x}}{2(-2)+4} = \frac{x e^{-2x}}{0} = \frac{x^2}{2} e^{-2x}$

7) Solve $(D^2+4D+13)y = 0$

Sol:- P.I $= \frac{1}{(D^2+4D+13)}$ The A.E is $m^2+4m+13=0$

$A=1,\ B=4,\ C=13.$

$m = \frac{-B \pm \sqrt{B^2-4AC}}{2A} = \frac{-4 \pm \sqrt{16-52}}{2}$

$= \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i.$

The roots are complex.

$\therefore$ C.F $= e^{-2x}(c_1 \cos 3x + c_2 \sin 3x).$

8) Find the complementary function of $(D^2-3D+2)y = e^x$

Sol:- The A.E is $m^2-3m+2=0$

$(m-1)(m-2)=0$

$m=1,2$

$\therefore$ The roots are different.

$\therefore$ C.F $= c_1 e^x + c_2 e^{2x}.$

9) Find the particular integral of $(D^2+1)y = \sin x$.

Sol:- P.I $= \frac{1}{(D^2+1)} \sin x = \frac{\sin x}{-1+1} = \frac{\sin x}{0}$

$= \frac{x \sin x}{2D} = \frac{x}{2} \int \sin x \, dx$

$\boxed{P.I = \frac{x}{2} \cos x.}$

10) Solve $\frac{d^2y}{dx^2} - 25y = 0$

Sol $(D^2-25)y=0$

The Auxiliary equation is $m^2-25=0$

$m = \pm 5$

$\therefore$ The roots are distinct.

Unit - IV

1. Define Analytic function.

Ans: A complex function $f(z)$ is said to be analytic at a point z_0, if $f(z)$ is differentiable at z_0 and at every point of some neighbourhood of z_0.

2. Write down the C-R equation.

Ans: C-R eqn is

$$u_x = v_y \quad \& \quad u_y = -v_x$$

3. Define singular point.

Ans: If a function 'f' fails to be analytic at a point z_0, but is analytic at some point in every neighbourhood of z_0, then z_0 is a singular point of 'f'.

4. State Cauchy's integral formula.

Ans: Let $f(z)$ be an analytic function inside and on a simple closed contour C, taken in the positive sense. If a is any point interior to C, then $f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} dz$.

5. Define Harmonic function.

Ans: A real function ϕ of two variables x & y is said to be harmonic in a domain D if it has continuous second order partial derivatives and satisfies the Laplace equation

i.e. $$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \quad (\text{or}) \quad \varphi_{xx} + \varphi_{yy} = 0$$

6. Show that $u = 2x - x^3 + 3xy^2$ is harmonic.

Sol:

$u_x = 2 - 3x^2 + 3y^2$ | $u_y = 6xy$

$u_{xx} = -6x$ | $u_{yy} = 6x$

$$\therefore u_{xx} + u_{yy} = -6x + 6x$$

$$\therefore u_{xx} + u_{yy} = 0$$

$u+iv = x - iy$

$u = x, \quad v = -y$

$u_x = 1 \quad v_x = 0$ $\qquad u_x \neq v_y \;\&\; u_y \neq -v_x$

$u_y = 0 \quad v_y = -1$

$f(z) = \bar{z}$ is not analytic

8. What is the necessary condition for the function $f(z)$ to be analytic.

<u>Ans</u>: $u_x = v_y \;\&\; u_y = -v_x$

9. Write down the different types of singularities.

<u>Ans</u>: (i) pole (ii) Removable singularity (iii) essential singularity

10. Define simple pole.

Pole of order one is called simple pole.

Unit – I

1) Define solenoidal vector.

<u>Ans</u>: If $\nabla \cdot \vec{F} = 0$, then $\vec{F}$ is called solenoidal vector.

2) Prove that the vector $\vec{F} = 3y^4 z^2 \vec{i} + 4x^3 z^2 \vec{j} - 3x^2 y^2 \vec{k}$ is solenoidal.

<u>Sol</u>: Given $\vec{F} = 3y^4 z^2 \vec{i} + 4x^3 z^2 \vec{j} - 3x^2 y^2 \vec{k}$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(3y^4 z^2) + \frac{\partial}{\partial y}(4x^3 z^2) + \frac{\partial}{\partial z}(-3x^2 y^2)$$

$$= 0 + 0 + 0 = 0$$

$\therefore \nabla \cdot \vec{F} = 0$, $\vec{F}$ is solenoidal.

3. Define Irrotational vector.

<u>Ans</u>: If $\nabla \times \vec{F} = 0$, then $\vec{F}$ is called irrotational vector

4. If $\phi = x + xy^2 + yz^2$, find grad ϕ.

<u>Sol</u>: grad $\phi = \nabla \phi = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}$

$$\nabla \phi = \vec{i}(1+y^2) + \vec{j}(2xy + z^2) + \vec{k}(2yz)$$

5. If $\vec{F} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$, then find $\nabla\cdot\vec{F}$.

Sol:- $\nabla\cdot\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

$\nabla\cdot\vec{F} = 2y + z^2 + x$

6. If $\vec{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$, then find $\operatorname{div}\vec{F}$.

Sol:- $\operatorname{div}\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

$\operatorname{div}\vec{F} = y^2 + 2x^2z - 6yz$

7. If $\vec{F} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$ then find $\operatorname{curl}\vec{F}$.

Sol $\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2z & z^2x \end{vmatrix} = \vec{i}[0-y^2] - \vec{j}[z^2-0] + \vec{k}[0-x^2]$

$= -\vec{i}y^2 - \vec{j}z^2 - \vec{k}x^2.$

$\therefore \operatorname{curl}\vec{F} = -y^2\vec{i} - z^2\vec{j} - x^2\vec{k}.$

8. State Gauss divergence theorem in a plane.

Statement: Gauss divergence theorem is,

$$\iint_S \vec{F}\cdot n\, ds = \iiint_V \nabla\cdot\vec{F}\, dV$$

9. State Green's theorem.

Statement: Green's theorem is,

$$\int_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy.$$

10. Define Line integral.

Ans: A Line integral of a vector point function $\vec{F}(\vec{r})$ over a curve C, where $\vec{r}$ is the position vector point on C, is defined by, PART A (6 marks) Unit - I

matrices

1. If $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ then find the eigen values of A^5 & A^{-1}

Sol. The given matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$ is an upper triangular

∴ The eigen values are its diagonals.

∴ The eigen values of A are 3, 4, 6.

∴ A^5 has eigen values $3^5, 4^5, 6^5$

A^{-1} has eigen values $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$

2. Find the eigen vectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Sol. Given matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

The given matrix is triangular.

∴ The eigen values of A are 1, 1.

The eigen vectors of A are given

$(A - \lambda I)X = 0$, where $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$\begin{pmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

When $\lambda = 1$

$$\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{l} 0x_1 + 2x_2 = 0 \\ 0x_1 + 0x_2 = 0 \end{array}\right\} \Rightarrow x_2 = 0$$

∴ x_1 may be any constant.

If $x_1 = 1$, ∴ one eigen vector is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$x_1 = -1$ another eigen vector is $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

3. Prove that the following vectors are linearly independent $(1,0,0), (0,1,0), (0,0,1)$.

Sol. Given vectors are $(1,0,0), (0,1,0), (0,0,1)$.

The matrix of the given vectors

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(∵ $|A| \neq 0$, then A is linearly independe)

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 1(1) - 0 + 0$

$|A| = 1 \neq 0$

∴ The matrix A linearly independent.

i.e) The vectors $(1,0,0), (0,1,0), (0,0,1)$ are linearly independent.

4. Check whether $A^3 - 20A + 8I = 0$, when $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

Sol.. Given $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

The characteristic equation of A is

$$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$$

Here $a_1 = 1+3-4$

$\boxed{a_1 = 0}$

$a_2 = 1(-12-12) + 3(-4+6) - 4(3-1)$

$= 1(-24) + 3(2) - 4(2)$

$= -24 + 6 - 8$

$a_2 = -26$

$a_3 = |A|$

$= \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{vmatrix}$

$= 1(-12-12) - 1(-4-6) + 3(-4+6)$

characteristic equation

$\lambda^3 - 0\lambda^2 - 20\lambda + 8 = 0$

∴ By Cayley Hamilton theorem

$A^3 - 20A + 8I = 0$

∴ $A^3 - 20A + 8I = 0$

5) Find the characteristic equation of the matrix

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 8 & 1 & -7 \\ -3 & 0 & 8 \end{pmatrix}$$

Sol:- Given $A = \begin{pmatrix} -1 & 0 & 3 \\ 8 & 1 & -7 \\ -3 & 0 & 8 \end{pmatrix}$

characteristic equation is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

where a_1 = sum of the main diagonal elements.

a_2 = sum of the minors of the main diagonal elements.

a_3 = det A or $|A|$.

$\therefore$ $a_1 = -1+1+8 = 8$

$$a_2 = \begin{vmatrix} 1 & -7 \\ 0 & 8 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ -3 & 8 \end{vmatrix} + \begin{vmatrix} -1 & 0 \\ 8 & 1 \end{vmatrix}$$

$= 8 \; - 8+9 \; -1$

$(-8-(-9)$

$(-8+9)$

$a_2 = 8$

$$a_3 = |A| = \begin{vmatrix} -1 & 0 & 3 \\ 8 & 1 & -7 \\ -3 & 0 & 8 \end{vmatrix} = -1(8-0) - 0(64-21) + 3(0+3)$$

$= -8+0+9$

$\boxed{a_3 = 1}$

$\therefore$ The required characteristic equation is

$$\lambda^3 - 8\lambda^2 + 8\lambda - 1 = 0$$

— x —

Unit – II

Differentiation and Integral calculus

1) Find the radius of curvature of the curve

$y = 3x^3 + 2x^2 - 3$ at $(0,-3)$.

Sol.- Radius of curvature $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$

Given $y = 3x^3 + 2x^2 - 3$ at $(0,-3)$.

$y_1 = \dfrac{dy}{dx} = 9x^2 + 4x$

3(0)

$y_2 = \dfrac{d^2y}{dx^2} = 18x + 4$

At $(0,-3)$

$y_1 = 9(0)^2 + 4(0)$ | $y_2 = 18(0) + 4$

$y_1 = 0$ | $y_2 = 4$

$\therefore \quad \rho = \frac{(1+0^2)^{3/2}}{4} = \frac{1^{3/2}}{4} = \frac{1}{4}$

$\therefore \boxed{\rho = \frac{1}{4}}$

(2) Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$, where m is the parameter.

Sol.- Given straight line is $y = mx + \frac{a}{m}$

$\Rightarrow y = \frac{m^2x + a}{m}$

$\Rightarrow m^2x - my + a = 0$ | $Am^2 + Bm + C = 0$

The envelopes are given by $B^2 = 4AC$

Here $A = x$

$B = -y$

$C = a$

$\therefore \quad (-y)^2 = 4(x)(a)$

$y^2 = 4ax$, which is the required envelope.

(3) Find the radius of curvature of the $y = e^x$ at $(0,1)$.

Sol.. Radius of curvature

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

Given $y = e^x$

$y_1 = e^x$

$y_2 = e^x$

At $(0,1)$

$\therefore \quad y_1 = e^0 = 1$ | $y_2 = e^0 = 1$

$$\therefore \rho = \frac{(1+1^2)^{3/2}}{1} = 2^{3/2} = 2\sqrt{2}$$

$$\therefore \boxed{\rho = 2\sqrt{2}}$$

4) Evaluate $\int_0^2 \int_0^3 \int_0^2 xyz \, dz \, dy \, dx$.

Sol

$$\int_0^2 \int_0^3 \int_0^2 xyz \, dz\,dy\,dx = \int_0^2 \int_0^3 \left[xy \int_0^2 z\,dz\right] dy\,dx$$

$$= \int_0^2 \int_0^3 xy \left[\frac{z^2}{2}\right]_0^2 dy\,dx = \int_0^2 \int_0^3 (xy\,(4/2))\, dy\,dx$$

$$= \int_0^2 \int_0^3 2xy \, dy\,dx = 2x\int_0^2 \left[\frac{y^2}{2}\right]_0^3 dx = 2x\int_0^2 \frac{9}{2}\,dx$$

$$= 9\int_0^2 x\,dx = 9\left[\frac{x^2}{2}\right]_0^2 = 9(4/2) = 18$$

5) Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dz\,dy\,dx$

Sol :-

$$\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dz\,dy\,dx = \int_0^1 \int_0^1 \left[\int_0^1 e^{x} \cdot e^{y} \cdot e^{z}\, dz\right] dy\,dx$$

$$= \int_0^1 \int_0^1 e^{x} \cdot e^{y} \left[e^{z}\right]_0^1 dy\,dx$$

$$= \int_0^1 \int_0^1 e^{x} \cdot e^{y} \left[e^{1} - e^{0}\right] dy\,dx = \int_0^1 \int_0^1 e^{x} \cdot e^{y}\,(e-1)\,dy\,dx$$

$$= \int_0^1 (e-1)\, e^{x}\left[\int_0^1 e^{y}\,dy\right] dx = \int_0^1 (e-1)\, e^{x}\left[e^{y}\right]_0^1 dx$$

$$= \int_0^1 (e-1)(e^{x})(e-1)\,dx = (e-1)^2 \int_0^1 e^{x}\,dx$$

$$= (e-1)^2\,[e-1] = (e-1)^3$$

$$\therefore \int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dz\,dy\,dx = (e-1)^3 \quad //$$

Unit - III

Differential Equation :-

1) Solve $(D^2-3D+2)\,y = e^{-5x}$.

Sol :- The given equation is $(D^2-3D+2)y = e^{-5x}$.

To find C.F, solve $(D^2-3D+2)\,y=0$

Auxiliary equation is $m^2-3m+2=0$

$(m-1)\,(m-2)=0$

$\therefore m=1,2$

$\therefore C.F = c_1e^{x}+c_2e^{2x}$

2, -1, -2, -3

To find P.I

$$P.I = \frac{1}{(D^2-3D+2)}\,e^{-5x}$$

$$= \frac{1}{(-5)^2-3(5)+2}\,e^{-5x} \quad \text{(Replace D by -5)}$$

$$= \frac{1}{25+15+2}\,e^{-5x} = \frac{e^{-5x}}{42}$$

$$\therefore P.I = \frac{e^{-6x}}{12}$$

$\therefore$ The General solution is $y = C.F+P.I$

$$\therefore y = c_1e^{x}+c_2e^{2x}+\frac{e^{-5x}}{42}.$$

(2) Solve $(D^2-2D+1)\,y = 3e^{2x}$

Sol. The given equation is $(D^2-2D+1)y = 3e^{2x}$

To find C.F, solve $(D^2-2D+1)y=0$

$\therefore$ A.E is $m^2-2m+1=0$

$\therefore (m-1)(m-1)=0$

$\therefore m=1,1$

1, -1, -1, -2

$\therefore C.F = (c_1+c_2x)\,e^{x}$

To find P.I

$$P.I = \frac{1}{\quad}\,3e^{2x}$$

(23)

$$= \frac{4e^{2x}}{(2)^2 - 4(2)+1} \quad \text{(Replace } D \text{ by } 2\text{)}$$

$$= \frac{4e^{2x}}{4-4+1} = 4e^{2x}$$

$P.I = 4e^{2x}$

the General solution is $y = C.F + P.I$

$\therefore y = (c_1 + c_2 x) e^{x} + 4e^{2x}$

Q) Solve $(x^2 D^2 + xD - 1)y = 0$

Sol. Put $x = e^z$ and $\theta = \frac{d}{dz}$, then $xD = \theta$, $x^2 D^2 = \theta(\theta - 1)$

$\therefore$ the equation $[\theta(\theta-1) + \theta - 1]y = 0$

$\Rightarrow (\theta^2 - 1)y = 0$

$\therefore$ A.E. is $m^2 - 1 = 0 \Rightarrow m = \pm 1$

the solution is $y = c_1 e^{z} + c_2 e^{-z}$

$$\boxed{y = c_1 x + c_2 x^{-1}}$$

Q) Solve $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$.

Sol. Given $\frac{dx}{dt} + y = \sin t$

$\frac{dy}{dt} + x = \cos t$

Put $D = \frac{d}{dt}$ & substitute in the previous eqn

$Dx + y = \sin t$

$Dy + x = \cos t$.

$Dx + y = \sin t \rightarrow (1)$

$x + Dy = \cos t \rightarrow (2)$

First eliminate y.

$D(\sin t)$

$① \times D \Rightarrow D^2x + \cancel{Dy} = \cancel{\cos t}$

$② \times 1 \Rightarrow x + \cancel{Dy} = \cancel{\cos t}$

$(-) \quad (-) \qquad (-)$

$(D^2x - x) = 0 \Rightarrow (D^2 - 1)x = 0$

$\therefore$ A-E is $m^2 - 1 = 0 \Rightarrow m^2 = 1$

$m = \pm 1$

$\therefore \boxed{x(t) = c_1 e^t + c_2 e^{-t}}$

$\because$ from ①, $y = -Dx + \sin t$

$y = -D(c_1 e^t + c_2 e^{-t}) + \sin t$

$y = -\frac{d}{dt}(c_1 e^t + c_2 e^{-t}) + \sin t$

$\therefore y = -(c_1 e^t - c_2 e^{-t}) + \sin t$

$\boxed{y = -c_1 e^t + c_2 e^{-t} + \sin t}$

5) Solve $(D^2 - 2D + 1)y = x$

Sol :- Given $(D^2 - 2D + 1)y = x$

$\therefore$ A-E is $m^2 - 2m + 1 = 0$

$\therefore (m-1)(m-1) = 0$

$m = 1, 1$

$\therefore$ C.F $= (c_1 + c_2 x)e^x$

To find P.I :-

$$P.I = \frac{1}{(D^2 - 2D + 1)} x$$

$$= \frac{1}{(1 + (D^2 - 2D))} x = (1 + (D^2 - 2D))^{-1} x$$

$$= [1 - (D^2 - 2D) + (D^2 - 2D)^2 - \dots]x$$

$$= [1 + 2D]x$$

Unit - IV

Analytic Function and complex Integration :-

(1) (i) Find the fixed points of the mapping $w = \frac{3-z}{1+z}$

(ii) Find the critical points of $f(z) = z^3$.

Sol :- Sol (i) Given $w = \frac{3-z}{1+z}$

The fixed points are given by $w = z$

$\therefore z = \frac{3-z}{1+z} \Rightarrow z + z^2 = 3 - z$

$\Rightarrow z^2 + 2z - 3 = 0$

$\Rightarrow (z+3)(z-1) = 0$

$\therefore z = -3, 1$ are the fixed points.

(ii) Sol ... given $f(z) = z^3$

Critical points are given by $f'(z) = 0$

$\therefore f'(z) = 3z^2 = 0$

$\Rightarrow z = 0$

$\therefore z = 0$ is the critical point.

(2) Determine the residue at the point $z = 2$ of

$$\frac{z+2}{(z+1)^2(z-2)}$$

Sol :- Let $f(z) = \frac{z+2}{(z+1)^2(z-2)}$

$\therefore$ The poles are given by denominator equal to be zero.

$\therefore (z+1)^2(z-2) = 0$

$\therefore z = -1, -1, | z = 2$

$\therefore z = -1$ is a pole of order 2.

$z = 2$ is a simple pole.

$$\left.\begin{array}{l}\text{Residue at} \\ z = 2\end{array}\right\} = R(2) = \lim_{z \to 2} (z-2) f(z)$$

$$= \lim_{z \to 2} (z-2) \frac{(z+2)}{(z+1)^2 (z-2)}$$

$$= \lim_{z \to 2} \frac{z+2}{(z+1)^2} = \frac{2+2}{(2+1)^2} = \frac{4}{9}$$

3) Find the residue of $f(z) = \dfrac{z}{(z-1)^2}$ as its poles.

Sol :- Let $f(z) = \dfrac{z}{(z-1)^2}$

$\therefore$ poles are given by $(z-1)^2 = 0$

$\Rightarrow z = 1, 1$

$\therefore$ $z = 1$ is a pole of order 2.

$\therefore$ If $z = a$ is a pole of order, then the residue at $z = a$ is

$$\boxed{\therefore R(a) = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right)}$$

Here $m = 2$, $a = 1$

$$\therefore R(1) = \frac{1}{(2-1)!} \lim_{z \to 1} \frac{d^{2-1}}{dz^{2-1}} \left[(z-1)^2 \frac{z}{(z-1)^2} \right]$$

$$= \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} (z)$$

$$= \lim_{z \to 1} (1) = 1$$

$\therefore$ $R(1) = 1$

(4) Using Cauchy's integral formula, Evaluate

$$\int_c \frac{3z^2+7z+1}{(z+1)} dz$$, where c is the circle $|z| = \frac{1}{2}$ -

Sol :- Singular point is given by $z+1=0$

$\therefore z=-1$

c is $|z| = \frac{1}{2}$

If $z=-1 \Rightarrow |z| = |-1| = 1 > 1/2$

$\therefore$ So $z=-1$ lies outside c.

$\therefore f(z) = \frac{3z^2+7z+1}{z+1}$ is analytic inside and on c.

$$\therefore \int_c f(z)\, dz = 0$$

(5) Evaluate $\int_c \frac{z}{z-2} dz$, where c is $|z-2| = 3/2$.

Sol :- singular point is given by $z-2=0$

$\Rightarrow z=2$.

$\therefore$ Let $f(z) = z$, c is $|z-2| = 3/2$

If $z=2 \Rightarrow |z-2| = |2-2| = 0 < 3/2$

$\therefore z=2$ lies inside c.

$f(z) = z$

$\boxed{f(2) = 2}$

$\therefore$ By Cauchy's integral formula,

$$\int_c \frac{f(z)}{z-2} dz = 2\pi i\, f(2) = 2\pi i(2) = 4\pi i.$$

Unit - V

Vector calculus

1) Find $\nabla\varphi$ at the point $(1,1,1)$ if $\varphi(x,y,z) = x^2y + y^2x + z^2$.

Sol:- Given $\varphi(x,y,z) = x^2y + y^2x + z^2$

$$\boxed{\nabla\varphi = \vec{i}\,\frac{\partial\varphi}{\partial x} + \vec{j}\,\frac{\partial\varphi}{\partial y} + \vec{k}\,\frac{\partial\varphi}{\partial z}} \rightarrow (1)$$

Given $\varphi(x,y,z) = x^2y + y^2x + z^2$

$$\frac{\partial\varphi}{\partial x} = 2xy + y^2, \quad \frac{\partial\varphi}{\partial y} = x^2 + 2xy, \quad \frac{\partial\varphi}{\partial z} = 2z$$

Substitute the above values in (1), we get

$$\therefore \nabla\varphi = \vec{i}\,(2xy + y^2) + \vec{j}\,(x^2 + 2xy) + \vec{k}\,(2z)$$

$$\therefore (\nabla\varphi)_{(1,1,1)} = \vec{i}\,(2(1)(1) + 1^2) + \vec{j}\,(1^2 + 2(1)(1)) + \vec{k}\,2(1)$$

$$= 3\vec{i} + 3\vec{j} + 2\vec{k}.$$

$$\therefore \boxed{(\nabla\varphi)_{(1,1,1)} = 3\vec{i} + 3\vec{j} + 2\vec{k}}$$

(2) Find the value of 'a', if $\vec{F} = ay^4z^2\vec{i} + 4x^3z^2\vec{j} + 5x^2y^2\vec{k}$ is solenoidal.

Sol:- Given $\vec{F} = ay^4z^2x\vec{i} + 4x^3z^2\vec{j} + 5x^2y^2\vec{k}$ is solenoidal.

$$\therefore \nabla\cdot\vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(ay^4z^2x) + \frac{\partial}{\partial y}(4x^3z^2) + \frac{\partial}{\partial z}(5x^2y^2) = 0$$

$$a + 0 + 0 = 0 \quad \Rightarrow a = 0$$

$\therefore$ ~~a can be any natural number.~~

$\therefore$ The value of a is $\underline{0}$.

(3) Find the value of 'a' if the vector $\vec{F} = (axy - z^2)\vec{i} + (x^2+2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational.

Sol :- Given $\vec{F} = (axy - z^2)\vec{i} + (x^2+2yz)\vec{j} + (y^2-axz)\vec{k}$ is irrotational.

$$\therefore \nabla \times \vec{F} = 0$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy-z^2 & x^2+2yz & y^2-axz \end{vmatrix} = 0$$

$$\Rightarrow \vec{i}\left[\frac{\partial}{\partial y}(y^2-axz) - \frac{\partial}{\partial z}(x^2+2yz)\right] - \vec{j}\left[\frac{\partial}{\partial x}(y^2-axz) - \frac{\partial}{\partial z}(axy-z^2)\right]$$

$$+\vec{k}\left[\frac{\partial}{\partial x}(x^2+2yz) - \frac{\partial}{\partial y}(axy-z^2)\right] = 0$$

$$\Rightarrow \vec{i}[(2y) - 2y] - \vec{j}[-az + 2z] + \vec{k}[2x - ax] = 0$$

$$\Rightarrow \vec{i}[0] - \vec{j}[-az+2z] + \vec{k}[2x-ax] = 0\vec{i} + 0\vec{j} + 0\vec{k}$$

Comparing the co-eff $\vec{j}$ or $\vec{k}$ on both sides we get

$-az + 2z = 0$ or $2x - ax = 0$

$\Rightarrow az = 2z$ | $2x = ax$

$\Rightarrow \boxed{a=2}$ | $a = 2$.

$$\therefore a = 2.$$

(4) Find $\nabla\varphi$ at the point $(1,-2,1)$ if $\varphi(x,y,z) = 3x^2y - y^3z$

Sol :- Given $\varphi(x,y,z) = 3x^2y - y^3z^2$

$$\nabla\varphi = \vec{i}\frac{\partial\varphi}{\partial x} + \vec{j}\frac{\partial\varphi}{\partial y} + \vec{k}\frac{\partial\varphi}{\partial z}$$

$$\therefore \frac{\partial\varphi}{\partial x} = 6xy, \quad \frac{\partial\varphi}{\partial y} = 3x^2 - 3y^2z^2, \quad \frac{\partial\varphi}{\partial z} = -2y^3z$$

$$\therefore \nabla\varphi = \vec{i}(6xy) + \vec{j}(3x^2-3y^2z^2) + \vec{k}(-2y^3z)$$

$$\therefore (\nabla\varphi)_{(1,-2,1)} = \vec{i}(6(1)(-2)) + \vec{j}(3(1)^2 - 3(-2)^2(1)^2) + \vec{k}(-2(-2)^3(1))$$

$$= \vec{i}(-12) + \vec{j}(-9) + \vec{k}(16)$$

$$\therefore \boxed{(\nabla\varphi)_{(1,-2,1)} = -12\vec{i} - 9\vec{j} + 16\vec{k}}$$

(5) If the vector $3x\vec{i} + (x+y)\vec{j} - az\vec{k}$ is solenoidal, then find a.

Sol:- Given $\vec{F} = 3x\vec{i} + (x+y)\vec{j} - az\vec{k}$ is solenoidal.

$$\therefore \nabla \cdot \vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(3x) + \frac{\partial}{\partial y}(x+y) + \frac{\partial}{\partial z}(-az) = 0$$

$$3 + 1 - a = 0$$

$$\Rightarrow 4 - a = 0 \quad \Rightarrow \boxed{a = 4}.$$

Part- C

Unit- I, Matrices (Concepts)

1. Verify Cayley- Hamilton Theorem, for the matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ and hence find A^{-1}.

Sol:-

Given matrix $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$

The ch. equ is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$.

where a_1 = {sum of leading diagonal elements

$= 1+2+2 = 5$.

a_2 = { sum of the minors of the Leading diagonal elements

$= \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}$

$= 4+0+2+0+2+0 = 8$.

$a_3 = \begin{vmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{vmatrix} = 1(4) - 0 - 2(0)$

$= 4$.

∴ The characteristic equ is $\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$.

Verification:- To verify cayley Hamilton theorem, we have to prove that,

$A^3 - 5A^2 + 8A - 4I = 0$.

Now,

$A^2 = A \times A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{pmatrix}$

$A^3 = A^2 \times A = \begin{pmatrix} 1 & 0 & -6 \\ 6 & 4 & 12 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{pmatrix}$

$A^3 - 5A^2 + 8A - 4I \Rightarrow$

$= \begin{pmatrix} 1 & 0 & -14 \\ 14 & 8 & 28 \\ 0 & 0 & 8 \end{pmatrix} + \begin{pmatrix} -5 & 0 & 30 \\ -30 & -20 & -60 \\ 0 & 0 & -20 \end{pmatrix} + \begin{pmatrix} 8 & 0 & -16 \\ 16 & 16 & 32 \\ 0 & 0 & 16 \end{pmatrix} + \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Hence Cayley Hamilton theorem is verified.

To Find A^{-1}:

we know that $A^3-5A^2+8A-4I=0$.

Multiplying by A^{-1},

$$A^{-1}A^3-5A^{-1}A^2+8A^{-1}A-4IA^{-1}=0.$$

$$A^2-5A+8I-4A^{-1}=0.$$

$$\therefore +4A^{-1}=+(A^2-5A+8I)$$

$$A^{-1}=\frac{1}{4}(A^2-5A+8I)$$

$$=\frac{1}{4}\left(\begin{pmatrix}1&0&-6\\6&4&12\\0&0&4\end{pmatrix}+\begin{pmatrix}-5&0&10\\-10&-10&-20\\0&0&-10\end{pmatrix}+\begin{pmatrix}8&0&0\\0&8&0\\0&0&8\end{pmatrix}\right)$$

$$=\frac{1}{4}\begin{pmatrix}4&0&4\\-4&2&-8\\0&0&2\end{pmatrix}$$

$$\therefore A^{-1}=\frac{1}{2}\begin{pmatrix}2&0&2\\-2&1&-4\\0&0&1\end{pmatrix}$$ //

2). Find the eigen values and eigen vectors of the matrix $A=\begin{pmatrix}1&2&0\\2&1&-6\\2&-2&3\end{pmatrix}$

Sol:

Given matrix $A=\begin{pmatrix}1&2&0\\2&1&-6\\2&-2&3\end{pmatrix}$

$\therefore$ the ch.equ is $\lambda^3-a_1\lambda^2+a_2\lambda-a_3=0$.

where $a_1=\{$ sum of leading diagonal elements.

$$=1+1+3=5.$$

$a_2=\{$ sum of the minors of the leading diagonal elements

$$=\begin{vmatrix}1&-6\\-2&3\end{vmatrix}+\begin{vmatrix}1&0\\2&3\end{vmatrix}+\begin{vmatrix}1&2\\2&1\end{vmatrix}=3-12+3+0+1-4=-9.$$

$$a_3=\begin{vmatrix}1&2&0\\2&1&-6\\2&-2&3\end{vmatrix}=1(3-12)-2(6+12)+0$$

$$=-9-36$$

$$=-45.$$

$\therefore$ ch.equ is $\lambda^3-5\lambda^2-9\lambda+45=0$. $\lambda=3$

To Find other Eigen values:

~~when $\lambda=3$,~~

$$\begin{array}{c|cccc} & 1 & -5 & -9 & 45\\ 3 & 0 & 3 & -6 & -45\\ \hline & 1 & -2 & -15 & \boxed{0}\end{array}$$

$$\lambda^2-2\lambda-15=0.$$

$$\lambda^2-5\lambda+3\lambda-15=0.$$

$$(\lambda-5)(\lambda+3)=0 \qquad \lambda=5,\ \lambda=-3.$$

$\therefore$ Eigen values are $\lambda=-3,3,5$.

To Find Eigen vector :-

case (i) :- when $\lambda = -3$,

then From the equation. $(A - \lambda I)X = 0$.

$$\begin{pmatrix} 4 & 2 & 0 \\ 2 & 4 & -6 \\ 2 & -2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$4x_1 + 2x_2 + 0x_3 = 0$$
$$2x_1 + 4x_2 - 6x_3 = 0$$
$$2x_1 - 2x_2 + 6x_3 = 0.$$

Considering First two equations and using cross Rule method,

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ \hline 2 & 0 & 4 & 2 \\ 4 & -6 & 2 & 4. \end{array}$$

$$\frac{x_1}{-12+0} = \frac{x_2}{0-24} = \frac{x_3}{16-4}.$$

$$\frac{x_1}{-12} = \frac{x_2}{-24} = \frac{x_3}{12}.$$

$$X_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$$

case (ii) :- when $\lambda = 3$,

then From the equation $(A - \lambda I)X = 0$.

$$\begin{pmatrix} -2 & 2 & 0 \\ 2 & -2 & -6 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$-2x_1 + 2x_2 + 0x_3 = 0.$$
$$2x_1 - 2x_2 - 6x_3 = 0.$$
$$2x_1 - 2x_2 + 0x_3 = 0.$$

Considering First two equations and using cross Rule method,

$$\begin{array}{cccc} & x_1 & x_2 & x_3 \\ \hline 2 & 0 & -2 & 2 \\ -2 & -6 & 2 & -2 \end{array}$$

$$\frac{x_1}{-12} = \frac{x_2}{-12} = \frac{x_3}{0}.$$

$$X_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Case (iii): $\lambda = 5$,

Then from the equation $(A-\lambda I)X = 0$.

$$\begin{pmatrix} -4 & 2 & 0 \\ 2 & -4 & -6 \\ 2 & -2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$-4x_1 + 2x_2 + 0x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0.$$
$$2x_1 - 2x_2 - 2x_3 = 0.$$

consider First two equations and using cross Rule Method,

x_1	x_2	x_3	
2	0	-4	2
-4	-6	2	-4.

$$\frac{x_1}{-12} = \frac{x_2}{-24} = \frac{x_3}{12}.$$

$$X_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Conclusion:-

characteristic equation : $\lambda^3 - 5\lambda^2 - 9\lambda + 45 = 0$.

Eigen values : $\lambda = -3, 3, 5$.

Eigen vectors : $X_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$, $X_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $X_3 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

=x=x=

3). Diagonalise the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

Sol:-

Given matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$

$\therefore$ the ch. equ is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$.

where $a_1 = \{$ sum of leading diagonal elements.

$= 3+5+3$

$= 11$.

$a_2 = \{$ Sum of minors of the leading diagonal elements.

$= \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix} = 15-1+9-1+15-1 = 36.$

$a_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 3(15-1)+1(-3+1)+1(1-5) = 3(14)+1(-2)+1(-4)$

$= 36.$

$\therefore$ The ch. equ is $\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$.

To Find Eigen values:-

when $\lambda = 2$, $(2)^3 - 11(2)^2 + 36(2) - 36 = 0$.

$$8 - 44 + 72 - 36 = 0.$$

$$0 = 0.$$

$\therefore \lambda = 2$ is a root,

$$\begin{array}{c|cccc} & 1 & -11 & 36 & -36 \\ 2 & 0 & 2 & -18 & 36 \\ \hline & 1 & -9 & 18 & \boxed{0} \end{array}$$

$$\lambda^2 - 9\lambda + 18 = 0.$$

$$\lambda^2 - 6\lambda - 3\lambda + 18 = 0.$$

$$(\lambda - 6)(\lambda - 3) = 0.$$

$$\lambda = 3, 6.$$

$\therefore$ Eigen values are $\lambda = 2, 3, 6$.

To Find Eigen vectors:-

case (i):- $\lambda = 2$.

then From the equ $(A - \lambda I)X = 0$.

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$x_1 - x_2 + x_3 = 0.$$
$$-x_1 + 3x_2 - x_3 = 0.$$
$$x_1 - x_2 + x_3 = 0.$$

Consider First two equations,

$$\begin{array}{cccccc} & x_1 & & x_2 & & x_3 \\ \hline -1 & & 1 & & 1 & -1 \\ 3 & & -1 & & -1 & 3 \end{array}$$

$$\frac{x_1}{-2} = \frac{x_2}{0} = \frac{x_3}{2}.$$

$$X_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Case (ii):- $\lambda = 3$

then From the equ $(A - \lambda I)X = 0$.

$$\begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$0x_1 - x_2 + x_3 = 0$$
$$-x_1 + 2x_2 - x_3 = 0$$

Consider First two equations

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \hline -1 & 1 & 0 & -1 \\ 2 & -1 & -1 & 2 \end{array}$$

$$\frac{x_1}{-1} = \frac{x_2}{-1} = \frac{x_3}{-1}$$

$$X_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Case (iii): $\lambda = 6.$

Then From the equ $(A - \lambda I)X = 0.$

$$\begin{pmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$-3x_1 - x_2 + x_3 = 0$$
$$-x_1 - x_2 - x_3 = 0$$
$$x_1 - x_2 - 3x_3 = 0.$$

Consider First two equations.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \hline -1 & 1 & -3 & -1 \\ -1 & -1 & -1 & -1. \end{array}$$

$$\frac{x_1}{2} = \frac{x_2}{-4} = \frac{x_3}{2}$$

$$X_3 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}.$$

∴ the Eigen vectors are $X_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $X_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $X_3 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

then Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$ +

− 1+3

+ −1

To Find B^{-1}:

$$\text{Adj. of } B = \begin{bmatrix} 3 & 0 & -3 \\ 2 & 2 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\det B = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{vmatrix} = 1(3) - 0 - 1(-3)$$
$$= 3 + 3$$
$$= 6$$

Hence Inverse of the matrix B is.

$$B^{-1} = \frac{Adj\ B}{|B|}.$$

$$= \begin{bmatrix} 1/2 & 0 & -1/2 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & -1/3 & 1/6 \end{bmatrix}$$

Now $AB = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 2 & 3 & 6 \\ 0 & 3 & -12 \\ -2 & 3 & 6 \end{pmatrix}$$

$D = B^{-1}AB.$

$$= \begin{bmatrix} -1/2 & 0 & -1/2 \\ 1/3 & 1/3 & 1/3 \\ 1/6 & -1/3 & 1/6 \end{bmatrix} \begin{bmatrix} 2 & 3 & 6 \\ 0 & 3 & -12 \\ -2 & 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Hence the diagonal matrix.

4). verify cayley- Hamilton theorem, For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find A^3.

Sol:- Given matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

the ch.equ is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$.

where $a_1 = \{$ sum of leading diagonal elements

$= 2+2+2 = 6.$

$a_2 = \{$ sum of the minors of the Leading diagonal elements.

$$= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$= 4-1+4-1+4-1$

$= 9.$

$$a_3 = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 6-1-1 = 4.$$

$\therefore$ the ch. equ is $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$.

Verification:-

By Cayley - Hamilton theorem

$$A^3 - 6A^2 + 9A - 4I = 0 \rightarrow (1).$$

$$A^2 = A \times A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}.$$

$$A^3 = A^2 \times A = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix}.$$

$A^3 - 6A^2 + 9A - 4I \Rightarrow$

$$= \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} + \begin{pmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{pmatrix} + \begin{pmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{pmatrix} + \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Hence verified Cayley - Hamilton theorem.

To Find A^3:-

$$A^3 - 6A^2 + 9A - 4I = 0.$$

$$A^3 = 6A^2 - 9A + 4I.$$

$$= \begin{pmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{pmatrix} + \begin{pmatrix} -18 & 9 & -9 \\ 9 & -18 & 9 \\ -9 & 9 & -18 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 36-18+4 & -30+9+0 & 30-9+0 \\ -30+9+0 & 36-18+4 & -30+9+0 \\ 30-9+0 & -30+9+0 & 36-18+4 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix}$$

5). Find the eigen values and eigen vector of the matrix

$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

Sol:-

Given matrix $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

the characteristic equ is. $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$.

$a_1 = \{$ sum of leading diagonal elements.

$= 1+2+3 = 6$.

$a_2 = \{$ sum of the minors of the leading diagonal elements.

$= \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$

$= 6-2+3+2+2+0$

$= 11$

$a_3 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix} = 1(6-2)+0-1(2-4)$
$= 4+2$
$= 6$.

$\therefore$ the ch-equ is $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$.

To Find Eigen values:-

Let $\lambda = 1$,

$(1)^3 - 6(1)^2 + 11(1) - 6 = 0$.

$1 - 6 + 11 - 6 = 0$

$0 = 0$.

$$\begin{array}{c|cccc} & 1 & -6 & 11 & -6 \\ 1 & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$\lambda = 1$ is a root,

$\lambda^2 - 5\lambda + 6 = 0$.

$\lambda^2 - 2\lambda - 3\lambda + 6 = 0$

$(\lambda-2)(\lambda-3) = 0$

$\lambda = 2, 3$.

6
$\wedge$
$-2 \quad -3$

$\therefore$ Eigen values $\lambda = 1, 2, 3$.

To Find Eigen vectors:- Consider the equ $(A-\lambda I)X = 0$.

Case (i): when $\lambda = 1$.

$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$.

$$0x_1 + 0x_2 - x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$
$$2x_1 + 2x_2 + 2x_3 = 0.$$

Consider First two equations.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \hline 0 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{0}$$

$$X_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Case (ii):- $\lambda = 2$, consider the equ $(A - \lambda I)X = 0$.

$$\begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$-x_1 + 0x_2 - x_3 = 0.$$
$$x_1 + 0x_2 + x_3 = 0.$$
$$2x_1 + 2x_2 + x_3 = 0.$$

Consider Last two equations.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \hline 0 & -1 & -1 & 0 \\ 2 & 1 & 2 & 2. \end{array}$$

$$\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{-2}.$$

$$X_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}.$$

Case (iii):- $\lambda = 3$ consider the equ $(A - \lambda I)X = 0$.

$$\begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

$$-2x_1 + 0x_2 - x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$
$$2x_1 + 2x_2 + 0x_3 = 0.$$

Consider First two equations.

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \hline 0 & -1 & -2 & 0 \\ -1 & 1 & 1 & -1 \end{array}$$

$$\frac{x_1}{-1} = \frac{x_2}{+1} = \frac{x_3}{+2} \; ; \; X_3 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Conclusion:-

Ch-equ: $\lambda^3 - 6\lambda^2 + 11\lambda - 6$

Eigen values: $\lambda = 1, 2, 3,$

Eigen vectors:-

$$X_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$
$$X_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$
$$X_3 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$

Part-C
unit-II

1. show that the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $(a/4, a/4)$ is $\left(x-\frac{3a}{4}\right)^2+\left(y-\frac{3a}{4}\right)^2=\frac{a^2}{2}$.

$\sqrt{x}+\sqrt{y}=\sqrt{a}$ —— (1)

Differentiating (1) w.r.to x

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\cdot y_1=0$$

$$\Rightarrow y_1=\frac{-1}{2\sqrt{x}}\times 2\sqrt{y}=-\frac{\sqrt{y}}{\sqrt{x}} \quad \text{—— (2)}$$

$$y_1\left(a/4, a/4\right)=-\frac{\sqrt{a/4}}{\sqrt{a/4}}=-1$$

$\boxed{y_1=-1}$

Squaring (2) on both sides

$$y_1^2=y/x \Rightarrow y_1^2 x=y$$

Differentiating

$$y_1^2(1)+x(2y_1y_2)=y_1$$

$$y_1^2+2xy_1y_2=y_1$$

$$1-2xy_2=-1$$

$$-2xy_2=-2$$

$$\boxed{y_2=1/x}$$

$$y_2(a/4, a/4)=\frac{1}{(a/4)}=4/a$$

$$\rho=\frac{[1+y_1^2]^{3/2}}{y_2}=\frac{(1+1)^{3/2}}{(4/a)}=\frac{2^{3/2}}{(4/a)}$$

$$\rho=\frac{2\sqrt{2}\times a}{4}=a\sqrt{2}/2$$

$$\rho=\frac{a}{\sqrt{2}}.$$

$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$

$= \frac{a}{4} - \frac{(-1)(1+1)}{(4/a)} = a/4 + a/4\,(2) = \frac{a+2a}{4} = \frac{3a}{4}.$

$\bar{x} = \frac{3a}{4}$

$\bar{y} = y + \frac{1}{y_2}(1+y_1^2)$

$= \frac{a}{4} + a/4\,(1+1) = \frac{3a}{4}$

$\bar{y} = \frac{3a}{4}.$

$\therefore$ The centre of curvature is $\left(\frac{3a}{4}, \frac{3a}{4}\right)$

circle of curvature is

$(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

$\left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = \left(\frac{a}{\sqrt{2}}\right)^2$

$\Rightarrow \left(x - \frac{3a}{4}\right)^2 + \left(y - \frac{3a}{4}\right)^2 = a^2/2.$

(2) Show that the equation of evolute of the parabola $x^2 = 4ay$ is $4(x-2a)^3 = 27ax^2$.

$x = 2at,\ y = at^2$ are co-ordinates

$\frac{dx}{dt} = 2a,\ \frac{dy}{dt} = 2at$

$y_1 = \frac{dy/dt}{dx/dt} = \frac{2at}{2a} = t$

$y_2 = \frac{d}{dt}(y_1)\frac{dt}{dx} = \frac{d}{dt}(t)\left(\frac{1}{2a}\right) = \frac{1}{2a}.$

$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$

$$= 2at - \frac{t}{\left(\frac{1}{2a}\right)}[1+t^2]$$

$$= 2at - 2at(1+t^2)$$

$$= 2at - 2at - 2at^3$$

$$\bar{x} = -2at^3 \text{ ——— } (1)$$

$$\bar{y} = y + \frac{1}{y_2}(1+y_1^2)$$

$$= at^2 + \frac{1}{\left(\frac{1}{2a}\right)}(1+t^2)$$

$$= at^2 + 2a(1+t^2) = 3at^2 + 2a$$

$$\therefore \bar{y} = 3at^2 + 2a \text{ ——— } (2)$$

To find the equation of the evalute we've to eliminate 't' from (1) & (2)

from (1) $\Rightarrow t^3 = -\frac{\bar{x}}{2a} \Rightarrow (t^3)^2 = \left(\frac{\bar{x}}{2a}\right)^2$ — (3)

from (2) $\Rightarrow t^2 = \frac{\bar{y} - 2a}{3a} \Rightarrow (t^2)^3 = \frac{(\bar{y} - 2a)^3}{27a^3}$ — (4)

(3) = (4) $\Rightarrow$

$$\left(\frac{\bar{x}}{2a}\right)^2 = \frac{(\bar{y} - 2a)^3}{27a^3}$$

$$\frac{\bar{x}^2}{4a^{\cancel{2}}} = \frac{(\bar{y} - 2a)^3}{27a^{\cancel{3}}}$$

$$\Rightarrow 27a\bar{x}^2 = 4(\bar{y} - 2a)^3$$

$\therefore$ Locus of $(\bar{x}, \bar{y})$ is $27ax^2 = 4(y-2a)^3$.

(5) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$, where 'a' and 'b' are connected by the relations $a+b=c$, 'c' being constant.

$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{——— (1)}$$

$$a+b=c \quad \text{——— (2)}$$

If 'a' and 'b' are functions of third variable 't'.

Differentiating (1) w.r.to 't'

$$-\frac{x}{a^2}\frac{da}{dt}-\frac{y}{b^2}\frac{db}{dt}=0$$

$$\Rightarrow \frac{x}{a^2}\frac{da}{dt}=\frac{-y}{b^2}\frac{db}{dt} \quad \text{— (3)}$$

Differentiating (2) w.r.to 't'

$$\frac{da}{dt}+\frac{db}{dt}=0$$

$$\Rightarrow \frac{da}{dt}=-\frac{db}{dt} \quad \text{——— (4)}$$

(3) ÷ (4)

$$\frac{x}{a^2}=\frac{y}{b^2} \Rightarrow \frac{\left(\frac{x}{a}\right)}{a}=\frac{\left(\frac{y}{b}\right)}{b}=\frac{\frac{x}{a}+\frac{y}{b}}{a+b}=\frac{1}{c}.$$

$$\therefore \frac{x}{a^2}=\frac{1}{c^2}. \Rightarrow a^2=cx \Rightarrow a=(cx)^{\frac{1}{2}}.$$

|||rly $\frac{y}{b^2}=\frac{1}{c^2} \Rightarrow b=(cy)^{\frac{1}{2}}$

We know that $a+b=c$

$$(cx)^{\frac{1}{2}}+(cy)^{\frac{1}{2}}=c$$

$x^{\frac{1}{2}}+y^{\frac{1}{2}}=c^{\frac{1}{2}}$ is the

req. envelope.

$\overset{5}{\text{37}}$

4) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{(1-x^2-y^2)-z^2}}$

$$\Rightarrow \int_0^1 \int_0^{\sqrt{1-x^2}} \left[\sin^{-1} \frac{z}{\sqrt{(1-x^2-y^2)}}\right]_0^{\sqrt{1-x^2-y^2}} dx\,dy$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left[\sin^{-1} \frac{\sqrt{1-x^2-y^2}}{\sqrt{1-x^2-y^2}} - \sin^{-1} 0\right] dx\,dy$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left[\sin^{-1}(1) - \sin^{-1}(0)\right] dx\,dy$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left[\pi/2 - 0\right] dx\,dy$$

$$= \pi/2 \int_0^1 \int_0^{\sqrt{1-x^2}} dx\,dy = \pi/2 \int_0^1 [y]_0^{\sqrt{1-x^2}} dx$$

$$= \pi/2 \int_0^1 \sqrt{1-x^2}\, dx$$

$$= \pi/2 \left[x/2 \sqrt{1-x^2} + 1/2 \sin^{-1} x\right]_0^1$$

$$= \pi/2 \left[0 + 1/2 \sin^{-1}(1) - 0\right]$$

$$= \pi/2 (1/2)\, \pi/2 = \pi^2/8$$

5) Evaluate $\int_0^a \int_0^b \int_0^c (x+y+z)\, dx\, dy\, dz$

$$= \int_0^a \int_0^b \left[\frac{x^2}{2} + yx + zx\right]_0^c dy\, dz$$

$$= \int_0^a \int_0^b \left[\frac{c^2}{2} + cy + cz\right] dy\, dz$$

$$= \int_0^a \left[\frac{c^2 y}{2} + \frac{cy^2}{2} + czy\right]_0^b dz$$

$$= \int_0^a \left[\frac{c^2 b}{2} + \frac{cb^2}{2} + czb\right] dz$$

$$= \left[\frac{c^2 bz}{2} + \frac{cb^2 z}{2} + \frac{cbz^2}{2}\right]_0^a$$

$$= \frac{abc^2}{2} + \frac{ab^2c}{2} + \frac{a^2bc}{2}$$

$$= \frac{abc}{2}(a+b+c).$$

Unit – III
(Differential equation)

1) Solve $(D^2 - 4D + 4)y = e^{-4x} + 5\cos 2x$

Sol:- The given equation is

$$(D^2 - 4D + 4)y = e^{-4x} + 5\cos 3x$$

To find the C.F, solve $(D^2 - 4D + 4)y = 0$

$\therefore$ The Auxiliary equation is $m^2 - 4m + 4 = 0$

$$\Rightarrow (m-2)(m-2) = 0$$

$$\Rightarrow m = 2, 2.$$

$$\therefore \; C.F = (c_1 + c_2 x) e^{2x}$$

To find $P.I_1$ & $P.I_2$

$$P.I_1 = \frac{1}{D^2 - 4D + 4} e^{-4x}$$

(Replace D by -4)

$$= \frac{e^{-4x}}{(-4)^2 - 4(-4) + 4} = \frac{e^{-4x}}{16 + 16 + 4}$$

$$\boxed{P.I_1 = \frac{e^{-4x}}{36}}$$

$$P.I_2 = \frac{5\cos 2x}{D^2 - 4D + 4}$$

(Replace D^2 by -2^2)

$$= \frac{5\cos 2x}{-2^2 - 4D + 4} = \frac{5\cos 2x}{-4 - 4D + 4}$$

$$= \frac{5\cos 2x}{\cancel{-5} - 4D} = \frac{5\cos 2x}{-4D}$$

$$= \frac{-5}{4} \frac{1}{D} \cos 2x = \frac{-5}{4} \int \cos 2x \, dx$$

$$= \frac{-5}{4} \frac{\sin 2x}{2} = \frac{-5}{8} \sin 2x$$

$\therefore$ The General solution is $y = C.F + P.I_1 + P.I_2$

$$y = (c_1 + c_2 x) e^{2x} + e^{-4x}/\ldots - 5/\ldots \sin 2x$$

(2) Solve $(D^2+6D+8)y = x^2$

Sol:- The given equation is $(D^2+6D+8)y = x^2$

To find C.F, Solve $(D^2+6D+8)y = 0$

∴ Auxiliary equation is $m^2+6m+8=0$

$$(m+2)(m+4)=0$$

$$m=-2,\ -4$$

8, 2, 4, 6

∴ $C.F = c_1 e^{-2x} + c_2 e^{-4x}$

To find P.I

$$P.I = \frac{x^2}{(D^2+6D+8)}$$

$$= \frac{x^2}{8\left[1+\left(\frac{D^2+6D}{8}\right)\right]} = \frac{1}{8}\left[1+\left(\frac{D^2+6D}{8}\right)\right]^{-1} x^2$$

$$= \frac{1}{8}\left[1-\left(\frac{D^2+6D}{8}\right)+\left(\frac{D^2+6D}{8}\right)^2+\cdots\right]x^2$$

$$= \frac{1}{8}\left[x^2 - \frac{D^2}{8}(x^2) - \frac{6D(x^2)}{8} + \frac{36D^2(x^2)}{64}\right]$$

$$= \frac{1}{8}\left[x^2 - \frac{1}{8}(2) - \frac{6}{8}(2x) + \frac{36}{64}(2)\right]$$

$$= \frac{1}{8}\left[x^2 - \frac{1}{4} - \frac{3x}{2} + \frac{9}{8}\right]$$

$$= \frac{1}{8}\left[x^2 - \frac{3}{2}x + \frac{7}{8}\right] = \frac{x^2}{8} - \frac{3}{2}x + \frac{7}{8}$$

∴ The General solution is $y = C.F + P.I$

$$\therefore\ y = c_1 e^{-2x} + c_2 e^{-4x} + \cancel{\frac{x^2}{8} - \frac{3x}{8} + \frac{93}{64}}$$

$$+ \frac{x^2}{8} - \frac{3}{2}x + \frac{7}{8}$$

(3) Solve $(D^2+4)y = \sec 2x$ by the method of variation of parameters.

Sol:- Given equation is $(D^2+4)y = \sec 2x$

To find C.F, solve $(D^2+4)y = 0$

Auxiliary equation is $m^2+4=0$

$\Rightarrow m^2 = -4$

$\Rightarrow m = \pm 2i$

$\therefore$ C.F $= c_1 \cos 2x + c_2 \sin 2x$ $\quad (\because \alpha = 0, \beta = 2)$

To find P.I, $= u(x) y_1 + v(x) y_2$

Here $y_1 = \cos 2x$, $y_2 = \sin 2x$, $R(x) = \sec 2x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2\cos^2 2x + 2\sin^2 2x$$

$$= 2(\cos^2 2x + \sin^2 2x) = 2(1) = 2.$$

$\therefore \boxed{W = 2}$

Now $u(x) = -\int \frac{y_2 R(x)}{W} dx$

$$= -\int \frac{\sin 2x \cdot \sec 2x}{2} dx$$

$$= \frac{-1}{2} \int \frac{\sin 2x}{\cos 2x} dx = \frac{1}{2} \int \frac{-\sin 2x}{\cos 2x} dx$$

$$= \frac{1}{2}\left[\frac{1}{2} \log \cos 2x\right] = \frac{1}{4} \log \cos 2x$$

$$\boxed{u(x) = \frac{1}{4} \log \cos 2x}$$

$v(x) = \int \frac{R_1(x)\, y_1}{W} dx$

$$v(x) = \int \frac{\cos 2x \cdot \sec 2x}{2} dx = \frac{1}{2}\int \cos 2x \cdot \frac{1}{\cos 2x} dx$$

$$= \frac{1}{2}\int dx = \frac{x}{2}$$

$$\therefore \boxed{v(x) = \frac{x}{2}}$$

$\therefore$ P.I $= u(x) y_1 + v(x) y_2$

$$= \left(\frac{1}{4} \log \cos 2x.\right) \cos 2x + \left(\frac{x}{2}\right) \sin 2x$$

$\therefore$ The General Solution is

$$y = C.F + P.I$$

$$= c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4} \log(\cos 2x) \cos 2x$$

$$+ \frac{x}{2} \sin 2x.$$

(4) Solve $y'' + y = \sec x$ by the method of variation of parameters.

Sol :- Given $y'' + y = \sec x$

This same as $(D^2+1) y = \sec x$.

$\therefore$ A.E is $m^2 + 1 = 0 \Rightarrow m^2 = -1$

$\Rightarrow m = \pm i$

$\therefore$ C.F $= c_1 \cos x + c_2 \sin x$ (Here $\alpha = 0$ $\beta = 1$)

To find P.I $= u(x) y_1 + v(x) y_2$

Here $y_1 = \cos x$, $y_2 = \sin x$ & $R(x) = \sec x$

$y_1' = -\sin x$ $y_2' = \cos x$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x = 1$$

$\therefore$ $\boxed{W = 1}$

Now $u(x) = -\int \frac{y_2 R(x)}{W} dx$

$= -\int \frac{\sin x \sec x}{1} dx$

$= -\int \frac{\sin x}{\cos x} dx = \int \frac{-\sin x}{\cos x} dx$

$$\boxed{u(x) = \log \cos x}$$

$v(x) = \int \frac{y_1(x) R(x)}{W} dx = \int \frac{\cos x \cdot \sec x}{1} dx$

$= \int \cos x \cdot \frac{1}{\cos x} dx = \int dx = x$

$\therefore$ $\boxed{v(x) = x}$

$\therefore$ $P.I = u(x) y_1 + v(x) y_2$

$P.I = \log(\cos x) \cos x + x \sin x$

$\therefore$ The General solution $y = C.F + P.I.$

$y = c_1 \cos x + c_2 \sin x + \log(\cos x) \cos x$

$+ x \sin x.$

(5) Solve $(D^2+1) y = \operatorname{cosec} x$ by the method of variation parameters.

Sol:- Given $(D^2+1) y = \operatorname{cosec} x$

To find C.F, solve $(D^2+1) y = 0$

A.E is $m^2+1=0$ $\Rightarrow m^2 = -1$

$\Rightarrow m = \pm i$ $\left[\begin{array}{l}\alpha = 0 \\ \beta = 1\end{array}\right.$

$\therefore$ $\boxed{C.F = c_1 \cos x + c_2 \sin x}$

To find $P.I = u(x) y_1 + v(x) y_2$

Here $y_1 = \cos x$, $y_2 = \sin x$, & $R(x) = \operatorname{cosec} x$

$\therefore y_1' = -\sin x$ $\quad y_2' = \cos x$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x$$

$$= 1$$

$$\therefore \boxed{W = 1}$$

Now $\quad u(x) = -\int \frac{y_2(x) R(x)}{W} dx = -\int \frac{\sin x \operatorname{cosec} x}{1} dx$

$$= -\int \cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} dx = -\int dx = -x$$

$$\therefore \boxed{u(x) = -x}$$

$$v = \int \frac{y_1 R(x)}{W} dx = \int \frac{\cos x \cdot \operatorname{cosec} x}{1} dx$$

$$= \int \cos x \cdot \frac{1}{\sin x} dx = \int \frac{\cos x}{\sin x} dx$$

$$\boxed{v = \log \sin x}$$

$\int \frac{f'(x)}{f(x)} dx$

$= \log f(x)$

$\therefore$ P.I $= u(x)\, y_1 + v(x)\, y_2$

P.I $= -x(\cos x) + \log(\sin x) \sin x$

$\therefore$ The General solution is

$$y = C.F + P.I$$

$$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log(\sin x)$$

Unit - IV

Analytic function and complex integration :-

1) Show that the function $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and hence find the corresponding analytic function $f(z)$.

Sol :- Given $u(x,y) = 3x^2y + 2x^2 - y^3 - 2y^2$

$u_x(x,y) = 6xy + 4x$ | $u_y = 3x^2 - 3y^2 - 4y$

$u_{xx}(x,y) = 6y + 4$ | $u_{yy} = -6y - 4$

$\therefore u_{xx} + u_{yy} = 6y + 4 - 6y - 4 = 0$

$\therefore u$ is harmonic.

WKT $f'(z) = u_x(x,y) - i\, u_y(x,y)$

By Milne Thompson method,

Replace x by z and y by 0

$$f'(z) = u_x(z,0) - i\, u_y(z,0) \longrightarrow (1)$$

$u_x(x,y) = 6xy + 4x$ | $u_y(x,y) = 3x^2 - 3y^2 - 4y$

$\boxed{u_x(z,0) = 4z} \rightarrow (2)$ | $u_y(z,0) = 3z^2 - 0 - 0$

$\boxed{u_y(z,0) = 3z^2} \rightarrow (3)$

Substitute (2) & (3) in (1) we get

$$f'(z) = 4z - i(3z^2)$$

Integrating both sides w.r. to z, we get

$$\int f'(z)\,dz = \int 4z\,dz - i\int 3z^2\,dz$$

$$f(z) = \frac{4z^2}{2} - 3i\frac{z^3}{3} + c$$

$$\boxed{f(z) = 2z^2 - iz^3 + c}$$

(2) construct the analytic Function $f(z)=u+iv$ given $u-v = e^{x}(\cos y - \sin y)$.

Sol:- $f(z) = u+iv \longrightarrow ①$

$\therefore if(z) = iu - v \longrightarrow ②$

$① + ② \Rightarrow (1+i)f(z) = u-v+i(u+v) \longrightarrow ③$

put $U=u-v$, $V=u+v$, and $F(z)=(1+i)f(z)$ in ③ we get,

$F(z) = U+iV$

$\because f(z)$ is analytic, $F(z)$ is analytic

Given $U = u-v = e^{x}(\cos y - \sin y)$

$\therefore U_x = e^{x}(\cos y - \sin y)$

$U_y = e^{x}(-\sin y - \cos y)$

Replacing x by z and y by 0, we get

$U_x(z,0) = e^{z}(\cos 0 - \sin 0) = e^{z}$

$U_y(z,0) = e^{z}(-\sin 0 - \cos 0) = -e^{z}$

$\therefore$ By Milne-Thomson method

$F'(z) = U_x(z,0) - iU_y(z,0)$

$= e^{z} + ie^{z} = (1+i)e^{z}$

$\therefore F'(z) = (1+i)e^{z}$

Integrating, $F(z) = (1+i)\int e^{z}dz$

$F(z) = (1+i)e^{z} + c$

$\Rightarrow (1+i)f(z) = (1+i)e^{z} + c$

$\Rightarrow f(z) = e^{z} + c'$, where $c' = \dfrac{c}{1+i}$

③ Evaluate $\int_C \frac{dz}{(z+1)^2(z-2)}$, where C is the circle $|z| = \frac{3}{2}$, by using cauchy's integral formula.

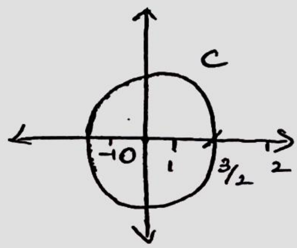
Sol:- Cauchy's integral formula for derivative is

$$\int_C \frac{f(z)}{(z-a)^2} = 2\pi i\, f'(a) \longrightarrow ①$$

Given $I = \int_C \frac{dz}{(z+1)^2(z-2)}$

The singular points are $z=-1$ and $z=2$.

C is the circle $|z| = \frac{3}{2}$, with centre $(0,0)$ and radius $= \frac{3}{2}$



If $z=-1$, then $|z| = |-1| = 1 < 3/2$

$\therefore z=-1$ lies inside C.

If $z=2$, then $|z| = |2| = 2 > 3/2$

$\therefore z=2$ lies outside C

Now $I = \int_C \frac{\frac{1}{z-2}}{(z+1)^2}\, dz$

Here $f(z) = \frac{1}{z-2}$ and $a = -1$

$f'(z) = \frac{-1}{(z-2)^2}$

$\therefore f'(a) = f'(-1) = -\frac{1}{(-1-2)^2} = -\frac{1}{9}$

$\therefore$ By ①, $I = 2\pi i\, f'(a)$

$= 2\pi i\, f'(-1)$

$= 2\pi i \left(-\frac{1}{9}\right)$

$$\boxed{I = \frac{-2\pi i}{9}}$$

④ Evaluate $\int_c \frac{z^2+1}{(z^2-1)} dz$, where c is the circle $|z-1|=1$, by using Cauchy's integral formula.

Sol:- Cauchy's integral formula is,

$$\int_c \frac{f(z)}{z-a} dz = 2\pi i \, f(a) \longrightarrow ①$$

Given $I = \int_c \frac{z^2+1}{z^2-1} dz = \int_c \frac{z^2+1}{(z+1)(z-1)} dz$

$\therefore z=-1$, $z=1$ are the singularities.

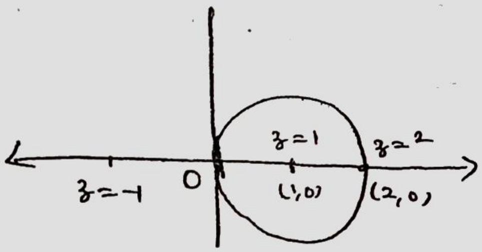
c is the circle $|z-1|=1$, with centre $(1,0)$ & radius $=1$

If $z=1$, $|z-1| = |1-1| = 0 < 1$, $z=1$ lies inside c.

If $z=-1 \Rightarrow |z-1| = |-1-1| = 2 > 1$, $z=-1$ lies outside c.

$$\therefore I = \int_c \frac{z^2+1}{(z-1)(z+1)} dz$$

$$= \int_c \frac{\frac{z^2+1}{z+1}}{z-1} dz$$



Here $f(z) = \frac{z^2+1}{z+1}$ is analytic inside and on c,

& $a=1$.

$$\therefore f(a) = f(1) = \frac{1+1}{1+1} = 1$$

$\therefore$ By ①, $I = 2\pi i \, f(a)$

$$= 2\pi i \, f(1)$$

$$= 2\pi i \, (1)$$

$$\boxed{I = 2\pi i}$$

Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed - to - be - University under section 3 of UGC Act 1956)

NAAC A

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY

Internal Assessment Test I, ODD Semester 2019

U18BSMA101/ U18BSMA102 – Engg Mathematics -1 / Mathematics -1 for Bio Engineer

Year/Sem : I / I — **Date: 04/09/2019**

Duration : 1 ½ Hour — **Max. Marks: 50**

	Part – A (6×2=12 Marks) Answer All Questions	CO Mapping	Pattern
1	State Cayley-Hamilton theorem.	C5	R
2	Find sum and product of the eigen values of the matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$	C5	E
3	If 1 and 2 are eigen values of a 2 x 2 matrix A, what are the eigen values of A^2 and A^{-1} ?.	C5	R
4	Write the matrix corresponding to the quadratic form $4x^2 + 2y^2 - 3z^2 + 2xy + 4zx$	C5	E
5	State any two properties of eigen values	C5	R
6	Find the characteristic equation of the matrix A= $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	C5	E
	Part – B (3×6=18 Marks) Any Three		
7	Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$	C5	R&E
8	Using Cayley-Hamilton theorem find the inverse of A= $\begin{matrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{matrix}$	C5	E
9	Discuss the nature of the quadratic form $6x^2 + 2y^2 + 3z^2 - 4xy + 8xz$.	C5	R&E
10	If A= $\begin{matrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{matrix}$ find the eigen values of A^4.	C5	U&E
	Part – C (2×10=20 Marks) Any Two		
11	Verify Cayley- Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^4.	C5	A&E
12	Diagonalise the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ by orthogonal transformation	C5	E
13	Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ into canonical form.	C5	E

Assessment Summary:

COs	Remember	Understand	Apply	Analyze	Evaluate	Create	Total
CO 5	5	1	1	-	11	-	18

Internal - I

~~Answer Key~~

4|9|2019

part - A

1) Every square matrix satisfies its own characteristic equation

2. Sum $= 1+3 = 4$
   Product $= 3 \cdot 8 = -5$

3. $A^2 = 1^2, 2^2$
   $A^{-1} = \frac{1}{1}, \frac{1}{2}$

4. $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

   $D_1 = 4$, $D_2 = 8-1 = 7$

   $D_3 = 4(6-2) - 1(3-4) + 2(1-4)$
   $= 16 + 1 - 6$
   $= 9$

   +ve. definite

5. Sum = diagonal
   Product = $|A|$

~~$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$~~

~~$\lambda^2 - S_1\lambda + S_2 = 0$~~

~~$\lambda^2 + 0\lambda - 4 = 0$~~

~~$\lambda^2 - 4 = 0$~~

~~$\lambda = \pm 2$~~

6. $A = \begin{pmatrix} 1 & 2 \\ 0 & +1 \end{pmatrix}$

   $\lambda^2 - S_1\lambda + S_2 = 0$

   $\lambda^2 - 2\lambda + 1 = 0$

7) Q.F is indefinite

10) $A^4 = 1, 1, 625$

~~proof~~

part - B

$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$

$\lambda = \pm 2$

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

part - C

12. $\lambda^3 - 18\lambda^2 + 45\lambda = 0$

    $\lambda = 0, 3, 15$

    $N = \frac{1}{2}\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$

    $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{pmatrix}$.

11. $A^3 - 6A^2 + 8A - 3I = 0$

    $A^4 = \begin{bmatrix} 129 & -123 & 122 \\ -95 & 96 & -93 \\ 95 & -95 & 127 \end{bmatrix}$

13. $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

    $\lambda^3 - 4\lambda^2 + 3\lambda = 0$

    $\lambda = 3, 1, 0$

    $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

    $0y_1^2 + y_2^2 + y_3^2 = 0$

Internal Test - I

Part - A

U19CS014

ADDALA BHANOJ

1. Given matrix, $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

The ch. eqn of Matrix A is $|A - \lambda I| = 0$.

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1-\lambda) - 0 = 0$$

$$1 - \lambda - \lambda + \lambda^2 = 0$$

$$\lambda^2 - 2\lambda + 1 = 0.$$

2

The ch. eqn of matrix A is $\lambda^2 - 2\lambda + 1 = 0$.

46/50

Part - B

7. Given matrix, $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$

The ch. eqn of matrix A is $|A - \lambda I| = 0$.

$$\begin{vmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1-\lambda) - 3 = 0$$

$$-1 - \lambda + \lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 4 = 0$$

$$\lambda = \pm 2.$$

The eigen values are $+2, -2$.

For eigen vectors.

$\lambda = +2.$

The eqn. for eigen vectors is $|A-\lambda I|X = 0.$

$$\begin{vmatrix} 1-2 & 3 \\ 1 & -1-2 \end{vmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$-x + 3y = 0 \rightarrow (1)$

$x - 3y = 0 \rightarrow (2)$

From eqn (1) $\Rightarrow$ $x = 3y$

$\frac{x}{3} = \frac{y}{1}$

6

$X_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(ii) $\lambda = -2$

$$\begin{vmatrix} 1+2 & 3 \\ 1 & -1+2 \end{vmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$3x + 3y = 0 \rightarrow (1)$

$x + y = 0 \rightarrow (2)$

From (1) $\Rightarrow$ $\frac{x}{1} = \frac{y}{1}$ $\qquad X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

Given matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

The ch. equ of given matrix, A is $|A - \lambda I| = 0$.

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0. \quad -(1)$$

$S_1 = 1+1+1 = 3$, $S_2 = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$

$= (1-0) + (1-0) + (1-0)$

$= 3.$

$S_3 = 1(1-0) - 2(0-0) + 3(0-0)$

$= 1 - 0 + 0 = 1.$

$(1) \Rightarrow \lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0.$

1	1	-3	3	-1
	↓	1	-2	1
	1	-2	1	0

$1\lambda^2 - 2\lambda + 1 = 0.$

$\lambda^2 - 1\lambda - 1\lambda + 1 = 0$

$\lambda(\lambda - 1) - 1(\lambda - 1) = 0.$

$\lambda = 1, 1.$

The eigen values are 1, 1, 1.

(19) Given matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

the ch. eqn is $|A - \lambda I| = 0$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$S_1 = 7$, $S_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$$= (6-2) + (4-1) + (6-2)$$

$$= 4 + 3 + 4 = 11$$

$$S_3 = 2(6-2) - 2(2-1) + 1(2-3)$$

$$= 8 - 2 - 1 = 5$$

$$\Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -7 & 11 & -5 \\ & & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & \boxed{0} \end{array}$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\lambda^2 - 1\lambda - 5\lambda + 5 = 0$$

$$\lambda(\lambda - 1) - 5(\lambda - 1) = 0$$

$$\lambda = 1, 5$$

Eigen values of A is 1, 1, 5.

Eigen values of A^4 is $1^4, 1^4, 5^4$.

Part - c

(11) Given matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The ch. eqn of A is $|A - \lambda I| = 0$.

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

$S_1 = 6$, $S_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= (4-1) + (4-2) + (4-1)$

$= 3 + 2 + 3 = 8$

$S_3 = 2(4-1) + 1(-2+1) + 2(+1-2)$

$= 6 - 1 - 2 = 3.$

$$\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0.$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & +8 & -3 \\ & & 1 & -5 & 3 \\ \hline & 1 & -5 & 3 & \boxed{0} \end{array}$$

$$\lambda^2 - 5\lambda + 3 = 0.$$

Prove $A^3 - 6A^2 + 8A - 3I = 0$.

$$A^2 = A \cdot A = \begin{bmatrix} \cancel{1} & \cancel{2} & \cancel{3} \\ \cancel{0} & \cancel{1} & \cancel{2} \\ \cancel{0} & \cancel{0} & \cancel{1} \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+6+9 & -7-12-9 & 14+6+18 \\ -10-6-6 & +5+12+6 & -10-6-12 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

$$A^3 - 6A^2 + 8A - 3I = 0$$

$$\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \lambda = 3.$

$$\begin{vmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$5x - 6y + 2z = 0 \quad —— (1)$

$-6x + 4y - 4z = 0 \quad —— (2)$

$2x - 4y + 3z = 0 \quad —— (3).$

(2) & (3)

$$\begin{array}{cccc} x & & y & & z \\ 4 & -4 & -6 & 4 \\ -4 & 3 & 2 & -4 \end{array}$$

$$\frac{x}{12-16} = \frac{y}{-8+18} = \frac{z}{+24-8}$$

$$\frac{x}{-4} = \frac{y}{10} = \frac{z}{16}$$

$$X_1 = \begin{bmatrix} -4 \\ 10 \\ 16 \end{bmatrix}.$$

ii) $\lambda = 15$

$$\begin{vmatrix} 8-15 & -6 & 2 \\ -6 & 7-15 & -4 \\ 2 & -4 & 3-15 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$-7x - 6y + 2z = 0$ — (4)

$-6x - 8y - 4z = 0$ — (5)

$2x - 4x - 12z = 0$ — (6)

(5) & (6)

$$\begin{array}{cccccc} & x & & y & & z \\ -8 & -4 & & -6 & -8 & \\ -4 & -12 & & 2 & -4 & \end{array}$$

$$\frac{x}{+96-16} = \frac{y}{-8-72} = \frac{z}{24+16}$$

$$\frac{x}{80} = \frac{y}{-80} = \frac{z}{40}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}.$$

4. Given $4x^2+2y^2-3z^2+2xy+4zx$.

$$\text{matrix } [A] = \begin{bmatrix} 4 & 2 & 4 \\ 2 & 2 & 0 \\ 4 & 0 & -3 \end{bmatrix}.$$

2

6. Given: $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Now $|A-\lambda I| = 0$

$$\left| \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

2

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(1-\lambda) - 0 = 0$

$1-\lambda-\lambda+\lambda^2 - 0 = 0$

$\lambda^2 - 2\lambda + 1 = 0$

1Q. Solution

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$a_1 = 18$

$a_2 = \begin{vmatrix} 7 & 4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$

$= |21-16| + |24-4| + |56-36|$

$= 5+20+20$

$= 45$

$a_3 = 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + 6\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$

$= 8|21-16| + 6|-18+8| + 2|24-14|$

$= 8(5) + 6(-10) + 2(10)$

$= 40-60+20$

$= 0$

$\therefore \lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$

$\lambda = 1$

$1 - 18\lambda^2 + 45\lambda - 0 = 0$

$\lambda = -1$

$1 + 18 + 45 \neq 0$

$\lambda = 2$

$8 - 18(4) + 45(2) \neq 0$

$8 - 72 + 90 \neq 0.$

$\lambda = 2$

$-8 - 72 - 90 \neq 0$

$\lambda = 3$

$27 - 18(9) + 45(3) \neq 0$

$27 - 162 + 135 = 0$

$\lambda = 3$

$$
\begin{array}{r|rrrr}
3 & 1 & -18 & 45 & 0 \\
  & 0 & 3 & -45 & \\
\hline
  & 1 & -15 & 0 &
\end{array}
$$

$\cancel{\lambda^2 - 15\lambda = 0}$

$\lambda^2 - 15\lambda = 0$

$\lambda(\lambda - 15) = 0$

$\lambda = 0, \ \lambda = 15$

To find eigen vectors

$\lambda = 0, 3, 15$

N. charan Re
U19CS142
CSE - B

1

Internal Assesment Test - I

Part - C

33/50

11. Sol

Given $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The characteristic equation is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$a_1 = 2+2+2 = 6.$

$a_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= 3 + 2 + 3$

$= 8$

$a_3 = |A| = 2(3) + 1(-1) + 2(-1)$

$= 6 - 1 - 2$

$= 3.$

$\therefore \lambda^3 - 6\lambda^2 + 8\lambda - 5 = 0.$

Now replace the λ with A is $A^3 - 6A^2 + 8A - 3 = 0$

$A^2 = A \cdot A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$

$= \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 14+6+9 & -7-12-9 & 14+6+18 \\ -10-6-6 & 5+12+6 & -10-6-12 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

Now $A^3 - 6A^2 + 8A - 3I$

$$\Rightarrow \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - \begin{bmatrix} 42 & -36 & 54 \\ -30 & 36 & -36 \\ 30 & -30 & 42 \end{bmatrix} + \cancel{\begin{bmatrix} 29 & -12 & 24 \\ -12 & 29 & 12 \\ 42 & -12 & 29 \end{bmatrix}}$$

$$\begin{bmatrix} 13 & -8 & 16 \\ -8 & 13 & -8 \\ 8 & -8 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow 0.$$

$\therefore \quad A^3 - 6A^2 + 8A - 3 = 0. \quad$ — ①

Now multiply the eq ① with A.

$$A^4 - 6A^3 + 8A^2 - 3A = 0$$

$$A^4 = 6A^3 - 8A^2 + 3A$$

$$= 6\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 8\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 3\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 174 & -168 & 228 \\ -132 & 138 & -168 \\ 132 & -132 & 174 \end{bmatrix} - \begin{bmatrix} 56 & -48 & 72 \\ -40 & 48 & -48 \\ 40 & -40 & 56 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{bmatrix}$$

$$\therefore A^4 = \begin{bmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{bmatrix}$$

10

Given matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The ch. eq is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$ 8×7

$a_1 = 8+7+3 = 18$

$a_2 = (21-16) + (24-4) + (56-36)$

$= 5 + 20 + 20$

$= 45$

$a_3 = 8(5) - 6(-18+8) + 2(24-14)$

$= 40 - 6(-10) + 2(10)$

$= 40+60+20$

$= 120$

$\therefore$ $\lambda^3 - 18\lambda^2 + 45\lambda - 120 = 0$, is the ch. eq.

Now the eigen values are.

13.

Sol

Given quadratic form $x_1^2+2x_2^2+x_3^2-2x_1x_2+2$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Now the ch. eq is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$a_1 = 1+2+1 = 4$

$a_2 = (2-1) + 1 + (2-1)$

$= 1+1+1$

$= 3$

$a_3 = 1 + 1(-1) + 0(-1)$

$= \cancel{1+1+1}$

$= 0$

$\therefore$ $\lambda^3 - 4\lambda^2 + 3\lambda = 0$. is the characteristic equ-

-tion.

Now the eigen values are

$1 - 4 + 3 -$

If $\lambda = 2 \Rightarrow 8 - 16 + 6$

$\Rightarrow 8 - 12 =$

$\Rightarrow -4 \neq 0$.

$\lambda = -2 \Rightarrow -8 - 16 - 9 - 2 \neq 0$.

If $\lambda = 3 \Rightarrow 0$.

2×2

Part-A

Cayley-Hamilton theorem:- Every matrix satisfies its own characteristic equation.

2

Given $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Sum of eigen values $= 4+2$

$= 6.$

product of eigen values $= |A|$

$= 8-3 = 5.$

2

Given $4x^2+2y^2-3z^2+2xy+4zx.$

matrix (A) $= \begin{bmatrix} 4 & 2 & 4 \\ 2 & 2 & 0 \\ 4 & 0 & -3 \end{bmatrix}$

Given $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

Now $|A-\lambda I| = 0.$

$\left|\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$

$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)(1-\lambda) - 0 = 0$

$1 - \lambda - \lambda + \lambda^2 - 0 = 0$

$\lambda^2 - 2\lambda + 1 = 0.$

$\therefore$ The characterstic equation is $\lambda^2 - 2\lambda + 1 = 0.$

Give

The

part - B

9. sop

Given $6x^2 + 2y^2 + 3z^2 - 4xy + 8xz.$

matrix (A) = $\begin{bmatrix} 6 & -4 & 8 \\ -4 & 2 & 0 \\ 8 & 0 & 3 \end{bmatrix}$

$D_1 = 6. > 0.$

$D_2 = 12 - 4 = 8 > 0$

$D_3 = 6(6) + 4(-12) + 8(-16)$

$= 36 - 48 - 128$

$= -12 - 128$

$= -140 < 0.$

$\therefore$ The nature of matrix is indefinite matrix.

-48
36
$12.$

12×4
48

$4,$
16×8
-128

128
12
140

$36.$
48
84

128
84
44

Sol.

Given $A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$

The charactristic equation is $\lambda^2 - a_1\lambda + a_2 = 0$

$a_1 = 1 - 1 = 0.$

$a_2 = (-1 - 3) = -4.$

$\therefore$ $\lambda^2 - 4 = 0$ is the charactristic equation.

Now $\lambda^2 - 4 = 0$

$\lambda^2 = 4$

$\lambda = \pm 2.$

3

$\therefore$ The eigen values are $+2, -2.$

Now the eigen vector $(A - \lambda I)x = 0.$

$\lambda = 2.$

$\Rightarrow \left[\begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$\begin{bmatrix} 1-2 & 3 \\ 1 & -1-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0.$

$\begin{bmatrix} -1 & 3 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0.$

$-x + 3y = 0.$

$x - 3y = 0.$

8.
Sol Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

The ch. eq is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$.

$a_1 = 3$

$a_2 = 1+1+1$

$= 3.$

$a_3 = 1(1) + 2(0) + 3(0)$

$= 1$

$\therefore$ $\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0.$

Now replace λ with A is. $A^3 - 3A^2 + 3A - 1 = 0.$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 3+4+3 \\ 0 & 1 & 2+2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

Now multiple ch. eq with A^{-1}

$$A^{-1}A^3 - 3A^2A^{-1} + 3A.A^{-1} - A^{-1} = 0.$$

$$A^2 - 3A + 3 - A^{-1} = 0$$

$$A^{-1} = A^2 - 3A + 3$$

$$= \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 10 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 9 \\ 0 & 3 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=$$

Given $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Now $A^2 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+2+1 & 4+6+2 & 2+2+2 \\ 2+3+1 & 2+9+2 & 1+3+2 \\ 2+2+2 & 2+6+4 & 2+2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 8 \end{bmatrix}$$

$A^4 = A^2 \cdot A^2$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 8 \end{bmatrix} \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 8 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 157 & 312 & 162 \\ 156 & 313 & 162 \\ 162 & 324 & 172 \end{bmatrix}$$

Part-A

3. Sol Given 1 & 2 are eigen values of A.

Now eigen values of A^2 is $(1)^2$ & $(2)^2$

$\Rightarrow$ 1 & 4.

eigen values of A^{-1} is $1/1$ & $1/2$

$= 1$ & $1/2$.

N. charan Reddy
U19CS142

Now

$$\begin{array}{c|cccc} & 1 & -4 & 3 & 0 \\ 3 & 0 & 3 & -3 & \\ \hline & 1 & -1 & 0 & \end{array}$$

$\lambda^2 - \lambda = 0.$

$\lambda(\lambda - 1) = 0$

$\lambda = 0, \lambda = 1$

$\therefore$ The eigen values are $\lambda = 3, 0, 1.$

The eigen vector $(A - \lambda I)x = 0$

$\underline{\lambda = 0.}$ $\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

5

$x - y + 0 = 0$

$-x + 2y + z = 0$

$\frac{x}{\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix}}$

$\frac{x}{-1} = \frac{-y}{1} = \frac{z}{1}$

$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$\lambda = 3.$

$(A - 3I)x = 0.$

$\begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = 0$

$-2x_3$

$\underline{\lambda = 1}$ $\Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$

$(0)x_2 - y_2 + (0)z_2 = 0$

$-x_2 + y_2 + z_2 = 0$

$\frac{x_2}{\begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}} = \frac{-y_2}{\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix}} = \frac{z_2}{\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix}}$

$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

CH Aniketh
CSE - M
U19CN062
Mathematics

24/09/19

Internal Assesment test - 3

Part - C

31/50

11 Given

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

We know that
The ch. eqn is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$a_1 = 2+2+2 = 6$

$a_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$= |4-1| + |4-2| + |4-1|$

$= 3+2+3$

$= 8$

$a_3 = |A|$

$= 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$

$= 2|4-1| + 1|-2+1| + 2|1-2|$

$= 2(3) + 1(-1) + 2(-1)$

$= 6-1-2$

$= 3.$

$\therefore$ The ch. eqn is.

$$\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$$

Cayley - Hamilton theorem

$$A^3 - 6A^2 + 8A - 3I = 0$$

$A^2 = A.A$

$$= \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+6+9 & -7-12-9 & 14+6+18 \\ -10-6-6 & +5+12+6 & -10-6-12 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

$$A^3 - 6A^2 + 8A - 3I = 0$$

$$\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - \begin{bmatrix} 42 & -36 & 54 \\ -30 & 36 & -36 \\ 30 & -30 & 42 \end{bmatrix} + \begin{bmatrix} 16 & -8 & 16 \\ -8 & 16 & -8 \\ 8 & -8 & 16 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 0$$

Rough

-45
-36
81
42
123

42
36
78
45
123

49
45
94
30
124

-42
36
45

63
63
126
36
162

$$=\begin{bmatrix} 29-42+16-3 & -28+36-8-0 & 38-54+16-0 \\ -22+30-8-0 & 23-36+16-3 & -28+36-8-0 \\ 22+30+8-0 & -22+30-8-0 & 29-42+16-3 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Verified

To find A^4

$$A^4 = A^2 . A^2$$

$$=\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 49+30+45 & -42-36-45 & 63+36+63 \\ -35-30-30 & +30+36+30 & -45-36-42 \\ 35+25+35 & -30-30-35 & 45+30+49 \end{bmatrix}$$

$$=\begin{bmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{bmatrix}$$

12 $\lambda = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$a_1 = 18$

$a_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$

$= |21-16| + |24-4| + |56-36|$

$= 5 + 20 + 20$

$= 45$

$a_3 = 8\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + 6\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}$

$= 8|21-16| + 6|-18+8| + 2|+24+14|$

$= 8(5) + 6(-10) + 2(10)$

$= 40 - 60 + 20$

$= 0.$

$\therefore \lambda^3 - 18\lambda^2 + 45\lambda - 0 = 0$

$\lambda = 1$

$1 - 18(1) + 45 = 0$

$\lambda = -1$

$1 + 18 + 45 \neq 0$

$\lambda = 2$

$8 - 18(4) + 45(2) \neq 0$

$8 - 72 + 90 \neq 0$

$\lambda = -2$

$-8 - 72 - 90 \neq 0$

$\begin{array}{r} {}^{3} \\ 18 \\ \times 4 \\ \hline 72 \end{array}$

for $\lambda = 3$

$27 - 18(9) + 45(3) \neq 0$

$27 - 162 + 135 = 0$

$\lambda = 3$

3	1	-18	45	-0
	0	3	-45	
	1	-15	0	

$\lambda^2 - 15\lambda = 0$

$\cancel{\lambda^2 = 15}$

$\cancel{\lambda = \pm\sqrt{15}}$

$\lambda^2 - 15\lambda = 0$

$\lambda(\lambda - 15) = 0$

$\lambda = 0, \lambda = 15$

To find eigen vectors

$\lambda = \cancel{3,0,15}.\ 0, 3, 15$

$\lambda = 3$

$[A - \lambda I](x_1) = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{24-8} = \frac{y_1}{-12+20} = \frac{z}{20-36}$$

$18 \times 9 = 162$

$45 \times 3 = 135$

162

$\lambda^2 - 15\lambda + 0 = 0$

$\lambda^2 - 15\lambda = 0$

$\lambda^2 = 15\lambda$

$\lambda(\lambda - 15)$

$\lambda = 0$

$5x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 4x_2 - 4x_3 = 0$

$2x_1 - 4x_2 + 0x_3 = 0$

x_1	y_1	1	
-6	2	5	-6
4	-4	-6	4

$$\frac{x_1}{24-8} = \frac{x_2}{-12+20} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{+8} = \frac{x_3}{-16}$$

$$x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$\underline{\lambda = 15}$

$$\left(\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -13 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-7x_1 - 6x_2 + 2x_3 = 0$$
$$-6x_1 - 8x_2 + 4x_3 = 0$$
$$2x_1 - 4x_2 - 13x_3 = 0$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 2 & -7 & -6 \\ -8 & 4 & -6 & -8 \end{array}$$

$$x_3 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$\frac{x_1}{-24+16} = \frac{x_2}{-12+28} = \frac{x_3}{+56-36}$$

$$\frac{x_1}{-8} = \frac{x_2}{16} = \frac{x_3}{16}$$

$\lambda = 0$

$$\begin{pmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{pmatrix} - 0\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = 0$$

$$\begin{pmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{pmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 0$$

$8x_1 - 6x_2 + 2x_3 = 0$

$-6x_1 + 7x_2 - 4x_3 = 0$

$2x_1 - 4x_2 + 3x_3$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{array}$$

$$\frac{x_1}{24-14} = \frac{x_2}{-12+32} = \frac{x_3}{56-36}$$

$$\frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20}$$

$$x_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$$

Normalized ..

$$N = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

$$n_1 = \begin{bmatrix} \frac{1}{\sqrt{1+4+4}} \\ \frac{2}{\sqrt{1+4+4}} \\ \frac{2}{\sqrt{1+4+4}} \end{bmatrix} \quad n_2 = \begin{bmatrix} \frac{2}{\sqrt{1+4+4}} \\ \frac{1}{\sqrt{1+4+4}} \\ \frac{-2}{\sqrt{1+4+4}} \end{bmatrix} \quad n_3 = \begin{bmatrix} \frac{-1}{\sqrt{1+4+4}} \\ \frac{2}{\sqrt{1+4+4}} \\ \frac{2}{\sqrt{1+4+4}} \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{3} & 2/3 & -1/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & 2/3 \end{bmatrix}$$

$$N^T = \begin{bmatrix} 1/3 & 2/3 & -1/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & -2/3 \end{bmatrix}$$

we have to find

$\cancel{NA^TN}$ N^TAN

$$= \begin{bmatrix} 1/3 & 2/3 & -1/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & -1/3 \\ 2/3 & 1/3 & 2/3 \\ 2/3 & -2/3 & 2/3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 8-12-2 & -6+14-4 & 2-8-3 \\ 16-6+4 & -12+7+8 & 4-4+6 \\ 16+12+4 & -12-14+8 & 4+8-6 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -6 & 4 & -9 \\ 14 & -17 & 6 \\ 32 & -32 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

8

16
12
4
32

$$= \frac{1}{3} \begin{bmatrix} -6+8-18 & -12+4+18 & +6+8-18 \\ 14-24+12 & 28-12-12 & -14-24+12 \\ 32+74-12 & 72-32-12 & -32-72-12 \end{bmatrix}$$

14
12
26
8

72
32
12
116

$$= \frac{1}{3} \begin{bmatrix} -16 & +10 & 16 \\ 2 & +4 & -26 \\ -44 & -44 & -116 \end{bmatrix}$$

10. $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$a_1 = 2+3+2$

$a_1 = 7$

$a_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= (6-2) + (4-1) + (6-2)$

$= 4+3+4$

$= 11$

$a_3 = 2\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$

$= 2(6-2) - 2(2-1) + 1(2-3)$

$= 2(4) - 2(+1) + 1(-1)$

$= 8 - 2 - 1$

$= 5.$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

Cayley Hamilton

$$A^3 - 7A^2 + 11A - 5I = 0$$

$A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+1 & 4+6+2 & 2+2+2 \\ 2+3+1 & 2+9+2 & 1+3+2 \\ 2+2+2 & 2+6+4 & 1+2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 5 \end{bmatrix}$$

$A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 5 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12+6 & 14+48+12 & 7+12+12 \\ 12+13+6 & 12+39+12 & 6+13+12 \\ 12+12+5 & 12+48+10 & 6+12+10 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 74 & 41 \\ 31 & 63 & 31 \\ 29 & 70 & 28 \end{bmatrix}$$

$A^3 - 7A^2 + 11A - 5I = 0$

$$\begin{pmatrix} 32 & 74 & 41 \\ 31 & 63 & 31 \\ 29 & 70 & 28 \end{pmatrix} - 7\begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 5 \end{bmatrix} + 11\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3

Verified

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ 13\times \\ \hline 156 \end{array}$$

$A^4 = A^2 \cdot A^2$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 5 \end{bmatrix} \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 5 \end{bmatrix}$$

$$\begin{array}{r} 12 \\ \times 6 \\ \hline 2 \end{array}$$

$$= \begin{bmatrix} 49 + \cancel{72} + 36 & 84 + 156 + 84 & 42 + 84 + 30 \\ 42 + 78 + 36 & 72 + 169 + 72 & 36 + 84 + 30 \\ 42 + 72 + 30 & 72 + 156 + 60 & 36 + 72 + 25 \end{bmatrix}$$

$$\begin{array}{r} 12 \\ \times 7 \\ \hline 84 \end{array}$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$(A - \lambda I) = 0$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(1-\lambda) = 0$$

$$1 - \lambda - \lambda + \lambda^2 = 0$$

$$\lambda^2 - 2\lambda + 1 = 0$$

$$6x^2 + 2y^2 + 3z^2 - 4xy + 8xz$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & 6 & -2 & 4 \\ y & -2 & 2 & 0 \\ z & 4 & 0 & 3 \end{array}$$

$$4x^2 + 2y^2 - 3z^2 + 2xy + 4zx = 0$$

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & 4 & 1 & 2 \\ y & 1 & 2 & 0 \\ z & 2 & 0 & -3 \end{array}$$

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & -3 \end{bmatrix}$$

$$=\begin{pmatrix} 49+72+36 & 84+156+84 & 42+84+70 \\ 42+78+36 & 72+169+72 & 36+78+70 \\ 42+72+30 & 72+156+60 & 36+72+25 \end{pmatrix}$$

(2) $A = \begin{bmatrix} \cancel{4} & 1 \\ 3 & 2 \end{bmatrix}$

Sum of eigen value

$\cancel{4+2+1+1(3-1)}$

(5). $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

Sum of eigen value of A = eigen value of A^T

$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$3+2=5$ $\quad = 3+2$

$= 5.$

~~Sum of the diagonal of matrix A~~

The product of matrin A is equal to matrin det A

$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ $\quad A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

product $3 \times 2 = 6$ $\quad$ det $A = |3 \times 2|$

$= 6.$

(a) $N \cdot \{$

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix}$$

$a_1 = 1 - 1$

$a_1 = 0$

$a_2 = -1 - 3$

$a_2 = -4$

$\lambda^2 - 0\lambda - 4 = 0$

$\lambda^2 - 4 = 0$

$\lambda = 2, 2$

$(A - \lambda I)(x_1) = 0$

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} x_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 + 3x_2 = 0$

$x_1 - 3x_2 = 0$

Bharath

INSTITUTE OF HIGHER EDUCATION AND RESEARCH

(Declared as Deemed - to - be - University under section 3 of UGC Act 1956)

NAAC A

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY

Internal Assessment Test II, ODD Semester 2019

U18BSMA101 Engg. Mathematics I & U18BSMA101 Mathematics -1 for Bio Engineering

Year/Sem : I / I — Date: 15/10/2019

Duration : 1 ½ Hour — Max. Marks: 50

	Part – A (6×2=12 Marks) Answer All Questions	CO Mapping	Pa rn
1	Evaluate $\lim_{x\to 2}\frac{x^2+x-6}{x-2}$	CO1	
2	Find $\frac{dy}{dx}$ $if\ y = \mathrm{Sin}\,(\sqrt{x})\ \mathrm{Cos}(x^3)$	CO1	
3	Find $\frac{dy}{dx}$ $if\ y = \cos^{-1}(\frac{1-x^2}{1+x^2})$	CO1	
4	Find $\frac{dy}{dx}$ $if\ x = a\cos\theta,\ y = a\cos\theta$	CO1	
5	State Lagrange's Mean Value Theorem	CO1	
6	Find the turning points of $f(x) = x^4 - 3x^3 + 3x^2 - x$	CO1	
	Part – B (3×6=18 Marks) Answer Any Three Questions		
7	Find $\frac{dy}{dx}$ $if\ x = a(\theta + \sin\theta)\ ,\ y = a(1-\cos\theta)$	CO1	
8	Find $\frac{dy}{dx}$ $if\ x^m y^n = (x+y)^{m+n}$	CO1	
9	Obtain Maclaurin's Series expansion of e^{2x}	CO1	
10	Verify Rolle's Theorem for the function $f(x) = \sin x\ \ 0 \le x \le \pi$	CO1	
	Part – C (2×10=20 Marks)) Answer Any Two Questions		
11	Differentiate $\tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$ with respect to $\tan^{-1}(\frac{2x}{1-x^2})$	CO1	
12	Find the maximum and minimum values of the function $f(x) = 2x^3 - 3x^2 - 36x + 10$	CO1	
13	Obtain Taylor's series expansion of $f(x) = \cos x$ at $(i)\ x = \frac{\pi}{3}\ (ii)\ x = 0$	CO1	

Assessment Summary:

COs	Remember	Understand	Apply	Analyze	Evaluate	Create
CO 1	12	4	8	6	26	10

1

Eng. Maths - I 15/10/2019

IA - 2

Answer key

PART-A

1. $\lim_{x\to 2} \frac{4+2-6}{2-2}$

$\lim_{x\to 2} \frac{0}{0}$

Apply L'Hospital Rule

$\lim_{x\to 2} \frac{2x+1}{1} = 5$

2. $\frac{dy}{dx} = \sin(\sqrt{x})\ 3\cos x^2\ (\sin x^3) + \cos(x^3)\ \cos(\sqrt{x})\ \frac{1}{2\sqrt{x}}$

3. $x = \tan\theta$

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta)$

$\frac{dy}{dx} = \frac{1}{1+x^2}$

4. $\frac{dy}{dx} = \frac{-a\cancel{\sin\theta}}{-a\cancel{\sin\theta}} = +1$

5. Suppose $f(x)$ is a fn that satisfies given below

i) $f(x)$ is continuous $[a,b]$

ii) $f(x)$ is differentiable (a,b)

iii) then there is number c such $a<c<b$

$f'(c) = \frac{f(b)-f(a)}{b-a}$

6. $f'(x) = 4x^3 - 9x^2 + 6x - 1$

$$\begin{array}{c|cccc} & 4 & -9 & 6 & -1 \\ 1 & 0 & 4 & -5 & 1 \\ \hline & 4 & -5 & 1 & \boxed{0} \end{array}$$

$4x^2 - 5x + 1 = 0$

$x = 1, 1, 1/4$

PART-B

7. $\frac{dx}{d\theta} = a(1+\cos\theta)$ $\quad \frac{dy}{d\theta} = a(0 + \sin\theta)$

$$\frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)}$$

$$= \frac{\cancel{2}\sin\theta/2 \, \cancel{\cos\theta/2}}{\cancel{2}\cos^2\theta/2} = \tan\theta/2$$

8. $x^m y^n = (x+y)^{m+n}$

Take log both side

$\log x^m y^n = \log(x+y)^{m+n}$

$m\log x + n\log y = (m+n)\log(x+y)$

Diff y w.r.to x

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{y}{x}$$

9. $f(x) = y = e^{2x}$ $\quad f(0) = y(0) = 0$

$f'(x) = 2e^{2x}$ $\quad f'(0) = 2$

10. $f(x) = \sin x \qquad 0 \le x \le \pi$

Step: 1 $f(x)$ is continuous $[0, \pi]$ &

differentiable in $(0, \pi)$

$f(0) = 0$

$f(\pi) = 0$

Rollo's thm $\quad f'(c) = 0$

$\cos c = 0$

$c = \pi/2$

$\boxed{\cos \pi/2 = 0}$

PART - C

11. $\tan^{-1}\left(\frac{\sqrt{-x^2} - 1}{x}\right)$ w. r to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$u = \tan^{-1}\left(\frac{\sqrt{1-x^2-1}}{x}\right)$

$x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$u = \tan^{-1}\left(\frac{\sqrt{1-\tan^2\theta} - 1}{x}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}(\tan \theta/2)$

$\frac{du}{dx} = \frac{1}{2(1+x^2)}$

$v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$v = \tan^{-1}\left(\frac{2\tan x}{1-\tan^2 x}\right)$

$= \tan^{-1}(\tan 2\theta)$

$= \frac{2}{1+x^2}$

$\boxed{\frac{du}{dv} = \frac{1}{4}}$

12. $f'(x) = 6x^2 - 6x - 36$

$f''(x) = 12x - 6$

$f'(x) = 0 \Rightarrow 6(x^2 - x - 6) = 0$

$x = 3, -2$

$f(3) = 54 - 27 - 108 + 10$

$= -71$

$f(-2) = -16 - 6 + 72 + 10$

$= +56$

13. $f(x) = \cos x$ $\quad f(0) = 1$

$f'(x) = -\sin x$ $\quad f'(0) = 0$

$f''(x) = -\cos x$ $\quad f''(0) = 1$

$$f(x) = 1 + \frac{x}{1!}(0) + \frac{x^2}{2!}(-1) + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4} + \cdots$$

15/10/2019 U19CS024

Mathematics - I Afkhan Nawaz Khan.

Internal Assesment Test - II

Part - A

26/50

1. $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

$= \lim_{x\to 2} \frac{(2)^2+2-6}{2-2}$

$= \frac{4-4}{2-2}$

$= 0$

2. $y = \sin(\sqrt{x}) \cos(x^3)$

$u = \sin\theta$

$v = (\sqrt{x} \cos x^3)$

$u' = \cos\theta.$

$v' = \sqrt{x} \frac{d}{dx} \cos(x^3) + \cos(x^3) \frac{d}{dx} \sqrt{x}$

$v' = \sqrt{x} \cdot d_1(-\sin x^3) \cdot 3x^2 + \cos(x^3) \cdot \frac{1}{2\sqrt{x}}.$

$= -3\sqrt{x} \cdot x^3 \cdot \sin x^3 + \frac{\cos x^3}{2\sqrt{x}}$

2

$\frac{dy}{dx} = u'v'$

$= \cos\left[-3\sqrt{x}\, x^2 \cdot \sin x^3 + \frac{\cos x^3}{2\sqrt{x}}\right]$

3. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

put $x = \tan\theta$

$\theta = \tan^{-1}(x)$

$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \cdots \left[\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)\right]$

$= \cos 2\theta$

$y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}x \ldots (\because \theta = \tan^{-1}x)$

$y = 2\tan^{-1}\theta$

$= 2 \frac{1}{1-x^2}$

2 $= \frac{2}{1-x^2}$

A. $x = a\cos\theta$

$\frac{dx}{d\theta} = -a\sin\theta$

$y = a\sin\theta$

$\frac{dy}{d\theta} = a\cos\theta$

$\frac{dy}{dx} = \frac{a\cos\theta}{a\sin\theta}$

2

$\frac{dy}{dx} = -\cot\theta$

PART - B

7.

$$x = a(\theta + \sin\theta)$$

$$y = a(1 - \cos\theta)$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta} a(\theta + \sin\theta)$$

$$= \frac{d}{d\theta} a\theta + a\sin\theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta} a - a\cos\theta$$

4

$$= a \sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a + a\cos\theta}$$

$$\frac{dy}{dx} = \frac{\sin\theta}{(1+\cos\theta)}$$

U19CS029

Akanksha Parvathaneni

Internal Assement Test - II

37/50

11) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Let $x = \tan\theta$

$\theta = \tan^{-1}(x)$

$\tan^{-1}\left(\frac{\sqrt{\sec^2\theta-1}}{\tan\theta}\right)$ ($\because$ Let $\tan^2\theta = \sec^2\theta$)

$\tan^{-1}\left(\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\theta/2}{2\sin\theta\cos2\theta/2}\right)$

$= \tan^{-1}\left(\frac{\cancel{2}\sin^2\theta/2}{\cancel{2}\sin\theta\cos2\theta/2}\right)$

$= \tan^{-1}\left(\tan\theta/2\right)$

$= \theta/2$

$= \frac{\operatorname{Tan}^{-1} x}{2}$

$= \frac{1}{2}\left(\frac{1}{1+x^2}\right)$

$= \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Let

$x = \tan\theta$

$\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$= 2$

$= 2\tan^{-1}x$

d. w.r.t with respect to x

$= 2\left(\frac{1}{1+x^2}\right) \rightarrow (2)$

$\frac{\frac{1/2}{2} \cdot \frac{1}{1+x^2}}{1+x^2} = \frac{1}{4}$ //

8

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(12) $f(x) = 2x^3 - 3x^2 - 36x + 10 \Rightarrow$ d. w. r. to x.

$F'(x) = 6x^2 - 6x - 36$

$F'(x) = x^2 - x - 6$

$= x^2 - 3x + 2x - 6$

$= x(x-3) + 2(x-3)$

$F(x) = (x-3)(x+2)$

$x = 3, 2$

$f''(x) = 12x - 6$

$f''(-2) = 12(-2) - 6$

$= 24 - 6$

$= -30$. maximum value of the function

$f''(3) = 12(3) - 6$

$= 36 - 6$

$= 30 > 0$ minimum value of the function

$f(x) = 2x^2 = 2x^3 - 3x^2 - 36x + 10$

$f(x) = 2(-2)^2 = 2(-2)^3 - 3(-2)^2 - 36(-2) + 10$

$= -16 - 12 + 72 + 10 = 54$ $f''(x) > 0$ it is minimum

$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 10$

$= 2(27) - 3(9) - 36(3) + 10$

$= 54 - 27 - 108 + 10$

$= -71$ $f''(x) < 0$ it is maximium

$f(-2)$ is minimum of 54

$f(3)$ is maximum of -71,

part - B

(7A) $\frac{dy}{dx} = x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

$x = a(\theta + \sin\theta)$

$\frac{dx}{d\theta} = a + \cos\theta$

$y = a(1 - \cos\theta)$

$y = \cancel{a\theta}\ a - \cos 2\theta$

d. w. r. t. θ

$\frac{dy}{d\theta} = (1 - \sin\theta)$

$= 2\sin\theta - \theta$

By (1) and (2)

$$\frac{\frac{dx}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a - \cos 2\theta}$$

$$\frac{dy}{dx} = \frac{a\sin\theta}{a + \cos 2\theta}$$

U19CB066

CSE(F) $\frac{8}{10}$

(8) $\frac{dy}{dx}$ if $x^m y^n = (x+y)^{m+n}$

Apply log on both sides

$\log(x^m y^n) = \log(x+y)^{m+n}$

$\log x^m + \log y^n = (m+n)\log(x+y)$

$m\log x + n\log y = (m+n)\log(x+y)$

d w.t.r.x

$m\,{}^1/_x + n\left(\frac{1}{y}\right)\frac{dy}{dx} + \cancel{n(\ )} = (m+n)\frac{1}{x+y}\frac{dy}{dx}$

$\frac{m}{x} + \frac{2}{3}\left(\frac{dy}{dx}\right) = \frac{m+n}{x+y}\left(\frac{dy}{dx}\right)$

$\frac{x}{y}\left(\frac{dy}{dx}\right)\left(\frac{m+n}{dx}\right) \frac{m+n}{x+y}\left(\frac{dy}{dx}\right) = \frac{m}{x}$

$\frac{dy}{dx}\left[\frac{x}{y} - \frac{m+n}{x+y}\right] = \frac{-m}{n}$

$\frac{dy}{dx} = \frac{-m}{n}\left[\frac{y}{n} - \frac{x+y}{m+n}\right]$

$= \frac{-m3}{nx} + \frac{m(x+y)}{n(x+n)}$

$= \frac{-my\,[x(m+n) + x(x+y)]\,nx}{n^2(m+n}$

$$\frac{m^2xy - mny + mnx^2 + mnxy}{mnx^2 + n^2x^2}$$

4

$$= \frac{y}{x}$$ //

(7) maclaurin's series expansion of e^{2x}

$$= f(x) = f'(x^2)^n + \frac{f''(x^2)}{2!} + \frac{f''(x^3)}{3!} \ldots\ldots + f^n(x^n)$$

$f(x) = e^{2x}$

$f'(x) = e^{2x}$

$f''(x) = 4e^2$

$f'''(x) = 8e^2$

$f^{iv}(x) = 16e^{2x}$

$= 2^x + 2e^{2x}(x)x$

4

$$= e^{2x} + \frac{2e^{2x}(x)(12)}{1!} + \frac{4e^{2x}}{2!}(4) + \frac{8e^{2x}}{3!}(8)^2 + \ldots$$

$$= e^{2x} + 2^x + \frac{2e^{2x}(x)(12)}{1!} + \frac{4e^{2x}}{2!}(4) + \frac{8e^{2x}}{3!}(8)^2 \ldots$$

$$= a^{2x}[1 + 4 + 38 + \ldots]$$

$\frac{cs}{15/10}$

Part - A

(4) $x = a\cos\theta, \quad y = a\cos\theta$

$$\frac{dx}{d\theta} = \cos\theta \cdot 0 + (\sin\theta) \cdot a \quad - (1)$$

d.w.r.to.r.x

$$\frac{dy}{d\theta} = \cos\theta + (-\sin\theta) \cdot a \rightarrow (2)$$

$$\frac{\frac{dx}{d\theta}}{\frac{dy}{d\theta}} = \frac{a\cancel{s}in\theta}{\cancel{-}a\sin\theta} = -1 \,//$$

(5) Lagrange's Mean value Theorem is then

Let the function F is to be continoues.

on (a,b) are different Lagrange's.

Mean value Theorem.

$$= \frac{F(b) - E(a)}{b - a}$$

(1) $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

$\lim_{x \to 2} \frac{x^2-2x+3x-6}{x-2}$

$\lim_{x \to 2} x(x-2)+3(x-2)$

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$

$\lim_{x \to 2} x+3$

$\Rightarrow 2+3=5$

(2) $\frac{dy}{dx}$ if $y = \sin(\sqrt{x}) \cos(x^3)$

d.w.to.r.x

$\frac{dy}{dx} = \cos x^3 = \cos\sqrt{x}\frac{1}{2x} + \sin(\sqrt{x}) - \sin(x^3) - 3x^2$

$= \frac{1}{2x} \cos x^3 \cos\sqrt{x} - \sin^2(\sqrt{x}) \cdot \sin(x^3)$

$= \frac{1}{2x} \cos x \Rightarrow \sqrt{x} - 3x^2 \ldots$

CS
15/10

(3A) $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Let $x = \tan\theta \Rightarrow \cos(\tan^{-1}(x))$

$y = \cos^{-1}\left(\frac{1-\tan 2\theta}{1+\tan 2\theta}\right)$

$y = \cos^{-1}(\cos 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}(x)$

$\frac{dy}{dx} = y\left(\frac{1}{1+x^2}\right)$

(6A)

$f(x) = x^4 - 3x^3 + 3x^2 - x$

$x^4 - 3x^3 + 3x^2 - x$

$f'(x) = 4x^3 - 6/x$

$f'(x$ $4x^3 - 9x^2 + 6x - 1$ — (1)

$f''(x) = 12x^2 + 18 + 6$

$12x^2 + 18$

$2x^2 - 2x - x - 4 = 0$ $(x-1)(4x-1) = x = 1; x = 1/4$

$x = 1, 1/4$

(17)

801

(6) ~~fdda~~

(b)

$$\begin{array}{r|rrrr} 1 & 4 & -9 & 6 & -1 \\ & 0 & 4 & -5 & 1 \\ \hline & 4 & -5 & 0 & \lfloor 0 \end{array}$$

$4x^2 - 4x - x + 1 = 0$

$4x(x-1) - 1(x-1) = 0$

$x = 1,\ x = 1/4$

U19CS022

Adla pavan Reddy

Internal Assessment Test - II.

12. Sol:

$f(x) = 2x^3 - 3x^2 - 36x + 10 \rightarrow (1)$

Now

Differentiate eqn (1)

$f'(x) = 6x^2 - 6x - 36$

$f''(x) = 12x - 6 \rightarrow (11)$

45/50

$x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x+2)(x-3) = 0$

$x = -2$ (or) $x = 3$.

$x = -2, 3$.

$f''(x) = 12x - 6$

$= 12(-2) - 6$

$= -24 - 6$

$f''(x) = -30 < $ maximum value is gives.

$f(x) = -2$

$\Rightarrow f(x) = 2x^3 - 3x^2 - 36x + 10$

$f(-2) = 2(-2)^3 - 3(-2) - 36(-2) + 10$

$= -16 - 8 - 12 + 72 + 10$

$= 82 - 28$

$f(-2) = 54$.

Sub $x = 3$ in eqn (ii)

$f''(x) = 12x - 6$

$= 12(3) - 6$

$= 36 - 6$

$f''(x) = 3$ minimum value

$f(3) = 2(3)^3 - 3(3)^2 - 36(3) + 10$

$= 2(27) - 3(9) - 108 + 10$

$= 54 - 27 - 108 + 10$

$= 27 - 108 + 10$

$f(3) = \underline{-71}$

10

$$(\cos x, \pi/3) = f(\pi/3) + \frac{(x-\pi/3)\, f'(\pi/3)}{1!} + \frac{(x-\pi/3)^2 f''(\pi/3)}{2!} + \cdots$$

$$\frac{(x-\pi/3)^3 f'''(\pi/3)}{3!} + \cdots$$

$$= \frac{1}{2} + \frac{(\cancel{x/3}\ x-\pi/3) \times -\sqrt{3}/2}{1} + \frac{(x-\pi/3)^2 \times -\frac{1}{2}}{1\times 2} + \cdots$$

$$+ \frac{(x-\pi/3)^3 \frac{\sqrt{3}}{2}}{1\times 2\times 3} + \cdots$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}(x-\pi/3) \cancel{+}\, -\frac{1}{2} \times \frac{1}{2}(x-\pi/3)^2 + (x-2\pi/3)^3 \times \frac{\sqrt{3}}{2} \times \frac{1}{6}$$

$$+ \cdots$$

$$= \frac{\sqrt{3}}{12}\left(x^3 - \frac{\pi^3}{27} - \cancel{3} \times x \times \pi/3\ (x-\pi/3) + \cdots\right.$$

$$= \frac{\sqrt{3}}{12} \cdot x^2\pi + \frac{\sqrt{3}}{36} x\pi^2$$

$$= \frac{1}{2} - \frac{\pi^2}{36}\frac{\sqrt{3}}{3} + \frac{\sqrt{3}\pi^3}{3} - \frac{\sqrt{3}}{2}x + \frac{\pi}{6}x + \frac{\sqrt{3}}{36}x^2 x - \frac{x^2}{4} - \frac{\sqrt{3}}{12}x^3$$

$$+ \frac{\sqrt{3}}{12}x^3$$

$$= \left(\frac{1}{2} - \frac{\pi^2}{36}\right)\frac{\sqrt{3}\pi}{3}$$

=

$(f(x)), c=0 \qquad c=0.$

$f(x) = \cos x$

$f(0) = \cos 0$

$f(0) = 1.$

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f'(0) = -\sin 0 = 0$

$f''(0) = -\cos 0 = -1.$

$f'''(0) = \sin 0 = 1$

$\frac{C_1}{15}$ | 10.

(10) Sol

Given funtion $f(x) = \sin x$ $(0 \le x \le \pi)$.

f(x) is differential on $(0, \pi)$

Since, we know every trigonometry funtions
~~continual and differential on~~ function

$\therefore$ sin x is a contious function

① f(x) is continue function is [0, π)

② f(x) is differential funtion in (0, π)

③ $f(0) = \sin 0 = 0$

$f(\pi) = \sin \pi = 0$

$\therefore f(0) = f(\pi)$

it satisfied all the condition of Rolle's.

then

$f'(x) = \cos x$

Now, $f'(x) = 0$.

$\cos x = 0$

$x = (2k+1)\pi/2$ $n \in R$

$x = (2 \times 0 + 1)\pi/2$

$x = \pi/2$, $\in [0, \pi]$

So, $x = \pi/2$ is the critical Point

hence, ~~sin x, verified~~ ~~$0 \le x \le \pi$,~~

6

7) Sol

Given $x = a(\theta + \sin\theta) \rightarrow ①$

$y = a(1-\cos\theta) \rightarrow ②$

diff eq ① w.r.t θ.

$$\frac{dx}{d\theta} = \frac{d(a\theta + a\sin\theta)}{d\theta}$$

$$\frac{dx}{d\theta} = a + a\cos\theta \quad — ③$$

differentiate ② w.r.t θ.

$$\frac{dy}{d\theta} = \frac{d(a-\cos\theta)}{d\theta}$$

$$= 0 - a(-\sin\theta) — (iv)$$

$$\frac{dy}{d\theta} = a\sin\theta$$

divide eq (iv) by (iii)

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a + a\cos\theta}$$

$$\frac{dy}{dx} = \frac{a\sin\theta}{a + a\cos\theta} = \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

$$\frac{dy}{dx} = \frac{2\sin\theta/2 \cos\theta/2}{2\cos^2\theta/2} = \frac{\sin\theta/2}{\cos\theta/2} =$$

$$\boxed{\frac{dy}{dx} = \tan\theta/2}$$

a) ~~obtain~~

$$f(x) = f(0) + \frac{f'(0)(x)}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$f(x) = e^{2x}$

$f(0) = e^{2\times 0} = e^0 = 1.$

$f'(x) = 2e^{2x}$	$f'(0) = 2\times e^0 = 2.$
$f''(x) = 4e^{2x}$	$f''(0) = 4\times e^0 = 4$
$f'''(x) = 8e^{2x}$	$f'''(0) = 8\times e^0 = 8.$
$f^{iv}(x) = 16e^{2x}$	$f^{iv}(0) = 16\times e^0 = 16.$

$\frac{c_1}{6/10}$

$(e^{2x}, 0) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{6} + \frac{f''''(0)x^4}{24}$

$= 1 + \frac{2x}{1!} + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \frac{16x^4}{4!}$ //

6

Part A

① $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

Applying L-Rule

$= \lim_{x \to 2} \frac{2x+1}{1}$

$= 2 \times 2 + 1$

$= 4 + 1 = 5$

$\therefore \lim_{x \to 2} \frac{x^2+x-6}{x-2} = 5$

2

② Given

$y = \sin(\sqrt{x})\cos(x^3)$

diff bothsides w.r.to x.

$\frac{dy}{dx} = \sin\sqrt{x}\,\frac{d(\cos x^3)}{dx} + \cos x^3\,\frac{d(\sin\sqrt{x})}{dx}$

$= \sin\sqrt{x}(-\sin x^3)3x^2 + \cos x^3 \cos\sqrt{x}\,\frac{1}{2\sqrt{x}}$

2

$\frac{dy}{dx} = -3x^2 \sin\sqrt{x}\sin x^3 + \frac{\cos x^3 \cos\sqrt{x}}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{\cos\sqrt{x}}{}$

$\boxed{\frac{d\,y}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}}$

3) sol

$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$x = \tan\theta$

diff both sides w.r.t x

$1 = \sec^2\theta \frac{d\theta}{dx}$

$\frac{dx}{d\theta} = \sec^2\theta$

$y = \cos^{-1}\frac{(1-\tan^2\theta)}{1+\tan^2\theta}$

$\boxed{\because \cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}}$

$y = 2\theta$

$y = 2\tan^{-1} x$

4) sol

$x = a\cos\theta,\ y = a\cos\theta.$

$x = a\cos\theta$

differentiate w.r.t

$\frac{dx}{d\theta} = (ax - \sin\theta)$

$\frac{dx}{d\theta} = -a\sin\theta \rightarrow ①$

$y = a\cos\theta$

diff w.r.t θ

$\frac{dy}{d\theta} = \frac{d(a\cos\theta)}{d\theta}$

$\frac{dy}{d\theta} = ax - \sin\theta$

$\frac{dy}{d\theta} = -a\sin\theta. \rightarrow (ii)$

divide eq ① & ②

$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{-a\sin} = 1$

$\frac{4}{5/10}$

c) Sol

Given $f(x) = x^4 - 3x^3 + 3x^2 - x$

find $f'(x)$

$f'(x) = 4x^3 - 9x^2 + 6x - 1.$

$f'(x) = 0$

$4x^3 - 9x^2 + 6x - 1 = 0$

$$\begin{array}{c|cccc} 1 & 4 & -9 & 6 & -1 \\ & 0 & 4 & -5 & 1 \\ \hline & 4 & -5 & 1 & 0 \end{array}$$

$\cancel{4x^2 - 5x + 1 = 0}$

$\cancel{(x-1)} = 0$

$(x-1)(4x^2 - 5x + 1) = 0$

$x = 1 \quad (4x^2 - 5x - x + 1) = 0$

$4x(x-1) - 1(x-1) = 0$

$(x-1)(x-1) = 0$

$4x - 1 = 0 \quad x - 1 = 0$

$x = 1/4 \quad x = 1.$

So, the function point of x is $1/4$ and 1.

Sol

(i) funtion must be contin in the interval

(ii) funtion must be differentiable in the interval

Assignment Questions

Mathematics - I

U18BSMA101

Part - A

1. State Cayley - Hamilton Theorem
2. Find the sum and product of the eigen values $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
3. State the properties of eigen values of 1 and 2. Prove it.
4. Form the matrix from $4x^2+3y^2+2z^2+3xy+4zx$
5. Properties of Eigen Values

Part B

6. Find the Eigen vectors of $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$
7. Write the matrix corresponding to the quadratic form $2x^2+y^2+z^2+2xy-4zy-6zx$
8. Prove cayley Hamilton Theorem for $A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
9. Find the nature of the quadratic form whose matrix is $A=\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$
10. Find A^4 for the matrix $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Part C

11. Verify cayley Hamilton theorem and find A^4 for $A=\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
12. Reduce to the Diagonal form by Diagonalization $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

MATHS PAPER

ASSIGNMENT

U19CS152

INDURI SUSHMITHA

CSE -B

PART - A

42/60

(1) Cayley - Hamilton Theorem :- Every square matrix statisifes it's own characterstic equantion.

(2) Sum of the eigen values = Trace of diagonal elements

$\Rightarrow$ $4+2=6$

product of eigen values is $|A|$

$|A| = 8-3 = 5$

(3) If 1 and 2 are the eigen values

Then: By property If λ_1^m, λ_2^m are eigen values of A^m, where 'M' is positive integer.

The eigen values of A^2 is $1^2, 2^2$ are 1, 4

Then, By the property of $\lambda_1, \lambda_2, \lambda_3$ are eigen values of 'A'. Then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$ are eigen values of A^{-1}. $\frac{1}{1}, \frac{1}{2}$ are eigen values of A^{-1}.

$4x^2+3y^2+2z^2+3xy+4zx.$

$a_{11} = 4$ $a_{12} = a_{21} = \frac{2}{2} = 1$

$a_{22} = 2$ $a_{23} = a_{32} = \frac{0}{2} = 0$

$a_{33} = 3$ $a_{13} = a_{31} = 4/2 = 2$

matrix is $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

① The eigen values of A and A^T are same.

② Sum of eigen values is the Trace of diagonal elements of Matrix.

③ product of eigen values is the determination of matrix 'A'

$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)(1-\lambda) - 0 = 0$

$(1-\lambda)^2 = 0.$

$1+\lambda^2 - 2\lambda = 0$

$\boxed{\lambda^2 + 1 - 2\lambda = 0}$

(7) The chagactersitc eqation is given by $[A - \lambda I] = 0$

$$\left| \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

3

$$\begin{vmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-1-\lambda) - 3(1) = 0$$

$$-1 - \cancel{\lambda} + \cancel{\lambda} + \lambda^2 - 3 = 0$$

The eigen values are $\lambda = -2, +2$.

$$\lambda^2 - 4 = 0$$

Case-i:- $\lambda = -2$ $[A - \lambda I]X = 0$

$$\left| \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left| \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$$\begin{bmatrix} -1 & 3 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$$-x_1 + 3x_2 = 0.$$

$$x_1 = 3x$$

$$\frac{x_1}{3} = \frac{x_2}{1} \qquad \therefore X_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

case-2 $[A - \lambda I]X = 0$

$\lambda = -2$

$$\left[\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 0 \\ 0 & +1 \end{vmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$3x_1 + 3x_2 = 0$ $\qquad$ $x_1 + x_2 = 0$

$\cancel{3}x_2 = -\cancel{3}x_1$ $\qquad$ $x_1 = x_2$

$x_2 = -x_1$

$\frac{x_1}{-1} = \frac{x_2}{1}$ $\qquad$ $X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

The eigen vectors are $X_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(8) characetristic equation is given by

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$D_1 = 1 + 1 + 1 = 3$

$$D_2 = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = (1-0) + (1-0) + (1-0).$$

$D_2 = 3.$

$$D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 1(1-0) - 2(0-0) + 3(0-0)$$

$= 1(1)$

$D_3 = 1$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0.$

By cayley Hamilton theorem λ changes to A.

$$A^3 - 3A^2 + 3A - I = 0$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{vmatrix} 1+0+0 & 2+2+0 & 3+4+3 \\ 0+0+0 & 0+1+0 & 0+2+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{vmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 2+4+0 & 3+8+10 \\ 0+0+0 & 0+1+0 & 0+2+4 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 6 & 21 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $A^3 - 3A^2 + 3A - I = 0$.

$$\Rightarrow \begin{bmatrix} 1 & 6 & 21 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 1-3+3-1 & 6-12+6-0 & 21-30+9-0 \\ 0-0+0-0 & 1-3+3-1 & 6-12+6-0 \\ 0-0+0-0 & 0-0+0-0 & 1-3+3-1 \end{vmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence cayley - Hamilton $= 0$ theorem verified

Take eq (2)

$$A^3 - 3A^2 + 3A - I = 0.$$

pre multiply by A^{-1}

$$A^2 - 3A + 3 - A^{-1} = 0$$

$$A^{-1} = A^2 - 3A + 3$$

$$A^{-1} = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+3 & 4-6+0 & 10-9+0 \\ 0-0+0 & 1-3+3 & 4-6+10 \\ 0-0+0 & 0-0+0 & 1-3+3 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$6x^2 - 2y^2 + 3z^2 - 4xy + 8zx = 0.$$

$a_{11} = 6$

$a_{12} = a_{21} = \frac{1}{2}(-4) = -2$

$a_{22} = -2$

$a_{23} = a_{32} = 0$

$a_{33} = 3$

$a_{13} = a_{31} = \frac{1}{2}(8) = 4$

$$A = \begin{bmatrix} 6 & -2 & 4 \\ -2 & -2 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$

$D_1 = 6 - 2 + 3 = 7$ (+ve)

$$D_2 = \begin{vmatrix} 6 & -2 \\ -2 & -2 \end{vmatrix} = -12+4$$

$= -16$ (−ve)

$$D_3 = \begin{vmatrix} 6 & -2 & 4 \\ -2 & -2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = 6(-6-0) + 2(-6-0) + 4(0+8)$$

$= -36 - 12 + 32$

$= -16$ (−ve)

D_1 & D_2 are +ve and −ve the quadratic form is indefinite.

(10) The characteristic equation is given by

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$D_1 = 2+3+2 = 7$

$D_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 4+3+4 = 11. \quad \therefore D_2 = 11$

$D_3 = 2(6-2) - 2(2-1) + 1(2-3)$

$= 2(4) - 2(1) + 1(-1)$

$= 8 - 2 + 1 = 8-3 \quad \Rightarrow D_3 = 5$

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0.$

by λ changes to A

$A^3 - 7A^2 + 11A - 5I = 0.$

$A^2 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 4+2+1 & 4+6+2 & 2+2+2 \\ 2+3+1 & 2+9+2 & 1+3+2 \\ 2+2+2 & 2+6+4 & 1+2+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix}$

8)

$A^3 = A^2 . A = \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 14+12+6 & 14+16+12 & 7+12+12 \\ 12+13+6 & 12+39+12 & 6+13+12 \\ 12+12+7 & 12+36+14 & 6+12+14 \end{bmatrix}$

$A^3 = \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix}.$

$A^4 = A^3 . A = \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$$\boxed{A^4 = \begin{bmatrix} 157 & 312 & 156 \\ 156 & 313 & 156 \\ 156 & 312 & 154 \end{bmatrix}}$$

PART - C

(11) $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The characteristic equation is given by

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0 \longrightarrow (1)$$

$$D_1 = 2+2+2 = 6$$

$$D_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = (4-1) + (4-2) + (4-1)$$

$$= 3+2+3 = \underline{8}$$

$$D_3 = \begin{vmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 2(4-1) + 1(-2+1) + 2(1-2)$$

$$= 2(3) + 1(-1) + 2(-1)$$

$$= 6-1-2$$

$$D_3 = 3$$

$$\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0.$$

λ changes as A

$$A^3 - 6A^2 + 8A - 3I = 0 \longrightarrow (2)$$

$$A^2 = A \times A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 14+6+9 & -7-12-9 & 14+6+18 \\ -10-6-6 & 5+12+6 & -10-6+2 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

Apply in eqn(2) $\quad A^3 - 6A^2 + 8A - 3I = 0$

$$\Rightarrow \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6\begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$A^3 - 6A^2 + 8A - 3I = 0$$

$$A^4(A^3 - 6A^2 + 8A - 3) = 0$$

$$A^4 - 6A^3 + 8A^2 - 3A = 0$$

$$= 6\begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 8\begin{bmatrix} 7 & -6 & 9 \\ -5 & -6 & 6 \\ 5 & -5 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & 2 \\ -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 174 & -168 & 228 \\ 132 & 138 & -168 \\ 132 & -132 & 174 \end{bmatrix} - \begin{bmatrix} 56 & -148 & 72 \\ -40 & 48 & -118 \\ 40 & -40 & 42 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 174-56+6 & -168+148-3 & 228-72+6 \\ 132+40-3 & 138-48+6 & -168+48-3 \\ 132-40+3 & -132+40-3 & 174-42+6 \end{bmatrix}$$

$$\boxed{A^4 = \begin{bmatrix} 124 & -123 & 162 \\ 169 & 96 & -123 \\ 95 & -175 & 124 \end{bmatrix}}$$

8

The characterstic equation is given by $\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$

$D_1 = 8 + 7 + 3 = 18$

$$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$$= 21 - 16 + 24 - 4 + 56 - 36$$

$$= 5 + 20 + 20$$

$$D_2 = 45$$

$$D_3 = 8(21-16) + 6(-18+8) + 2(24-14)$$

$$= 8(5) + 6(-10) + 2(10) = 40 - 60 + 20$$

$$D_3 = 0.$$

$$\lambda^3 - 18\lambda^2 + 45\lambda + 0 = 0.$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$\underline{\lambda = 0}$

$$\lambda^2 - 18\lambda + 45 = 0.$$

$$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$$

$\therefore$ The eigen values are $\lambda = 0, 3, 15$

<u>case-i</u> $\lambda = 0$ $(A - \lambda I)x = 0.$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$8x - 6y + 2z = 0.$$

$$-6x + 7y - 4z = 0.$$

$$2x - 4y + 3z = 0.$$

$$\frac{x}{[illegible]} = \frac{-y}{[illegible]} = \frac{z}{[illegible]}$$

$$\frac{x}{5} = \frac{-y}{-10} = \frac{z}{10}$$

$$\Rightarrow \quad \frac{x}{1} = \frac{y}{2} = \frac{z}{2}$$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \qquad X_1^{N} = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

Case - II

$\lambda = 3 \qquad (A - \lambda I)X = 0$

$$\left(\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$5x - 6y + 2z$

$-6x + 4y - 4z = 0$

$2x - 4y + 0z = 0$

$$\frac{x}{-16} = \frac{-y}{8} = \frac{z}{16}$$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \qquad X_2^{N} = \begin{bmatrix} 2/3 \\ 1/3 \\ -1/3 \end{bmatrix}$$

Case - III

$\lambda = 15$

$(A - \lambda I)X = 0$

$$\left(\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-7x - 6y + 2z = 0$$

$$-6x - 8y - 4z = 0$$

$$2x - 4y - 12z = 0$$

$$\frac{x}{96-16} = \frac{-y}{72+8} = \frac{z}{24+16}$$

$$\Rightarrow \frac{x}{80} = \frac{-y}{80} = \frac{z}{40}$$

$\div(40)$ $$\Rightarrow \frac{x}{2} = \frac{-y}{2} = \frac{z}{1}$$

$$X_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \quad X_3^N = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

$$N = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \quad N^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$N^T A N = D$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 8+2+4 & -6+14-8 & 2-8+6 \\ 16-6-4 & -12+7+8 & 4-4-6 \\ 16+12+2 & -12-14-4 & 4+8+3 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 0 & 0 & 0 \\ 6+6-12 & 12+3+12 & 12-6-6 \\ 30-60+30 & 60-30-30 & \cancel{+60}+\cancel{60}+15 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 135 \end{bmatrix}$$

$$\boxed{D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}}$$

(12)

$x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$

$a_{11} = 1 \qquad a_{12} = a_{21} = \frac{1}{2}(-2) = -1$

$a_{22} = 2 \qquad a_{23} = a_{32} = \frac{1}{2}(2) = 1$

$a_{33} = 3 \qquad a_{13} = a_{31} = \frac{1}{2}(0) = 0$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$D_1 = 1+2+1 = 4$

$$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 2-1+1-0+2-1$$

$= 1+1+1$

$D_2 = 3$

$$D_3 = \begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0$$

$\lambda^3 - 4\lambda^2 + 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda + 3) = 0$

$\lambda = 0, \qquad \lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 3\lambda - \lambda + 3 = 0$

$\lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$(\lambda - 1)(\lambda - 3) = 0$

$\lambda = 1, \quad \lambda = 3$

The eigen values are $0, 1, 3$.

for a diagonal matrix of a matrix A. The diagonal elements are eigen - values of A.

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

canonical form can be given by

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 0y_1^2 + y_2^2 + 3y_3^2 + 0y_1y_2 + 0y_2y_3 + 0y_3y_1$$

$$= y_2^2 + 3y_3^2.$$

canonical form of $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ is

$$y_2^2 + 3y_3^2.$$

AKULA VENKATESH
U19CS036
CSE

MATHS PAPER ASSIGNMENT

Part - A

15/60

(1) Cayley - Hamilton Theorem:

Every square matrix Satisfies it's own characteristic equation.

2

(2) Sum of the eigen Values = Trace of diagonal elements

$\Rightarrow$ ~~4+2~~ = 6

1

(3) Product of eigen Values is |A|

|A| = 8-3 = 5

1

(3) If 1 and 2 are the eigen Values

Then:

By property if λ_1^m, λ_2^m are eigen Values of A^m, where 'm' is positive integer.

The eigen Values of A^2 is $1^2, 2^2$ are 1, 4

Then, by the property of $\lambda_1, \lambda_2, \lambda_3$ are eigen Values of 'A'. Then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$ are eigen Values of A^{-1}. $\frac{1}{1}, \frac{1}{2}$ are eigen Values of A^{-1}.

(4.) $4x^2 + 3y^2 + 2z^2 + 3xy + 4zx$.

$a_{11} = 4$ $\quad$ $a_{12} = a_{21} = \frac{\cancel{2}}{\cancel{2}} = 1$

$a_{22} = 2$ $\quad$ $a_{23} = a_{32} = \frac{0}{2} = 0$

$a_{33} = 3$ $\quad$ $a_{13} = a_{31} = \frac{\cancel{4}^{2}}{\cancel{2}} = 2$

matrix is $\begin{bmatrix} 4 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$

(5) (1) The eigen values of A and A^T are same.

(2) Sum of eigen values is the trace of diagonal elements of matrix.

(3) product of eigen values is the determination of matrix 'A'.

(6) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 1-\lambda \end{bmatrix} = 0$

$(1-\lambda)(1-\lambda) = 0$

$1-\lambda^2 = 0$

$1 + \lambda^2 - 2\lambda = 0$

$\boxed{\lambda^2 - 2\lambda + 1 = 0}$

(7) The characteristic equation is given by $[A-\lambda I]=0$.

$$\left|\begin{bmatrix}1 & 3\\ 1 & -1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right|=0.$$

$$\left|\begin{bmatrix}1 & 3\\ 1 & -1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right|=0$$

$$\begin{vmatrix}1-\lambda & 3\\ 1 & -1-\lambda\end{vmatrix}=0$$

$$(1-\lambda)(-1-\lambda)-3(1)=0.$$

$$-1\cancel{-\lambda}\cancel{+\lambda}+\lambda^2-3=0$$

$$\lambda^2-4=0$$

The eigen values are $\lambda=-2,+2$.

Case: (i) $\lambda=-2$ $\quad [A-\lambda I]X=0$.

$$\left|\begin{bmatrix}1 & 3\\ 1 & -1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right|\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=0$$

$$\left|\begin{bmatrix}1 & 3\\ 1 & -1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right|\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

$$\begin{bmatrix}-1 & 3\\ 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

$$-x_1 + 3x_2 = 0$$

$$x_1 = 3x_2$$

$$\frac{x_1}{3} = \frac{x_2}{1}$$

$$\therefore X_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Case-2: $[A - \lambda I]\, X = 0$

$$\lambda = -2$$

$$\left[\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - (-2) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left[\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$3x_1 + 3x_2 = 0$$

$$\cancel{3}x_2 = -\cancel{3}x_1$$

$$x_2 = -x_1$$

$$\frac{x_1}{-1} = \frac{x_2}{1}$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The eigen vectors are $x_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

8. Characteristic equation is given by

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$D_1 = 1+1+1 = 3$

$$D_2 = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= (1-0) + (1-0) + (1-0).$$

$D_2 = 3.$

$$D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1(1-0) - 2(0-0) + 3(0-0)$$

$$= 1(1)$$

$D_3 = 1$

$$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0,$$

By Cayley Hamilton theorem λ changes to A.

$$A^3 - 3A^2 + 3A - I = 0$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 2+2+0 & 3+4+3 \\ 0+0+0 & 0+1+0 & 0+2+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 2+4+0 & 3+8+10 \\ 0+0+10 & 0+1+0 & 0+2+4 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 6 & 21 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $A^3 - 3A^2 + 3A - I = 0.$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 21 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 1-3+3-1 & 6-12+6-0 & 21-30+9-0 \\ 0-0+0-0 & 1-3+3-1 & 6-12+6-0 \\ 0-0+0-0 & 0-0+0-0 & 1-3+3-1 \end{vmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence Cayley Hamilton theorem verified

Take equ (2).

$$A^3 - 3A^2 + 3A - I = 0.$$

Pre multiply A^{-1} =

$$A^2 - 3A + 3 - A^{-1} = 0$$

$$A^{-1} = A^2 - 3A + 3$$

$$A^{-1} = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+3 & 4-6+0 & 10-9+0 \\ 0-0+0 & 1-3+3 & 4-6+10 \\ & & \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}$$

9. $6x^2 - 2y^2 + 3z^2 - 4xy + 8zx = 0$

$a_{11} = 6$ $\qquad a_{12} = a_{21} = \frac{1}{2}(-4) = -2$

$a_{22} = -2$ $\qquad a_{23} = a_{32} = 0$

$a_{33} = 3$ $\qquad a_{13} = a_{31} = \frac{1}{2}(8) = 4$

$$A = \begin{bmatrix} 6 & -2 & 4 \\ -2 & -2 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$

$D_1 = 6 - 2 + 3 = 7 \quad (+ve)$

$D_2 = \begin{vmatrix} 6 & -2 \\ -2 & -2 \end{vmatrix} = -12 - 4 = -16 \quad (-ve)$

$D_3 = \begin{vmatrix} 6 & -2 & 4 \\ -2 & -2 & 0 \\ 4 & 0 & 3 \end{vmatrix} = 6(-6-0) + 2(-6-0) + 4(0-8)$

$= -36 - 12 + 32$

$= -16 \quad (-ve)$

D_1 & D_2 are +ve and -ve the quadratic form is indefinite.

10. The characteristic equation is given by.

$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0.$

$D_1 = 2 + 3 + 2 = 7$

$D_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 4 + 3 + 4$

$= 11$

$D_2 = 11$

$$D_3 = 2(6-2) - 2(2-1) + 1(2-3)$$

$$= 2(4) - 2(1) + 1(-1)$$

$$= 8 - 2 + 1) = 8 - 3 \qquad \underline{\underline{D_3 = 5}}$$

$$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

by λ changes to A

$$A^3 - 7A^2 + 11A - 5I = 0$$

$$A^2 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+1 & 4+6+2 & 2+2+2 \\ 2+3+1 & 2+9+2 & 1+3+2 \\ 2+2+2 & 2+6+4 & 1+2+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12+6 & 14+16+12 & 7+12+12 \\ 12+13+6 & 12+39+12 & 6+13+12 \\ 12+12+7 & 12+36+14 & 6+12+14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix}$$

$$A^4 = A^3 . A = \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 157 & 312 & 156 \\ 156 & 313 & 156 \\ 156 & 312 & 154 \end{bmatrix}$$

part-C

11. $$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

The characteristic is given by

$$\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0 \rightarrow (1)$$

$$D_1 = 2+2+2 = 6.$$

$$D_2 = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= (4-1) + (4-2) + (4-1)$$

$$= 3+2+3$$

$$= 8$$

$$D_3 = \begin{vmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 2(4-1) + 1(-2+1) + 2(1-2)$$

$$= 2(3) + 1(-1) + 2(-1)$$

$$= 6-1-2$$

$$\boxed{D_3 = 3}$$

$$\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$$

λ Changes as A

$$A^3 - 6A^2 + 8A - 3I = 0 \rightarrow (2)$$

$$A^2 = A \times A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+2 & -2-2-2 & 4+1+4 \\ -2-2-1 & 1+4+1 & -2-2-2 \\ 2+1+2 & -1-2-2 & 2+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+6+9 & -7-12-9 & 14+6+18 \\ -10-6-6 & 5+12+6 & -10-6+2 \\ 10+5+7 & -5-10-7 & 10+5+14 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix}$$

10

Apply in eqn (2) $A^3 - 6A^2 + 8A - 3I = 0$.

$$\Rightarrow \begin{bmatrix} 29 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -22 & 29 \end{bmatrix} - 6 \begin{bmatrix} 7 & -6 & 9 \\ -5 & 6 & -6 \\ 5 & -5 & 7 \end{bmatrix} + 8 \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = A^3 . A$$

$$A^3 - 6A^2 + 8A - 3I = 0$$

$$A(A^3 - 6A^2 + 8A - 3) = 0$$

$$A^4 - 6A^3 + 8A^2 - 3A = 0$$

$$= 6\begin{bmatrix} 30 & -28 & 38 \\ -22 & 23 & -28 \\ 22 & -23 & 29 \end{bmatrix} - 8\begin{bmatrix} 7 & -6 & 9 \\ -5 & -6 & 6 \\ 5 & -5 & 7 \end{bmatrix} - 3\begin{bmatrix} 2 & 1 & 2 \\ -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 174 & -168 & 228 \\ 132 & 138 & -168 \\ 132 & -138 & 174 \end{bmatrix} - \begin{bmatrix} 56 & -48 & 72 \\ -40 & 48 & -48 \\ 40 & -40 & 42 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 6 \\ -3 & 6 & -3 \\ -3 & -3 & 6 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 124 & -123 & 162 \\ 69 & 96 & -123 \\ 96 & -125 & 124 \end{bmatrix}$$

(b) The characteristic equation is given by $\lambda^3 - D_1\lambda^2 + D_2\lambda - D_3 = 0$.

$D_1 = 8+7+3 = 18$

$$D_2 = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}$$

$= 21-16+24-4+56-36$

$= 5+20+20$

$D_2 = 45$

$D_3 = 8(21-16)+6(-18+8)+2(24-14)$

$= 8(5)+6(-10)+2(10)$

$= 40-60+20$

$D_3 = 0.$

$\lambda^3 - 18\lambda^2 + 45\lambda + 0 = 0$

$\lambda(\lambda^2 - 18\lambda + 45) = 0$

$\lambda = 0$

$$\lambda^2 - 18\lambda + 45 = 0$$

$$\lambda(\lambda - 15) - 3(\lambda - 15) = 0$$

$\therefore$ The eigen values are $\lambda = 0, 3, 15.$

Case i: $\lambda = 0$ $(A - \lambda I)X = 0$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right| \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0.$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$8x - 6y + 2z = 0.$$

$$-6x + 7y - 4z = 0.$$

$$2x - 4y + 3z = 0.$$

$$\frac{x}{21 - 16} = \frac{-y}{-18 + 8} = \frac{z}{24 - 14}$$

$$\frac{x}{5} = \frac{-y}{-10} = \frac{z}{10}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{2} = \frac{z}{2}$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad x_1 M = \begin{bmatrix} \cancel{1}\,3 \\ 2/3 \\ 2/3 \end{bmatrix}$$

Case (ii) $\lambda = 3$ $(a - \lambda I)x = 0$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right| \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$5x - 6y + 2z = 0$

$-6x + 4y - 4z = 0$

$2x - 4y + 0z = 0$

$$\frac{x}{-16} = \frac{-y}{8} = \frac{z}{16}$$

$$x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \quad x_{2N} = \begin{bmatrix} 2/3 \\ 1/3 \\ -1/3 \end{bmatrix}$$

Case (iii) $\lambda = 15$ $\cancel{(A - \lambda I)x = 0}$

$$\left| \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix} \right| \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-7x - 6y + 2z = 0$

$-6x - 8y - 4z = 0$

$2x - 4y - 12z = 0$

$$\frac{x}{96 - 16} = \frac{-y}{72 + 8} = \frac{z}{24 + 16} \Rightarrow \frac{x}{80} = \frac{-y}{80} = \frac{z}{40} \quad \div 10$$

$$\Rightarrow \frac{x}{2} = \frac{-y}{2} = \frac{z}{1}$$

$$x_3 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \qquad x_{3N} = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}$$

$$N = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \qquad N^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & +1 \end{bmatrix}$$

$$N^T A N = D$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & +1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 8+2+4 & -6+14-8 & 2-8+6 \\ 16-6-4 & -12+7+8 & 4-4-6 \\ 16+12+2 & -12-14-4 & 4+8+3 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

10

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 0 & 0 & 0 \\ 6+6-12 & 12+3+12 & 12-6-6 \\ 30-60+30 & 60-30-30 & 60+60+15 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 135 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

QP CODE: *U18BSMA101*

Reg.No								

Bharath Institute of Higher Education and Research

Declared as deemed to be University under section 3 of UGC act 1956

173,Agaram Main Road,Selaiyur,Chennai-600073,Tamil Nadu.

UNIVERSITY EXAMINATIONS-NOV/DEC-2019

Regulation-2018

U18BSMA101 Engineering Mathematics-I

Duration: Three Hours Max.Marks:100

COURSE OUTCOME:

CO1	To apply both the limit definition and rules of differentiation to differentiate functions. Also they will have a basic understanding of Rolle's theorem that is fundamental to application of analysis to Engineering problems.
CO2	To apply definite integrals of algebraic and trigonometric functions using formulae and substitutions. Also they will have a basic understanding of Beta and Gamma functions.
CO3	To apply differential and integral calculus to notations of curvature. Also apply differentiation to find maxima and minima of functions.
CO4	To apply multiple integrals to compute area and volume over curves surface and domain in two dimensional and three dimensional spaces.
CO5	Identify Eigen value problems from practical areas using transformations; Diagonalising the matrix would render the Eigen Values.

	PART-A (10 × 2 = 20) MARKS ANSWER ALL QUESTIONS	**BT**	**CO**	**Marks**
1	Evaluate $\lim_{x\to 0}\frac{(1+x)^4-1}{x}$	U	CO1	2
2	Find $\frac{dy}{dx}$ $if\ x^3+y^3=a^3$.	R	CO1	2
3	Evaluate $\int Cos^3 x\ dx$	Ap	CO2	2
4	Evaluate $\int_0^1 (x^{10}+10^x)\,dx$	U	CO2	2
5	Find the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ if $x = rcos\theta$; $y = rsin\theta$	An	CO3	2
6	Find $\frac{du}{dt}$ $w\square en\ u = x^2+y^2,\ x = at^2\ and\ y = 2at$	E	CO3	2
7	Evaluate $\int_2^a\int_2^b \frac{dx\,dy}{xy}$.	U	CO4	2
8	Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,dx\,dy\,dz$	Ap	CO4	2
9	State Cayley Hamilton Theorem	R	CO5	2
10	Write the matrix of the Quadratic form $10{x_1}^2+2{x_2}^2+5{x_3}^2+6x_2x_3-10x_1x_2-4x_1x_2$.	U	CO5	2

PART-B (5 × 6 = 30) MARKS ANSWER Either (a) Or (b) FROM EACH QUESTIONS		BT	CO	Marks
11a	Show that the function $f(x)$ given by $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ Is continuous at $x = 0$	E	CO1	6
11 b	Find the maxima and minima of the function $2x^3 + 5x^2 - 4x$	Ap	CO1	6
12a	Evaluate $\int \frac{dx}{\sqrt{x^2-8x+20}}$	R	CO2	6
12 b	Evaluate $\int x \tan^{-1} x\, dx$	C	CO2	6
13a	If $u = \tan^{-1}(\frac{x^3+y^3}{x+y})$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = Sin2u$	Ap	CO3	6
13 b	If $u = f(x-y, y-z, z-x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	An	CO3	6
14a	Using double integral, find the area of the Circle.	R	CO4	6
14 b	Using double integral, find the area of the Cardioids $r = a(1 + Cos\theta)$	C	CO4	6
15a	Find the Eigen Values and Eigen Vectors of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$	U	CO5	6
15 b	If A = $\begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$, express A^3in terms of A and using Cayley Hamilton Theorem	Ap	CO5	6

PART-C (5 × 10 = 50) MARKS ANSWER ANY FIVE QUESTIONS		BT	CO	Marks
16	Verify Rolle's Theorem and Mean Value theorem for the function $f(x) = x^2 + 2x = 8\ at\ \ (-4,2)$	U	CO1	10
17	Evaluate the Riemann sums for $f(x) = x^3 - 6x$,taking the sample points to be the right end Points and a = 0, b = 3 and n = 6. Also evaluate $\int_0^3 (x^3 - 6x)\, dx$	An	CO2	10
18	If $x + y + z = u, y + z = uv\ and\ z = uvw$ find the $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	E	CO3	10
19	A rectangular box open at the top, is to have a volume of 32 cc. Find the dimension of the box, that requires the least material for its construction	C	CO3	10
20	Using triple integral, find the Volume of the tetrahedron bounded by the co-ordinates plane and the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$	An	CO4	10
21	Verify Cayley-Hamilton theorem and find its inverse of the matrix $\begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$	Ap	CO5	10
22	Reduce the given quadratic form Q to its canonical form using orthogonal transformation $Q = 3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy$	C	CO5	10

Bharath Institute of Higher Education & Research

Sub code: U18BSMA101

Time: 3 Hrs
Marks: 100

Sub Name: Eng. Mathematics-I

Part-A

10×2=20

Answer All the Questions.

1) $\lim_{x \to 0} \frac{(1+x)^4 - 1}{x}$

By Applying L'Hospital Rule

$\lim_{x \to 0} \frac{(1+x^4)-1}{x} = \frac{1}{3}.$

2) $x^3 + y^3 = a^3$

$3x^2 + 3y^2 \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x^2}{y^2}$

3. $\int \cos^3 x \, dx = \int \cos^2 x . \cos x \, dx$

$= \int (1 - \sin^2 x) \cos x \, dx$

$= \frac{4}{5}$.

4. $\int_0^1 (x^{10} + 10^x) \, dx$

$= \int_0^1 (x^9 . x + 10^x . 10^0) \, dx$

$= 47.$

5. Jacobian $\frac{\partial(x, y)}{\partial(r\theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial y} & \frac{\partial u}{\partial x} \end{vmatrix}$

6. $\frac{du}{dt} = 2x + 3y$ $\quad x = at^2$

$y = 2at$

PART-B

Answer Either (a) (or) b :

11) a) $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & x \neq 0 \\ 2 & x = 0 \end{cases}$

$f(x) = \left\{ \frac{\sin x}{x} + \cos x \right.$

$= \operatorname{cosec}^2 x + 2$

11) b) $2x^3 + 5x^2 - 4x$

$\frac{dy}{dx} = 3 - 2x^2 + 10x$

$\frac{d^2y}{dx^2} = -4x + 10 < 0$

at $(1, 2)$

max value : 7.

Minimum value : 2

7. $$\int_2^a \int_2^b \frac{\partial x \, \partial y}{xy}$$

$$\int_2^a \int_2^b \frac{\partial}{\partial y}\left[x\right] dy$$

$$\int_2^a \left[\frac{x^2}{2}\right]_2^b dy$$

$$= 7/2$$

8. $$\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} \, dx\,dy\,dz$$

$$= 8/7$$

9. Cayley Hamilton Thm:
Every Square matrix Satisfies its own Characteristic form.

10. Quadratic Form:

$$\begin{pmatrix} 10 & 2 & 3 \\ 1 & 6 & 4 \\ 1 & 4 & 4 \end{pmatrix}$$

12.a) $\int \frac{dx}{\sqrt{x^2 - 8x + 20}}$

$\int \frac{dx}{\sqrt{(x^2+1)(x^2-1)}}$

$\int \frac{dx}{\sqrt{(x+1)^2(x^2+1)(x^2-1)}}$

$= 8/4$

12.b) $\int x \tan^{-1} x \, dx$

By ~~applying~~ Int. Formula

$\int x \tan^{-1} x \, dx = x \left(\frac{1}{1+x^2}\right) - \tan^{-1} x$

on substituting the limits,

we get $\pi/2$.

13) a) $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

$\tan u = \left(\frac{x^3+y^3}{x-y}\right)$

$\frac{\partial u}{\partial x} = \sec u \left[\frac{(x-y)3x^2 - y(3x^2)}{(x-y)^2}\right]$

$\frac{\partial^2 u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$

15) b) $u = f(x-y, y-z, z-x)$

$X = x-y$

$Y = y-z$

$Z = z-x$

$\frac{\partial u}{\partial x} = 1$; $\frac{\partial u}{\partial y} = 1 - \frac{\partial x}{\partial x}$; $\frac{\partial u}{\partial z} = 1$

Adding, we get

$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

PART-C

5×10=50

Answer Any Five

16) $f(x) = x^2+2x-8$ at $(-4, 2)$

$f'(x) = 2x+2$

$f''(x) = 2$

Min : $f'(x) = 0$

17) $\int_0^3 (x^3-6x)\,dx$

$f'(x) = 3x^2-6$

$a=0$; $b=3$ $n=6$

$\int_0^3 (x^3-6x)\,dx = 8/9$

18) $x+y+z=u,$

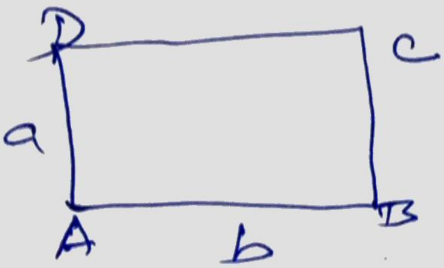
$y+z \; tz = uw$

$z = uvw$

$$\frac{\partial(xyz)}{\partial(uvw)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial u}{\partial y} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \end{vmatrix}$$

$= 3z.$

19)



32 cu.m.

20. $\frac{x}{2}+\frac{y}{3}+\frac{z}{4} = \int_0^3 \left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right) dx\,dy\,dz$

$= 8/7.$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 8 \sin 2u,$$

4) a) Area of circle

Let $x = \sin\theta$; $y = \cos\theta$

$\frac{dx}{d\theta} = \cos\theta$ $\quad \frac{dy}{d\theta} = -\sin\theta$

on Jacobian we have

πr^2.

(4) b) $r = a(1 + \cos\theta)$

$\frac{dr}{d\theta} = a(1 - \sin\theta)$

$\frac{d^2r}{d\theta^2} = a(1 - \cos\theta)$

(5) a)

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

Eigen values are 1, 2, 5)

Eigen vector: $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \\ 8 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}$

(5b) $A = \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$

$A^3 = A^2 \cdot A$

$= \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$

$= \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$

21).

$$\begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

$$A^3 + A^2 - 4A + 5I = 0.$$

where $A = \begin{pmatrix} 5 & 2 & 0 \\ 4 & 7 & 1 \\ 3 & 2 & 7 \end{pmatrix}$

22. $Q = 3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$

$$A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 5 & 7 \\ 1 & 6 & 5 \end{pmatrix}$$

$$Q.F = A^1 \; A^2 = A^3$$

$$= 819.$$

Text Books and Reference Books followed

TEXTBOOKS

1. Grewal B. S, Higher Engineering Mathematics, Khanna Publisher, Delhi – 2014.
2. Kreyszig. E, Advanced Engineering Mathematics, 10th edition, John Wiley & Sons, Singapore, 2012.

REFERENCE BOOKS

1. Veerarajan T, Engineering Mathematics, II edition, Tata McGraw Hill Publishers, 2008.
2. Kandasamy P &co., Engineering Mathematics, 9th edition, S. Chand & co Pub., 2010.
3. N.P.Bali and Manish Goyal, A text book of Engineering Mathematics, Laxmi Publications, Reprint, 2010.
4. Narayanan S., Manicavachagam Pillai T.K., Ramanaiah G., Advanced Mathematics for Engineering students, Volume I (2nd edition), S.Viswanathan Printers and Publishers,
5. George B. Thomas ,Jr ,Maurice D.Weir, Joel Hass., Thomas' Calculus ,Twelfth Edition Addison-Wesley, Pearson.

BMA101

(only above code to be shaded in the Answer book #)

Bharath University, Chennai - 73

B.Tech, I Semester, Nov 2010

BMA101 - Mathematics - I

(common to all branches) (2008 to 2010 batches)

Time: 3 Hrs JEETU KUMAR Maximum: 100 Marks

(10 x 2 = 20)

Part A

Answer All Questions

1. The eigen values of an idempotent matrix are either zero or unity.
2. Determine the nature of the quadratic form $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2$
3. Find the equation of sphere whose centre (2, – 3, 4) and r = 3
4. Find the equation of cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 0$
5. Show that the radius of curvature of any point of catenary $y = c\cosh(\frac{x}{c})$ is y^2/c
6. Find the centre of curvature $y = x^2$ at the origin.
7. If $f(x,y) = \log\sqrt{x^2+y^2}$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$
8. If $u = y^2/x$ and $v = x^2/y$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
9. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} f(x,y)\,dy\,dx$
10. Evaluate $\int_0^a \int_0^b \int_0^c xyz\,dz\,dy\,dx$

1

Part B (5 x 16 = 80)

Answer either (a) or (b) from each question

11. (a) Reduce the quadratic form $x^2+y^2+z^2-2xy-2yz-2zx$ to Canonical form. Also discuss its nature.

(OR)

(b) (i) Use Cayley Hamilton theorem to find eigen value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$, if

$$A=\begin{pmatrix}2&1&1\\0&1&0\\1&1&2\end{pmatrix}$$ hence compute A^{-1}

(ii) A Square matrix A and its transpose A^T have same eigen values.

12. (a) (i) Show that the equation of R.C. cylinder described on the circle through three points (1, 0, 0) , (0, 1, 0), (0, 0, 1) as the guiding curve is $x^2+y^2+z^2-yz-zx-xy=1$

(ii) Find the equation of sphere having the circle $x^2+y^2+z^2-10y-4z-8=0,\ x+y+z=3$ as a great circle.

(OR)

(b) (i) Find the equation of Cone whose vertex is (1, 2, 3) and guiding curve is the circle $x^2+y^2+z^2=4,\ x+y+z=1$

(ii) Find the equation of R.C cone whose vertex is (1, -2, -1) the semi-vertical angle 60° & the axis is the line $\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z+1}{5}$

$64-80+28-5$

2

$92-85$

13. (a) Find the evolute of the Ellipse.

(OR)

(b) Find the circle of curvature of :

(i) $\sqrt{x}+\sqrt{y}=\sqrt{a}\ at\left(\frac{a}{4},\frac{a}{4}\right)$

(ii) Find the evolute of $y^2=4ax$ considering it as the envelope of the normals.

14. (a) (i) A rectangular box open at the top is to have volume 32 cm³. Find the dimension of box requiring least material for its construction.

(ii) If $u=u\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$ show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$

(OR)

(b) (i) Expand $e^x\log(1+y)$ in powers of 'x' and 'y' upto terms of second degree.

(ii) Find the maxima & minima for the function

$f(x,y)=x^3+y^3-3x-12y+20$

15. (a) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

(OR)

(b) (i) Evaluate $\iint xy(x+y)\,dxdy$ over the region bounded by $x^2=y\ \&\ y=x$

(ii) Evaluate $\iint r^2\sin\theta\,dr\,d\theta$ where 'R' is the semicircle $r=2a\cos\theta$ about the initial line.

****

$\cancel{1-5+7-3}$

$1-5+7-3$

3

$8-20+14-3$

$27-45+21-3$

48

BMA101

(only above code to be shaded in the Answer book #)

Bharath University, Chennai – 73

B.Tech, I Semester, May 2011

BMA101 – Mathematics I

(common to 2008 to 2010 batches) (common to all branches)

Time: 3 Hrs | Maximum: 100 Marks

(10 x 2 = 20)

Part A

Answer All Questions

1. Define Eigen values and Eigen vectors.
2. Write down the quadratic form corresponding to the matrix.

$$\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & -2 & 3 \end{bmatrix}$$

3. Find the angle between the planes $2x - y + z = 3, \quad x + y + 2z = 7$
4. Find the equation of the sphere having centre is (7, 4, -3) and radius 6.
5. Define curvature and radius of curvature.
6. Define evolute and involute.
7. Find $\frac{dz}{dt}$ *Where* $z = xy^2 + x^2 y \qquad x = at^2, \; y = 2at$.
8. If $x = r\cos\theta, \quad y = r\,Sin\theta, \; z = z$ find $J\left(\frac{x, y, z}{r, \theta, z}\right)$.
9. Evaluate $\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^4 \sin\phi \, dr \, d\phi \, d\theta$.
10. Change the order of integration. $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$

Part B **(5 x 16 = 80)**

Answer either (a) or (b) from each question

11. (a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and also use it to find A^{-1}.

(OR)

(b) Reduce the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1 x_3$ to canonical form by orthogonal reduction. Find also the nature of the quadratic form.

12. (a) Find the centre and radius of the circle given by :

$x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$ and $x + 2y + 2z - 20 = 0$

(OR)

(b) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9, \quad x + y + z = 3$

13. (a) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ for the curve

$x^3 + y^3 = 3axy$

(OR)

(b) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where 'a' & 'b' are connected by the relation : (i) a + b = c (ii) ab = c^2 where 'c' is a constant.

14. (a) Find the Taylor's series expansions of $e^x \sin y$ near the $pt\left(-1, \pi/4\right)$ upto the third degree terms.

(OR)

(b) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimension of the box that requires the least material for its construction.

15. (a) Change the order of integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\, dx$ and then evaluate it.

(OR)

(b) Evaluate $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xyz\, dx\, dy\, dz$

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19DA02

BMA101

(only above code to be shaded in the Answer book #)

Bharath University, Chennai - 73

B.Tech, All Branches, I Semester, Nov 2011

BMA101 - Mathematics I

(2011 batch)

Time: 3 Hrs | Maximum: 100 Marks

(10 x 2 = 20)

Part A

Answer All Questions

1. If -1 is an eigen value of the matrix $A=\begin{pmatrix}1 & -2\\ -3 & 2\end{pmatrix}$, find the eigen values of A^4 using properties.
2. Use Cayley-Hamilton $A^4-8A^3-12A^2$ when $A=\begin{pmatrix}5 & 3\\ 1 & 3\end{pmatrix}$
3. Find the equation of the sphere having the points $(2,-3,4)$ and $(-1,5,7)$ as the ends of a diameter.
4. Find the equation of the right circular cone of revolution with vertex at the origin, axis the y-axis and semi-vertical angle 30^0.
5. Find the envelope of the lines $y=m\pm\sqrt{a^2m^2+b^2}$ where m is the parameter.
6. What is the radius of curvature of the curve $x^4+y^4=2$ at the point $(1,1)$
7. If $u=x^3y^2+x^2y^3$ where $x=at^2$ and $y=2at$ then find $\frac{du}{dt}$.
8. If $x=u^2-v^2$, $y=2uv$ evaluate the Jacobian of x, y with respect to u, v
9. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.
10. Evaluate $\int_0^1\int_0^2\int_0^3 xyz \quad dzdydx$.

Part B

(5 x 16 = 80)

Answer either (a) or (b) from each question

11. (a) (i) Show that the matrix $A=\begin{pmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{pmatrix}$ satisfies the characteristics equation and hence find its inverse.

(ii) Find all the eigen values and eigen vectors of the matrix $\begin{pmatrix}1 & -1 & 4\\ 3 & 2 & -1\\ 2 & 1 & -1\end{pmatrix}$

(OR)

(b) Reduce the Quadratic Form $2x_1^2+x_2^2+x_3^2+2x_1x_2-2x_1x_3-4x_2x_3$ into a Canonical form by an o:thogonal transformation.

12. (a) (i) Find the centre and radius of the circle given

$$x^2+y^2+z^2+2x-2y+4z-19=0 \quad \text{and} \quad x+2y+2z+7=0$$

(ii) Find the equation of the cone with vertex at $(1,1,1)$ and passing through the curve of intersection of $x^2+y^2+z^2=1$ and $x+y+z=1$.

(OR)

(b) (i) Find the two tangent planes to the sphere $x^2+y^2+z^2-4x-2y-6z+5=0$ which are parallel to the plane $x+4y+8z=0$. Find their point of contact.

(ii) Find the equation of the right circular cylinder of radius 3 and axis

$$\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$$

13. (a) (i) Find the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta),\ y=a(1-\cos\theta)$

(ii) Find the equation of circle of curvature of the parabola $y^2=12x$ and the point $(3, 6)$

(OR)

(b) (i) Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$ where the parameters a and b are connected by the relation $a^2+b^2=c^2$, c being constant.

(ii) Find the equation of the evolute of the parabola $y^2=4ax$

14. (a) (i) If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that

(1) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$ and (2) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\tan^2 u$

(ii) Find the maximum value of $x^m y^n z^p$ subject to the condition $x+y+z=a$

(OR)

(b) (i) Find the Taylor series expansion of $e^x \sin y$ at the point $(-1, \pi/4)$ up to 3rd degree terms.

(ii) Find the extreme values of the function $f(x,y)=x^3+y^3-3x-12y+20$

15. (a) (i) Find the area enclosed by the curves $y^2=4ax$ and $x^2=4ay$ using double integration.

(ii) Change the order of integration and evaluate $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}dxdy$.

(OR)

(b) (i) Express $\int_0^a\int_y^a \frac{x^2}{(x^2+y^2)^{3/2}}dxdy$ in polar coordinates & then evaluate it.

(ii) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate plane $x=0,\ y=0,\ z=0$.

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BMA101

Bharath University, Chennai – 73
B.Tech, I Semester, May 2010
BMA101 – Mathematics - I
(common to all branches) (2008 & 2009 batches)

Time: 3 Hrs **Maximum: 100 Marks**

(10 x 2 = 20)

Part A

Answer All Questions

1. Find the Eigen values of the matrix A^{-1} if $A = \begin{pmatrix} 1 & 5 \\ 0 & 4 \end{pmatrix}$
2. Show that the matrix $P = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
3. Test whether the plane x =3 touches the sphere $x^2 + y^2 + z^2 = 9$.
4. Give the general equation of the cone passes through the origin.
5. Find the envelope of the family given by $x = my + \frac{1}{m}$, 'm' being the parameter.
6. Write down the formula for radius of curvature interms of Parametric coordinates systems.
7. Find $\frac{du}{dt}$, $\;$ *if* $u = x^2 + y^2, x = at^2, y = 2at$.
8. If $u = \log(x^2 + xy + y^2)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$
9. Evaluate $\int_0^{\pi/2}\int_0^{\cos\theta} r^2\, dr\, d\theta$.
10. Change the order of Integration of $\int_0^a\int_y^a f(x,y)dx\,dy$.

Part B (5 x 16 = 80)

Answer either (a) or (b) from each question

11. (a) (i) Find the Eigen values and Eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(ii) Verify Cayley – Hamilton for the Matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

(OR)

(b) Find the canonical form of the quadratic form :

$$2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$$

12. (a) (i) Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z = 3$ as a great circle.

(ii) Find the equation of the right circular cylinder whose axis is the line $x = 2y = -z$ and radius 4.

(OR)

(b) (i) Find the equation of the right circular cone whose vertex is at the origin and axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi vertical angle of 30°.

(ii) Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$.

13. (a) (i) Find the radius of curvature at the point (0, c) on the curve

$$y = c\cosh\left(\frac{x}{c}\right)$$

(ii) Find the evolute of the parabola $y^2 = 4ax$.

(OR)

(b) (i) Find the circle of curvature at (3, 4) on $xy = 12$.

(ii) Find the envelop of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$ subject to the condition $a + b = 1$.

14. (a) (i) If $u = \frac{y^2}{2x}$ and $V = \frac{x^2 + y^2}{2x}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.

(ii) Expand $e^x \log(1 + y)$ in powers of x and y upto terms of third degree.

(OR)

(b) (i) Find the extreme value of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

(ii) Find the maximum value of $x^m y^n z^p$ when $x + y + z = a$.

15. (a) (i) Change the order of integration in $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} y \, dx \, dy$ and then evaluate it.

(ii) Express $\int_0^a \int_y^a \frac{x^2 \, dx \, dy}{(x^2 + y^2)^{3/2}}$ in polar coordinates and then evaluate it.

(OR)

(b) (i) Find the area enclosed by the curve $y = x^2$ and $x + y - z = 0$

(ii) Evaluate $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)\, dx\, dy\, dz$.

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Arrear

BMA101

Bharath University, Chennai – 73
B.Tech, I Semester, May 2009
BMA101 – Mathematics – I (2008 batch)
(common to all branches except B.Arch)

Time: 3 Hrs | **Maximum: 100 Marks**

(10 x 2 = 20)

Part A

Answer All Questions

1. Find the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$
2. Show that the matrix $\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & -\cos\theta \end{pmatrix}$ is orthogonal.
3. Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
4. Define a right circular cylinder.
5. Write the formula for radius of curvature in Cartesian form.
6. Find the envelope of the family of lines

   $y = mx + \sqrt{1+m^2}$, m being the parameter.
7. If $z(x + y) = x^2 + y^2$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$
8. Find the Jacobian of the transformation $x = r\cos\theta$; $y = r\sin\theta$
9. Evaluate $\int_1^2 \int_1^3 xy^2\,dxdy$.
10. Express the area bounded by the circle $x^2 + y^2 = 1$ interms of double integral.

Part B (5 x 16 = 80)

Answer either (a) or (b) from each question

11. (a) Verify Cayley–Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

and hence find A^{-1}

(OR)

(b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form and specify the matrix of transformation.

12. (a) Find the centre and radius of the circle :

(i) $x^2 + y^2 + z^2 - 2y - 4z = 11, \quad x + 2y + 2z = 15.$

(ii) Find the equation of the sphere which touches the plane $x - 2y - 2z = 7$ at the point (3, -1, -1) and passes through the point (1, 1, - 3).

(OR)

(b) (i) Find the equation of the right circular cone generated when the straight line $2y + 3z = 6, x=0$ revolves about the z-axis.

(ii) The radius of a normal section of a right circular cylinder is 2 units, the axis lies along the straight line

$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-2}{5}$, find the equation of the right circular cylinder.

13. (a) (i) Find the radius of curvature at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ of the curve $x^3 + y^3 = 3axy$.

(ii) Show that the radius of curvature at any point on the curve $x = a\cos^3 t;\ y = a\sin^3 t$ is $-3a \sin t \cos t$

(OR)

(b) Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is another equal cycloid.

14. (a) (i) If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-1}{2}\cot u.$

(ii) Expand $x^2y + 3y - z$ inpowers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem.

(OR)

(b) Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$.

15. (a) (i) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$

(ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy$ by changing to polar coordinates and hence show that $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

(OR)

(b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dxdydz$

(ii) Calculate the volume of the solid bounded by the planes $x = 0, y = 0, \quad x + y + z = 1$ and $z = 0$.

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BMA101

(only above code to be shaded in the Answer book #)

Bharath University, Chennai – 73

B.Tech, I Semester, Nov 2012

BMA101 – Mathematics I

(common to all branches) (2012 batch)

Time: 3 Hrs — Maximum: 100 Marks

(10 x 2 = 20)

Part A

Answer All Questions

1. Using the properties of eigen values, find the eigen values of Adj (A) if the matrix $(A)_{3x3}$ has two eigen values 1, 1 and det (A) = 4.
2. State any two uses of cayley – Hamilton theorem.
3. Find the equation of the sphere of centre at (1, 2, 3) and touching a plane at (2, 1, 3).
4. Define Right circular cone.
5. Find 'ρ' at any point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$.
6. Find the envelope of $y = mx + \sqrt{1+m^2}$ where m is a parameter.
7. Using Euler's theorem, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$.
8. If $x = u(1+v)$, $y = v(1+u)$, find the value of $\frac{\partial(x,y)}{\partial(u,v)}$.
9. Sketch the region of integration of the integral $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} f(x,y)\, dy\, dx$.

10. Change the following double integral into Polar form $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dx\,dy}{x^2+y^2}$

Part B (5 x 16 = 80)

Answer either (a) or (b) from each question

11. (a) (i) Diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ by means of an orthogonal transformation.

(ii) Using Cayley - Hamilton theorem, find the inverse of the matrix A where $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$

(OR)

(b) (i) Find the eigen vectors of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(ii) Find the nature of the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ also find signature.

12. (a) (i) Find the centre and radius of the circle given by $x^2+y^2+z^2+2x-2y-4z-19=0$ and $x+2y+2z+7=0$

(ii) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ also find the point of contact.

(OR)

(b) (i) Find the equation of the cone whose vertex is the point (1, 1, 0) and whose base is the curve $x^2+z^2=4, y=0$

(ii) Find the equation of the right circular cylinder whose guiding circle is $x^2+y^2+z^2=9,\ x-y+z=3$

13. (a) (i) Find the evolute of $y^2=4x$ considering it as the envelope of its normals.

(ii) Find the equation of the circle of curvature at (c, c) on xy = c²

(OR)

(b) Find the equation of the evolute of the curve $x^{2/3}+y^{2/3}=a^{2/3}$

14. (a) (i) If $v=(x^2+y^2+z^2)^{-1/2}$ then find $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}$

(ii) Expand $f(x, y)=e^x \cos y$ in powers of x and y as far as the terms of third degree using Taylor's series.

(OR)

(b) (i) If $x+y+z=u,\ y+z=uv,\ z=uvw$, Prove $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$

(ii) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

15. (a) (i) Evaluate $\int_0^a \int_0^x \int_0^{x-y} e^{x+y+z}\,dx\,dy\,dz$

(ii) Find by double integration, the area inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$

(OR)

(b) (i) Change the order of integration in $\int_0^1 \int_y^{2-y} xy\,dx\,dy$ and hence evaluate it.

(ii) Transform the integral into polar co-ordinates and hence evaluate

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$$

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QUESTION BANK FOR *MATHEMATICS I*(BMA101)

PART-A

UNIT-I MATRICES

1. Find sum and product of the eigen values of the matrix $A=\begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$
2. Find sum and product of the eigen values of the matrix $A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$
3. If 1 and 2 are eigen values of a 2 x 2 matrix A, what are the eigen values of A^2 and A^{-1}.
4. If 2 and 3 are eigen values of $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, what are the eigen values of A^{-1} and A^3.
5. State Cayley-Hamilton theorem.
6. State two uses of Cayley-Hamilton theorem.
7. Write the matrix corresponding to the quadratic form $2x^2 + y^2 + z^2 + 2xy - 4zy - 6zx$.
8. Write the matrix corresponding to the quadratic form $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 6x_2x_3$.
9. Find the nature of the quadratic form $2x_1^2 + 2x_1x_2 + 3x_2^2$.
10. Find the nature of the quadratic form whose matrix is $\begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$.

UNIT-II THREE DIMENSIONAL ANALYTICAL GEOMETRY

1. Find the equation of sphere whose centre is (2,-1,0) and radius 4.
2. Find the centre and radius of the sphere $x^2+y^2+z^2-4x+6y+4=0$.
3. Find the equation of sphere on the line joining the points (1,2,3) and (0,4,-1) as diameter.
4. Test whether the plane x=3 touches the sphere $x^2+y^2+z^2=9$.
5. Show that the spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ intersect at right angles.
6. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$.
7. Find the equation of the sphere described on the line joining the points (-5,5,1) and (4,1,7) as diameter.

8. Find the equation of the cone whose vertex is at the origin and guiding curve is $\frac{x^2}{4}+\frac{y^2}{9}+z^2=9; x+y+z=1$.
9. Define right circular cone.
10. Write the equation of right circular cylinder.

UNIT-III DIFFERENTIAL CALCULUS

1. Write down the formula for radius of curvature in Cartesian form.
2. What is the curvature of a straight line?
3. Find the radius of curvature of $x^2+y^2+4x-12y+4=0$.
4. Find the radius of curvature of $y=e^x$ at (0,1)
5. Find the radius of curvature of the curve given by $x=3+2\cos\theta, y=4+2\sin\theta$.
6. Write the formula for circle of curvature.
7. Define evolute.
8. Define envelope of family of curves.
9. Find envelope of family of straight lines $y=mx+\frac{1}{m}$.
10. Find the envelope of $y=mx+\sqrt{a^2m^2+b^2}$.

UNIT-IV FUNCTIONS OF SEVERAL VARIABLES

1. If $u=x^4+y^4+6x^3y^3$, find $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}$.
2. If $u=\log(x^2+xy+y^2)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2$.
3. State Euler's theorem.
4. If $u=x^2+y^2$, $x=at^2, y=2at$, find $\frac{du}{dt}$.
5. If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
6. If $u=x-\frac{y}{2}, v=\frac{y}{2}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
7. Define stationary point.
8. Find stationary points of $x^2-xy+y^2-2x+y=0$.
9. Write anyone property of Jacobian.
10. Write down Taylor's series expansion.

UNIT-V MULTIPLE INTEGRALS

1. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$

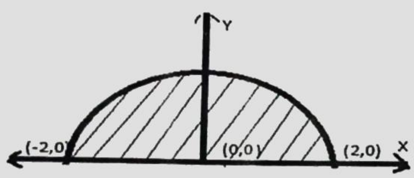
2. Evaluate $\int_0^1\int_0^x dydx$

3. Evaluate $\int_0^1\int_0^2 e^{x+y}dxdy$

4. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} rdrd\theta$

5. Evaluate $\int_0^1\int_0^2\int_0^3 xyzdzdydx$

6. Evaluate $\iint_R dxdy$ where R is the shaded region in the figure.



7. Sketch roughly the region of integration for $\int_0^a\int_y^a f(x,y)dxdy$

8. Sketch roughly the region of integration for $\int_0^1\int_0^x f(x,y)dydx$

9. Change the order of integration in $\int_0^1\int_x^1 \frac{x}{x^2+y^2}dydx$

10.Change the order of integration in $\int_0^1\int_x^1 dydx$.

PART-B

UNIT-I MATRICES

1.Find the eigen values and eigen vectors of the matrix A= $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

2. Verify Cayley Hamilton theorem for $A=\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence compute its inverse.

3. Verify Cayley Hamilton theorem for $A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and hence find A^4.

4. Diagonalise the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ by orthogonal transformation.

5. Reduce the quadratic form into canonical form $x^2+y^2+z^2-2xy-2yz-2xz$.

UNIT-II THREE DIMENSIONAL ANALYTICAL GEOMETRY

1. Find the centre and radius of the circle given by $x^2+y^2+z^2+2x-2y-4z-19=0$ and $x+2y+2z+7=0$.
2. Find the equation of tangent plane to the sphere $x^2+y^2+z^2-4x-2y-6z+5=0$ which are parallel to plane $x+4y+8z=0$. Find also their point of contact.
3. Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$; $x+y+z=3$ as a great circle.
4. Find the equation of the sphere that passes through the circles $x^2+y^2+z^2+x-3y+2z-1=0$; $2x+5y-z+7=0$ and cuts orthogonally the sphere $x^2+y^2+z^2-3x+5y-7z-6=0$.
5. Find the equation of the cone with vertex (1,1,1) and passing through the curve of intersection of $x^2+y^2+z^2=1$ and $x+y+z=1$.

UNIT-III DIFFERENTIAL CALCULUS

1. Find the radius of curvature at the point (a,a) on the curve $x^3+y^3=2a^3$.
2. Find the equation of circle of curvature at (c,c) on $xy=c^2$.
3. Find the evolute of the parabola $y^2=4ax$.
4. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$; $y=a(1-\cos\theta)$ is another cycloid.
5. Find the envelope of $\frac{x}{a}+\frac{y}{b}=1$ where a and b are connected by the relation $a^2+b^2=c^2$, where 'c' is a constant.

UNIT-IV FUNCTIONS OF SEVERAL VARIABLES

1. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ prove that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$ and

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$

2. Expand $e^x \log(1+y)$ in powers of x and y up to 3^{rd} degree terms.
3. Examine $f(x,y) = x^3 + y^3 - 3x - 12y + 20$, for its extremes values.
4. A rectangular box open at the top is to have volume of 32cc. Find the dimensions of the box that requires the least material for its construction.
5. Find the greatest and least distance of the point (3,4,12) from the unit sphere whose centre is at the origin.

UNIT-V MULTIPLE INTEGRALS

1. Change the order of integration and hence evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$.

2. Find the area between the parabolas $3y^2 = 25x$ and $5x^2 = 9y$.

3. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$.

4. Find the volume of the tetrahedron bounded by the co-ordinate planes and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

5. Change the order of integration in $\int_0^{\infty}\int_0^{y} y e^{\frac{-y^2}{x}} dx\,dy$ and hence evaluate.

Bharath

INSTITUTE OF HIGHER EDUCATION AND RESEARCH

(Declared as Deemed - to - be - University under section 3 of UGC Act 1956)

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

STUDENT NAMELIST - CSE

Serial No	Register No.	Student Name	Attendance Percentage(%)
1.	U19CS001	AADHISWARAN T	88
2.	U19CS002	AAKAVARAM KIRAN	89.5
3.	U19CS003	ABBURI SAMPATH KUMAR	85
4.	U19CS004	ABDUL SHARUKH	92
5.	U19CS005	ABHISHEK GANDEPALLI	92.3
6.	U19CS006	ABHISHEK KUMAR SINGH	95
7.	U19CS007	ABINAYA A	89
8.	U19CS008	ACHAM SUKUMAR	94
9.	U19CS009	ACHUTHA TEJA	96.5
10.	U19CS010	ADABALA PUSHKAR RAMA SAI PRAVE	92
11.	U19CS011	ADAPA KRISHNAVARDHAN NAIDU	98
12.	U19CS012	ADAPALA CHAITANYA NAGA SAI	89
13.	U19CS013	ADAPALA CHUPAK PHANI SAI	88
14.	U19CS014	ADDALA BHANOJ	93
15.	U19CS015	ADDANKI CHINNARAO	93
16.	U19CS016	ADEPU VENKATESH	95
17.	U19CS017	ADHITHIYAN K	94
18.	U19CS018	ADITHYA KANNAN K	96
19.	U19CS019	ADITYA RAJ	94
20.	U19CS020	T G S ADITYA	96
21.	U19CS021	ADLA HASINI	93.5
22.	U19CS022	ADLA PAVAN REDDY	97
23.	U19CS023	ADUSUMALLI RAGA VENKATA MANI KISHORE	98
24.	U19CS024	AFKHAN NAWAZ KHAN	94.2
25.	U19CS025	AIDA PRANEETH	98
26.	U19CS026	AINALA KARTHIK	94

27.	U19CS027	AISHWARYA J	94.5
28.	U19CS028	AJAY KUMAR M	95
29.	U19CS029	AKANKSHA PARVATHANENI	94
30.	U19CS030	AKASH S	96
31.	U19CS031	AKASH T	94
32.	U19CS032	AKULA VENKATESH	95
33.	U19CS033	AKULA VINAYAK	95.3
34.	U19CS034	AKUNURI KRISHNASAI	94
35.	U19CS035	ALA ABHISHEK KUMAR	95
36.	U19CS036	ALA NIKHIL KUMAR REDDY	96
37.	U19CS037	ALAMURU LIKHITHA	98
38.	U19CS038	ALAVALA SAI CHANDU	95.2
39.	U19CS039	ALETI ANJI REDDY	96
40.	U19CS040	ALLENKI USHA REDDY	97
41.	U19CS041	ALLURI VIJAY	94
42.	U19CS042	ALURI SANDEEP	98
43.	U19CS043	ALUVALA KEERTHAN CHAND	92
44.	U19CS044	AMANAGANTI VIKAS	96
45.	U19CS045	AMARAVARAPU CHENNAKESAVA RAYUDU	98
46.	U19CS046	AMARAVARAPU NAGA VAMSI	94
47.	U19CS047	AMARTYA KUMAR	95
48.	U19CS048	AMBADIPUDI CHARETHARDHA	94
49.	U19CS049	AMI REDDY VARSHITHA REDDY	96
50.	U19CS050	AMRIT SAH	94.2
51.	U19CS051	AMRITHA VARSHINI G	94.8
52.	U19CS052	AMUDALAPALLI SIVAKIRAN	98
53.	U19CS053	ANAGANI HARSHAVARDHAN	93
54.	U19CS054	ANAKAPALLI CHANDRABABU NAIDU	99
55.	U19CS055	ANBARASU A	94
56.	U19CS056	ANIMIREDDY GUNA SEKHAR	89
57.	U19CS057	ANKAM VISHNUVARDHAN BABU	94
58.	U19CS058	ANKIPALLI SIDDARDHA	89
59.	U19CS059	ANMOL KUMAR SONI	94
60.	U19CS060	ANNAM VAMSI	85
61.	U19CS061	ANNAM YASWANTH	90
62.	U19CS062	CH. ANIKETH	91

63.	U19CS063	ANNE JAYA KRISHNA	92.3
64.	U19CS064	ANNEBOINA RAHUL GOUD	94.4
65.	U19CS065	ANTHAM ROHITH REDDY	95.2
66.	U19CS066	ANTHATI UDAY GOUD	94
67.	U19CS067	ARAMALLA YASHWANTH REDDY	95.6
68.	U19CS068	ARETI SUPRIYA	94.4
69.	U19CS069	ARIGELA SRINIVASARAO	95.3
70.	U19CS070	ARJUN J	93
71.	U19CS071	ARRA SAI PRASANNA	97
72.	U19CS142	N. CHARAN REDDY	98
73.	U19CS152	INDURI SUSMITHA	94
74.	U19CS074	ARUKONTHAM DEVENDER REDDY	95
75.	U19CS075	ASLAM SAFIQ A	96

Bharath

INSTITUTE OF HIGHER EDUCATION AND RESEARCH

(Declared as Deemed-to-be University under section 3 of UGC Act, 1956)

(Vide Notification No. F.9-5/2000 - U.3, Ministry of Human Resource Development, Govt. of India, dated 4th July 2002)

Department of Mathematics

U18BSMA101 - ENGINEERING MATHEMATICS I (2019-2020)

END SEMESTER EXAM											
CO	CO ATTAINMENT AVERAGE FROM ASSESSMENT TEST	MARKS ALLOTTED	MARKS OBTAINED	CO ATTAINMENT AVERAGE FROM END SEMESTER EXAM	DIRECT CO ATTAINMENT	INDIRECT CO ATTAINMENT (OBTAINED FROM EXIT SURVEY)	TOTAL CO ATTAINMENT (%)	TARGET [CLASS AVERAGE] (%)	CO ATTAINMENT GAP [TARGET - ATTAINMENT](%)	Actions Proposed to bridge the Gap (Gap >0)	Modification of Target when achieved (Gap <=0)
CO1	69	16	14	88	80	81	80	75	-5	Target Attained	Target Increased to 85
CO2	68	16	13	81	76	73	75	72	-3	Target Attained	Target Increased to 80
CO3	71	16	13	81	77	84	79	75	-4	Target Attained	Target Increased to 83
CO4	67	16	12	75	72	81	74	70	-4	Target Attained	Target Increased to 78
CO5	77	16	11	69	72	84	74	70	-4	Target Attained	Target Increased to 80
CO6	92	20	14	70	79	87	80	75	-5	Target Attained	Target Increased to 85

Co attainment score

S.No	Reg.No	Name						
1	U19CS001	AADHISWARAN T	71	64	79	73	60	100
3	U19CS003	ABBURI SAMPATH KUMAR	71	86	96	73	100	100
4	U19CS004	ABDUL SHARUKH	79	73	86	91	100	100
6	U19CS006	ABHISHEK KUMAR SINGH	71	86	79	73	60	100
7	U19CS007	ABINAYA A	93	64	71	86	100	100
8	U19CS008	ACHAM SUKUMAR	79	0	79	68	100	100
9	U19CS009	ACHUTHA TEJA	0	82	71	82	60	100
10	U19CS010	ADABALA PUSHKAR RAMA SAI PI	75	64	57	95	100	100
11	U19CS011	ADAPA KRISHNAVARDHAN NAIDU	89	59	61	73	100	100
12	U19CS012	ADAPALA CHAITANYA NAGA SAI	93	82	82	73	100	100
13	U19CS013	ADAPALA CHUPAK PHANI SAI	57	77	89	77	100	100
14	U19CS014	ADDALA BHANOJ	82	68	82	91	100	100
15	U19CS015	ADDANKI CHINNARAO	68	68	71	68	100	100
17	U19CS017	ADHITHIYAN K	71	68	79	73	60	100
18	U19CS018	ADITHYA KANNAN K	79	82	86	77	100	100
19	U19CS019	ADITYA RAJ	57	0	68	86	100	100
20	U19CS020	T G S ADITYA	0	59	79	64	100	100
21	U19CS021	ADLA HASINI	54	73	79	73	100	100
23	U19CS023	ADUSUMALLI RAGA VENKATA M.	64	82	86	55	100	100
24	U19CS024	AFKHAN NAWAZ KHAN	89	86	0	0	100	100
25	U19CS025	AIDA PRANEETH	71	77	79	73	60	100
26	U19CS026	AINALA KARTHIK	86	59	82	86	60	100
27	U19CS027	AISHWARYA J	64	68	86	59	100	100
28	U19CS028	AJAY KUMAR M	54	73	82	68	100	100
29	U19CS029	AKANIKSHA PARVATHANENI	79	0	68	86	100	100
30	U19CS030	AKASH S	0	82	68	82	100	100
31	U19CS031	AKASH T	96	55	75	82	100	100
32	U19CS032	AKULA VENKATESH	93	50	93	64	100	100
33	U19CS033	AKULA VINAYAK	64	55	89	59	100	100
34	U19CS034	AKUNURI KRISHNASAI	93	45	68	64	100	100
35	U19CS035	ALA ABHISHEK KUMAR	68	86	82	68	100	100
36	U19CS036	ALA NIKHIL KUMAR REDDY	71	86	79	73	60	100
37	U19CS037	ALAMURU LIKHITHA	86	77	86	82	100	100
38	U19CS038	ALAVALA SAI CHANDU	79	77	68	82	100	100
39	U19CS039	ALETI ANJI REDDY	79	64	86	77	100	100
40	U19CS040	ALLENKI USHA REDDY	79	82	86	73	100	100
41	U19CS041	ALLURI VIJAY	75	86	86	82	60	100
42	U19CS042	ALURI SANDEEP	71	59	79	73	60	100
44	U19CS044	AMANAGANTI VIKAS	75	73	71	59	100	100
45	U19CS045	AMARAVARAPU CHENNAKESAVA	64	68	71	41	100	100
46	U19CS046	AMARAVARAPU NAGA VAMSI	68	64	75	77	100	100
47	U19CS047	AMARTYA KUMAR	89	0	93	64	100	100
48	U19CS048	AMBADIPUDI CHARETHARDHA	0	55	79	82	100	100
49	U19CS049	AMI REDDY VARSHITHA REDDY	71	73	86	77	100	100
50	U19CS050	AMRIT SAH	71	59	75	82	100	100

51	U19CS051	AMRITHA VARSHINI G	89	68	89	68	100	100
53	U19CS053	ANAGANI HARSHAVARDHAN	82	64	86	59	100	100
54	U19CS054	ANAKAPALLI CHANDRABABU NA	64	73	64	73	100	100
55	U19CS055	ANBARASU A	68	68	75	64	100	100
57	U19CS057	ANKAM VISHNUVARDHAN BABU	0	86	86	82	100	100
58	U19CS058	ANKIPALLI SIDDARDHA	79	91	93	86	100	100
59	U19CS059	ANMOL KUMAR SONI	89	59	0	0	60	100
60	U19CS060	ANNAM VAMSI	68	82	86	68	100	100
61	U19CS061	ANNAM YASWANTH	75	55	71	77	100	100
62	U19CS062	ANIKETH	46	86	82	36	100	100
63	U19CS063	ANNE JAYA KRISHNA	93	0	79	91	100	100
64	U19CS064	ANNEBOINA RAHUL GOUD	0	55	93	82	100	100
65	U19CS065	ANTHAM ROHITH REDDY	71	55	86	77	100	100
66	U19CS066	ANTHATI UDAY GOUD	93	50	93	64	100	100
67	U19CS067	ARAMALLA YASHWANTH REDDY	64	55	89	59	100	100
68	U19CS068	ARETI SUPRIYA	93	45	68	64	100	100
69	U19CS069	ARIGELA SRINIVASARAO	68	86	82	68	100	100
70	U19CS070	ARJUN J	71	86	79	73	60	100
71	U19CS071	ARRA SAI PRASANNA	86	77	86	82	100	100
72	U19CS142	CHARAN REDDY	79	77	68	82	100	100
73	U19CS152	INDURI SUSMITHA	79	64	86	77	100	100
74	U19CS074	ARUKONTHAM DEVENDER REDD	79	82	86	73	100	100
75	U19CS075	ASLAM SAFIQ A	75	0	86	82	60	100
		average	70	64	77	71	92	98
		No of students above average	52	51	53	50	58	69

CO attainment through students Performance

Department of Mathematics

Year	I year	Semester	I
Subject code	U18BSMA101	Subject	Engineering Mathematics I
Test	All test	Strength	75

	CO1	CO2	CO3	CO4	CO5	CO6
Average Mark	70	64	77	71	92	98
No.of students above average	52	51	53	50	58	69
Total no. of students	75	75	75	75	75	75
% CO attainment	69.3	68.0	70.7	66.7	77.3	92.0

CO INDIRECT ATTAINMENT – SURVEY REPORT

CO	No. of 5's	No. of 4's	No. of 3's	No. of 2's	No. of 1's	CO %
CO1	19	23	19	9	5	81.3
CO2	21	18	16	10	10	73.3
CO3	18	23	22	6	6	84.0
CO4	20	22	19	8	6	81.3
CO5	18	24	21	7	5	84.0
CO6	23	24	18	4	6	86.7
Total	119	134	115	44	38	

PO mapping against CO							Aver. PO
	CO1	CO2	CO3	CO4	CO5	CO6	
PO1	3	3	3	3	3	2	76.82
PO2	3	3	3	3	3	3	77.00
PO3		1	2	2	2		75.57
PO4	2	1	2	1			76.00
PO5		2	1	2	1		75.17
PO12	1	2	2	3	3	3	76.57
%CO TOTAL Attainment	80.0	75.0	79.0	74.0	74.0	80.0	

All the PO's are above the set value(50%)