Bharath

INSTITUTE OF HIGHER EDUCATION AND RESEARCH

Declared as **Deemed-to-be-University** u/s 3 of the UGC Act, 1956

B.Tech Civil Engineering

U20MABT03 - Transforms and Boundary Value Problems

Course File

School of Civil and Infrastructure Engineering

Vision and Mission of the Department

Vision

The Department of Civil Engineering is striving to become as a world class academic centre for quality education and research in diverse areas of civil engineering, with a strong social commitment.

Mission

Mission of the department is to achieve international recognition by:

M1: Producing highly competent and technologically capable professionals.

M2: Providing quality education in undergraduate and post graduate levels, with strong emphasis on professional ethics and social commitment.

M3: Developing a scholastic environment for the state – of –art research, resulting in practical applications.

M4: Undertaking professional consultancy services in specialized areas of civil engineering.

Program Educational Objectives (PEOs)

PEO1: PREPARATION

Civil Engineering Graduates are in position with the knowledge of Basic Sciences in general and Civil Engineering in particular so as to impart the necessary skill to analyze, synthesize and design civil engineering structures.

PEO2: CORE COMPETENCE

Civil Engineering Graduates have competence to provide technical knowledge, skill and also to identify, comprehend and solve problems in industry, research and academics, related to recent developments in civil and environmental engineering.

PEO3: PROFESSIONALISM

Civil Engineering Graduates are successfully work in various Industrial and Government organizations, both at the National and International level, with professional competence and ethical administrative insight so as to be able to handle critical situations and meet deadlines.

PEO4: SKILL

Civil Engineering Graduates have better opportunity to become a future researchers/ scientists with good communication skills so that they may be both good team-members and leaders with innovative ideas for a sustainable development.

PEO5: ETHICS

Civil Engineering Graduates are framed to improve their technical and intellectual capabilities through life-long learning process with ethical feeling so as to become good teachers, either in a class or to juniors in industry.

PROGRAMME OUTCOMES (POs)

On completion of B.Tech in Civil Engineering Programme, Graduates will have to

1) **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization for the solution of complex civil engineering problems

2) **Design/Development of Solutions:** Design solutions for complex civil engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety, and cultural, societal, and environmental considerations.

3) **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

4) **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

5) **Problem analysis:** Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

6) **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

7) **Communication:** Communicate effectively on complex engineering activities with the engineering community and with t h e society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

8) **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

9) **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

10) **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

11) **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

12) **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

Bharath INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed - to - be - University under section 3 of UGC Act 1956)

COURSE FILE

FACULTY	Dr. S. ANUSUYA	FACULTY DEPT	MATHEMATICS
SUBJECT	TRANSFORMS AND BOUNDARY VALUE PROBLEMS	SUBJECT CODE	U20MAB T03
YEAR	2020 – 2023	SEMESTER	odd
DEG &BRANCH	CSE – B.Tech	DURATION	

SL.NO	DETAILS IN COURSE FILE	REMARKS
1.	LEARNING OUTCOMES	
2.	LESSON PLAN &CO-PO MAPPING	
3.	SYLLABUS WITH COURSE OUTCOMES	
4.	INDIVIDUAL TIME TABLE	
5.	TEXT BOOK AND REFERENCE BOOK	
6.	LECTURE NOTES (FOR ALL UNITS)	
7.	INTERNAL ASSESSMENT I - QUESTION PAPER	
8.	INTERNAL ASSESSMENT I - KEY	
9.	INTERNAL ASSESSMENT I – SAMPLE ANSWER SHEETS	
10.	INTERNAL ASSESSMENT II - QUESTION PAPER	
11.	INTERNAL ASSESSMENT II - KEY	
12.	INTERNAL ASSESSMENT II-SAMPLE ANSWER SHEETS	
13.	INTERNAL ASSESSMENT III- QUESTION PAPER	
14.	INTERNAL ASSESSMENT III - KEY	
15.	INTERNAL ASSESSMENT III– SAMPLE ANSWER SHEETS	
16.	INTERNAL ASSESSMENT IV–ASSIGNMENT QUESTIONS	
17.	SAMPLE ASSIGNMENTS	
18.	END SEMESTER QUESTION PAPER	
19.	END SEMESTER ANSWER KEY	
20.	STUDENT PERFORMANCE RECORD	
21.	STUDENT ATTENDANCE RECORD	
22.	COURSE END SURVEY	
23.	CO ATTAINMENT	

STAFF   HOD   DEAN S&H

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH

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173, Agaram Main Road, Selaiyur, Chennai – 600 073,Tamil Nadu

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH

Department of Science and Humanities/ Mathematics

LEARNING OUTCOMES:

Degree and Programme	B.E & Common for all branches
Year and Semester:	II year & III Semester
Subject Code and Subject Title:	U20MABT03 / BS and Transforms and Boundary Value problem
Prerequisite:	Diploma
Course Category	BS
L T P C	3 1 0 4
Date of commencement:	
Faculty Incharge:	Dr. S. Anusuya

Transforms and boundary value problems are fundamentals to virtually all of higher mathematics and its applications in the natural, social, and management sciences. These topics, therefore, form the core of the basic requirements in mathematics both for mathematics majors and for students of science and engineering

Course Objectives:

- Expand given function using the knowledge of Fourier Series and frequently needed practical harmonic analysis that an Engineer may have to make from discrete data..
- Solve PDE and Higher order with constant co-efficient and physically interpret the results.
- Apply PDE in Boundary Value Problems and Analyze the solution involving PDE.
- Solve many problems in Engineering by applying Fourier Transforms with the possible special cases with attention to their applications.
- Apply the basics of Z- Transforms in its applicability to discretely varying functions gained the skill of formulate certain problems in terms of difference equation and solve them using the Z- Transforms techniques.

LESSON PLAN & CO-PO MAPPINGS:

CO-PO Mappings

Mapping of Course Outcome with Programme Outcomes (PO) & PSO
(H/M/L indicates strength of correlation) H – High, M – Medium, L – Low

CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PS01	PS02	PS03
CO1	3	3	-	-	-	-	-	-	-	-	-	1			
CO2	3	3	-	-	1	-	-	-	-	-	-	2			
CO3	3	3	-	-	2	-	-	-	-	-	-	2			
CO4	3	3	2	-	1	-	-	-	-	-	-	1			
CO5	3	3	1	-	-	-	-	-	-	-	-	2			

(Tick mark or level of correlation: 3-High, 2-Medium, 1-Low)

Course Plan

Content delivery methods:

- ✓ Lecture interspersed with discussion (chalk and board)
- ✓ Online through PDF and Video Conference
- ✓ Presentation slides (PPT)

Assessment methods:

- ✓ Internal Assessment test
- ✓ Assignments (Problems, Seminars)

LEARNING RESOURCES

TIME TABLE ACADEMIC YEAR 2021 - 2022 (ODD SEMESTER)

1. Text Books:

Reference Code	Description
R1	Kandasamy, P., etal., Engineering Mathematics, Vol. II & Vol. III (4th revised edition), S.Chand & Co., New Delhi, 2000
R2	Grewal B.S, "Higher Engg Maths", Khanna Publications, 42nd Edition, 2012.
R3	Kreyszig.E, "Advanced Engineering Mathematics", 10th edition, John Wiley & Sons. Singapore,2012
R4	Sivaramakrishna Das P. and Vijayakumari.C, A text book of Engineering Mathematics III, Viji"s Academy,2010
R5	Narayanan. S., Manickavachagom Pillay. T . and Ramanaiah, G., Advanced Mathematics for Engineering students, Volume II & III (2nd edition), S.Viswanathan Printers and Publishers, 1992
R6	Venkataraman, M,K., Engineering Mathematics - Vol.III - A & B (13th edition), National Publishing Co., Chennai, 1998.
R7	Veerarajan, T., „Engineering mathematics", Tata McGraw-Hill (Education) India Pvt.Ltd, 2006
R8	P.A.Anand, QUANTITATIVE APTITUDE for competitive examinations, Wiley Publications,2016

2. Teaching Tool Planned:

Type Code	Teaching Tool Planned
T1	Black board etc.
T2	Power Point Presentation
T3.	Tutorial and Problem Solving
T4	Video Presentation
T5	Notes

3. Other Resources (Online Resources or others):

- Partial differential equations (https://www.math.uni-leipzig.de/~miersemann/pdebook.pdf)
- Fourier Series (https://www.studocu.com/row/document/makerere-university/electrical-engineering/fourier-series-engineering-mathematics/6364142)
- Boundary value problems (https://sites.ualberta.ca/~niksirat/ODE/chapter-7ode.pdf)
- Fourier Transforms (https://www.thefouriertransform.com/)
- Z – Transforms (https://learn.lboro.ac.uk/archive/olmp/olmp_resources/pages/workbooks_1_50_jan2008/Workbook21/21_2_bscs_z_trnsfm_thry.pdf)

INDIVIDUAL TIME TABLE

Year / Sem: II / III

Individual Timetable

Day/ Period	I 9.00 AM – 9.50 AM	II 9.50 AM – 10.40AM		III 10.50 AM – 11.40 AM	IV 11.40 AM – 12.30 PM		V 1.30 PM – 2.20 PM	VI 2.20 PM – 3.10 PM	VII 3.10 PM – 4.00 PM
MON	S6 SA303		BREAK	S18 SA311		LUNCH			S24 AM302
TUE	S18 SA311			S6 SA303					S24 AM302
WED		S18 SA311			S6 SA303		S24 AM302		
THUR		S6 SA303			S24 AM302		S18 SA311		
FRI	S6 SA303			S24 AM302			S18 SA311		

ONLINE TIME TABLE 2021
TERM-III (Odd Semester)

Day/ Period	I 9.30 AM – 10.30 AM	II 10.40 AM – 11.40AM	III 11.50 AM – 12.50 AM	IV 1.50 PM – 2.50 PM	V 3.00 PM – 4.00 PM
MON	L SA311		C SA303		N AM302
TUE	N AM302	C SA303		L SA311	
WED	L SA311		N AM302	C SA303	
THUR	N AM302		L SA311		C SA303
FRI	C SA303		N AM302		L SA311

	SUMMARY OF COURSE CONTENT	Hrs	CO's
01	**UNIT I - FOURIER SERIES** Introduction to Differentiation, integration and Basic Formulas.	1	CO1
02	Periodic functions, Dirichlet's Condition and Definition of Fourier Series	1	CO1
03	To Knows the definition of Fourier series and problems under Fourier series.	1	CO1
04	Problems in the intervals (0,2l) and (0, 2π).	1	CO1
05	Solving More Problems in the intervals (0,2l) and (0, 2π).	1	CO1
06	Problems to find expansion in the interval (-l, l) and (-π, π).	1	CO1
07	Problems under odd and even function in the intervals (-l,l) and (-π, π).	1	CO1
08	Tutorial	1	CO1
09	Half Range sine and Cosine series in (0, π) and (0,l)	1	CO1
10	Tutorial	1	CO1
11	Harmonic analysis and Solving Problems under Harmonic analysis.	1	CO1
12	Tutorial	1	CO1
13	**UNIT II PARTIAL DIFFERENTIAL EQUATIONS** Introduction to Partial Differential Equations, Formation of PDE by elimination of arbitrary constants -Problems	1	CO2
14	Formation of PDE by elimination of arbitrary functions-problems	1	CO2
15	Formation of PDE by elimination of arbitrary functions in $\phi(u,v)=0$	1	CO2
16	Methods to solve the standard types of Partial differential equations – Type -1, 2 and 3	1	CO2

17	Methods to solve the standard types of Partial differential equations – Type - 4, 5 and 6	1	CO2
18	Tutorial	1	CO2
19	Methods to solve the first order partial differential equations with constant coefficient -Type-1, Type-2 and Type -3.	1	CO2
20	Methods to solve the first order partial differential equations with constant coefficient -Type-4,and Type 5	1	CO2
21	Tutorial	1	CO2
22	Lagrange's Linear Equations-Method of Grouping	1	CO2
23	Lagrange's Linear Equations- Method of Multipliers.	1	CO2
24	Tutorial	1	CO2
25	**UNIT III -BOUNDARY VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS** Classification of 2nd order linear partial differential equations.	1	CO3
26	Introduction to one dimensional Wave Equation.	1	CO3
27	Initial value theorem and final value theorem and solving problems.	1	CO3
28	One dimensional Wave Equation Boundary and initial value Problems with zero velocity.	1	CO3
29	Boundary and initial value Problems with zero velocity - problems.	1	CO3
30	Boundary and initial value Problems with Non-zero velocity.	1	CO3
31	Tutorial	1	CO3
32	One dimensional heat equation - problems with zero boundary values.	1	CO3
33	Tutorial	1	CO3
34	Steady state conditions and Non-zero boundary conditions	1	CO3
35	Steady and transient states - problems	1	CO3
36	Tutorial	1	CO3

37	**UNIT IV-FOURIER TRANSFORMS** Introduction to Fourier transforms - statement of Fourier integral theorem.	1	CO4
38	Problems on Fourier Transforms in $(-\infty,\infty)$	1	CO4
39	Problems in inverse Fourier Transforms in $(-\infty,\infty)$	1	CO4
40	Tutorial	1	CO4
41	Properties of Fourier transforms	1	CO4
42	Properties of Fourier sine & cosine Transforms	1	CO4
43	Problems on Fourier sine & cosine Transforms in $(0,\infty)$	1	CO4
44	Transforms of simple functions	1	CO4
45	Tutorial	1	CO4
46	Convolution Theorem	1	CO4
47	Parseval's Identity; Integral equations	1	CO4
48	Tutorial	1	CO4
49	**UNIT V- Z-TRANSFORMS AND DIFFERENTIAL EQUATIONS**Introduction to Z-transforms	1	CO5
50	Properties of Laplace transform	1	CO5
51	Problems based on Z- transform and its properties	1	CO5
52	Inverse Z-transform, related problems, long division method	1	CO5
53	Tutorial	1	CO5
54	Inverse Z-transform - residue theorem method	1	CO5
55	Solving problems on general Inverse Z-transform	1	CO5
56	Convolution theorem and Based Problems	1	CO5
57	Solving Problems on Convolution theorem based problems	1	CO5
58	Tutorial	1	CO5
59	Solution of linear difference equations with constant coefficients using Z-transform	1	CO5
60	Tutorial	1	CO5

CONTENT OF THE COURSE

SYLLABUS

U20MABT03 / BS Transforms and Boundary Value problem

UNIT I FOURIER SERIES (9+3)

Dirichlet's conditions–General Fourier series – Half range Sine and Cosine series–Parseval's Identity– Harmonic Analysis.

UNIT II PARTIAL DIFFERENTIAL EQUATIONS (9+3)

Formation–Solutions of standard types of first order equations–Lagrange's linear equations – Linear partial differential equation of second and higher order with constant coefficients.

UNIT III BOUNDARY VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS (9+3)

Classifications second order linear partial differential equations – Solution of one dimensional wave equation – One dimensional heat equation – Steady state solution of two dimensional heat equation –Fourier series solutions in Cartesian coordinates.

UNIT IV FOURIER TRANSFORMS (9+3)

Fourier integral theorem (without proof) – Fourier transform pairs – Fourier sine and cosine transform – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

UNIT V Z- TRANSFORMS AND DIFFERENCE EQUATIONS (9+3)

Z – Transform – Elementary properties – Inverse Z – Transform – Convolution theorem – Formation of difference equations– Solution of difference equations using Z–Transform.

COURSE OUTCOMES

CO's NO.	Course Outcomes	Blooms Level
CO1	Expand given function using the knowledge of Fourier Series and frequently needed practical harmonic analysis that an Engineer may have to make from discrete data.	3
CO2	Solve PDE and Higher order with constant co-efficient and physically interpret the results.	3
CO3	Apply PDE in Boundary Value Problems and Analyze the solution involving PDE.	3
CO4	Solve many problems in Engineering by applying Fourier Transforms with the possible special cases with attention to their applications.	3
CO5	Apply the basics of Z- Transforms in its applicability to discretely varying functions gained the skill of formulate certain problems in terms of difference equation and solve them using the Z- Transforms techniques.	3

U20MABT03	TRANSFORMS AND BOUNDARY VALUE PROBLEMS	L	T	P	C
		3	1	0	4
	Total Contact Periods: 60				
	Prerequisite–U20MABT01 and U20MABT02 or Diploma				
	Department:-Department of Mathematics				

COURSEOBJECTIVES:

- Grasp the Fourier series expansion for given periodic function in specific intervals and their different forms.
- Learn techniques of solving the standard types of first order and second order partial differential equations.
- Learn solving wave and heat equation using Fourier series.
- Understand the problems using Fourier transform and their properties.
- Understand the problems using Z-transform and their properties.

Course Outcome(COs)	
CO1	Expand given function using the knowledge of Fourier Series and frequently needed practical harmonic analysis that an engineer may have to make from discrete data.
CO2	Solve PDE and higher order with constant coefficients and physically interpret the results.
CO3	Apply partial differential equations in boundary Value Problems and analyze the solutions involving partial differential equations.
CO4	Solve many problems in engineering by applying Fourier transform with the possible special cases with attention to their applications.
CO5	Apply the basics of Z– Transform in its applicability to discretely varying functions, gained the skill of formulate certain problems in terms of difference equations and solve them using the Z–Transform techniques.

Mapping of Course Outcomes with Program outcomes (POs)(1/2/3indicatesstrength of correlation)3-High,2-Medium,1-Low

1	COs/POs	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PS O1	PS O2	PS O3
2	CO1	3	3										1			
	CO2	3	3			1							2			
	CO3	3	3			2							2			
	CO4	3	3	2		1							1			
	CO5	3	3	1									2			
3	Category	Basic Sciences (BS)														
4	Approval	47th Meeting of Academic Council														

UNIT I FOURIER SERIES (9+3)

Dirichlet's conditions–General Fourier Series–Half range Sine and Cosine series–Parseval's Identity– Harmonic Analysis.

UNIT II PARTIAL DIFFERENTIAL EQUATIONS (9+3)

Formation–Solutions of standard types of first order equations–Lagrange's linear equations – Linear partial differential equation of second and higher order with constant coefficients.

UNIT III BOUNDARY VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS (9+3)

Classifications of second order linear partial differential equations – Solution of one dimensional wave equation – One dimensional heat equation – Steady state solution of two dimensional heat equation –Fourier Series solutions in Cartesian coordinates.

UNIT IV FOURIER TRANSFORMS (9+3)

Fourier integral theorem (without proof) – Fourier transform pairs – Fourier sine and cosinetransform – Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.

UNIT V Z– TRANSFORMS AND DIFFERENCE EQUATIONS (9+3)

Z – Transform – Elementary properties – Inverse Z – Transform – Convolution theorem – Formation of difference equations– Solution of difference equations using Z–Transform.

TEXTBOOK:

1. B.S.Grewal,Higher Engineering Mathematics,KhannaPublishers, 42nd Edition,2016.
2. Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, John Willie & Sons,2006.

REFERENCES:

1. R. Haberman, Elementary Applied partial differential equations with Fourier Series and BoundaryValue Problems, 4th Ed., PrenticeHall,1998.
2. Manish Goya and .N.P Bali l, Transforms and Partial Differential Equations, University SciencePress, Second Edition, 2010.
3. Venkataraman. M. K. "Engineering Mathematics Volume III" , 13th Edition NationalPublishingCompany, Chennai, 1998.
4. George B. Thomas Jr., Maurice D. Weir, Joel R. Hass., Thomas' Calculus, 12th Edition,Addison-Wesley, Pearson.
5. S. J. Farlow, Partial Differential Equations for Scientist and Engineers, DoverPublications 1993.
6. Shanmugam,T.N.:http:.www.annauniv.edu/shan/trans.h

BHARATH INSTIUTE OF HIGHER EDUCATION AND RESEARCH

Staff Name : Dr. S. ANUSUYA
Designation : Assistant Professor
Department : Mathematics
Subject : Transforms and Boundary value problems – U20MABT03 (II – CSE-SA301, CSE-SA306, CSE-SA114,)

TIME TABLE 2021
TERM-III (Odd Semester)

Day/ Period	I 9.00 AM – 9.50 AM	II 9.50 AM – 10.40AM		III 10.50 AM – 11.40 AM	IV 11.40 AM – 12.30 PM		V 1.30 PM – 2.20 PM	VI 2.20 PM – 3.10 PM	VII 3.10 PM – 4.00 PM
MON	S6 –MATHS (SA301)						S12 –MATHS (SA306)		
TUE		S12 –MATHS (SA306)	B		S6 –MATHS (SA301)	L		CB-MATHS (SA114)	
WED	S6 –MATHS (SA301)		R E	S12 –MATHS (SA306)		U N			CB-MATHS (SA114)
THUR	S12 –MATHS (SA306)		A K		CB-MATHS (SA114)	C H	S6 –MATHS (SA301)		S12 –MATHS (SA306)
FRI				CB-MATHS (SA114)			S6 –MATHS (SA301)		CB-MATHS (SA114)

Signature of the Staff

(S. ANUSUYA)

DEPARTMENT OF MATHEMATICS

COURSE FILE – ACADEMIC YEAR – 2021-2022

SEMESTER / TERM / YEAR	:	ODD / I / II
COURSE CODE	:	U20MABT03
COURSE NAME	:	TRANSFORMS AND BOUNDARY VALUE PROBLEMS

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Biotechnology

INTERNAL ASSESSMENT -1
U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 12.10.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 60 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8)Answer all questions			
1	Write down the Fourier series formula.	2	CO1	2
2	Find a_0 and a_n, if $f(x) = x, in -\pi < x < \pi$.	2	CO1	2
3	State Dirichlet conditions.	2	CO1	2
4	Write down the Parseval's identity formula.	2	CO1	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) Find a_0 and a_n the fourier series for the function $f(x) = x^2, in\ (0, 2l)$.	6	CO1	2
	(Or)			
	(b) Find a_0 and a_n the fourier series for the function $f(x) = x(2\pi - x), in\ (0, 2\pi)$.	6	CO1	2
6	(a) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0, l)$.	6	CO1	2
	(Or)			
	(b) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0, \pi)$.	6	CO1	2
	PART – C (1X10 =10) Answer either or question			
7	(a) Find the Fourier series for the function in f(x) = x+x2 in (π,-π) and deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.	10	CO1	2
	(Or)	10	CO1	2
	(b)Find the fourier series expansion of period 2π for the function y= [see table below] f(x) which is defined in (0,2π) by means of the table of the values given below. Find the series up to to third harmonic.	10	CO1	2

X	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

CO	Weightage
CO1	30
CO2	

CO3	
CO4	
CO5	
CO6	
Total	50

Prepared by	Staff Name H.SASIKALA	Signature
Verified by	HoD Dr.S.V. Manemaran	Signature

U20MABT03 - Transform & Boundary Value Problems

Internal Assessment - I

A)

1) $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx)$

2). $a_0 = 0$, $a_n = 0$.

3). $f(x)$ is periodic, continuous,

4) $\frac{1}{2\pi} \int_0^{2\pi} [f(x)]^2 dx = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

B) 5) a) $a_0 = \frac{8l^2}{3}$, $a_n = -\frac{4}{n^2\pi^2}$.

b) $a_0 = \frac{4l^2}{3}$, $a_n = -\frac{4l^2}{n^2\pi^2}$.

6) a) $a_0 = l$, $a_n = \frac{2l}{n^2\pi^2} [(-1)^n - 1]$

b) $a_0 = \pi$, $a_n = \frac{2}{\pi n^2} [(-1)^n - 1]$

c) 7) a) $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{n^2}(-1)^{n+1}$, $b_n = \frac{2}{n}(-1)^{n+1}$

b) $f(x) = 1.45 + (-0.36 \cos x + 0.173 \sin x) + (-0.1 \cos 2x - 0.057 \sin 2x) + (0.033 \cos 3x$

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY

Department of Biotechnology

INTERNAL ASSESSMENT -I1

U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 21.11.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 90 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8) Answer all questions			
1	Classify the differential equation $3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}-u=0.$	2	CO3	2
2	State the Fourier integral theorem.	2	CO4	2
3	If $F\{f(x)\}=F(s)$, then $F\{f(x)\cos ax\}=$.............	2	CO4	2
4	What are the various solutions of one dimensional wave equation?	2	CO3	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $k(lx-x)$, then show that $y(x,t)=\frac{8kl^3}{a\pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{4}\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$. (OR)	6	CO3	2
	(b) Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & \lvert x\rvert>1.\end{cases}$		CO4	2
6	(a) State and prove shifting theorem. (OR)	6	CO4	2
	(b) State and prove Modulation theorem.		CO4	2
	PART – C (1X110 =10) Answer either or question			
7	(a) A metal bar 30cm has its end A and B 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature distribution function u(x,t) taking x=0 at A .		CO3	2
	(b) Show that the Fourier transform of $f(x)=\begin{cases}1-x^2 & \lvert x\rvert<1\\ 0 & \lvert x\rvert>1>0\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s-s\cos s}{s^3}\right)$. Hence deduce that $\int_0^{\infty}\frac{\sin s-s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$.	10	CO4	2

CO	Weightage
CO1	
CO2	
CO3	10
CO4	20
CO5	
CO6	
Total	30

Prepared by	Staff Name H.Sasikala	Signature
Verified by	HoD Dr.S.V. Manemaran	Signature

TBVP_ U20MABT03.

CLA - II.

1) $B^2-4AC=-56$, Ellipse.

2) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{i(x-t)s}\, dt\, ds.$

3) $F\{f(x)\cos ax\} = \frac{1}{2}\left[f(s+a)+f(s-a)\right]$

4) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c e^{-pat})$

$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$

$y(x,t) = (c_1 x + c_2)(c_3 t + c_4).$

5) b) $f(x) = \sqrt{\frac{2}{\pi}}\left(\frac{\sin s}{s}\right).$

6) (a) a) Shifting thm

$F[f(x)] = F(s)$ then

$F[f(x-a)] = e^{isx} F(s)$

b) Modulation thm

$F[f(x)] = F(s)$ then

$F[f(x)\cos ax] = \frac{1}{2}[F(s+a)+F(s-a)].$

7) a) $c_2 = 20$, $c_1 = \frac{60}{l}$ $\quad p = \frac{n\pi}{r}$, $b_n = \frac{40}{n\pi}\left[1+4(-1)^{n}\right]$

b) $F(s) = 2\sqrt{2}\left[\frac{\sin s - s\cos s}{s^3}\right]$

DEPARTMENT OF ABT, IBT, GENETICS

CONTINUOUS LEARNING ASSESSMENT – III

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date : 26.12.2022

Academic Year / Semester :2022-2023/ODD

Duration :1 hour 15 mins

Instructions : Part A- Answer all questions

Part B - Answer either A or B for the questions 5and 6

Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (5X2=10)			
1	Form the partial differential equation by eliminating the arbitrary function in Z = f($x^2 + y^2$).	2	CO2	*R*
2	Solve pe^y=qe^x.	2	CO 2	*R*
3	Prove that Z[n] = $\frac{z}{(z-1)^2}$.	2	CO 5	*U*
4	Find Z[$\frac{1}{n}$].	2	CO 5	*U*
	PART B (2x6=12)			
5	(a) Solve Z = px + qy + $p^2 - q^2$ **(OR)** (b) Solve the equation $\left(D^2 - 2DD' + D'^2\right)z = \cos(x - 3y)$..	6	CO2	*U*
6	(a)Find the Z – transform of the following, i) Z [1] ii) Z [a^n] **(OR)** (b) Solve $(mz - ny)p + (nx - lz)q = ly - mx$	6	CO 5	*U*
	PART C (1x10=10)			
7	a)Solve $(D^2 - DD' + 2D'^2)z = 2x + 3y + e^{3x+4y}$ b)Find $Z^{-1}[\frac{z}{z^2+5z+6}]$	10	CO 2	*U*

CO	**Weightage**
CO1	
CO2	20
CO3	
CO4	
CO5	10
CO6	-
Total	30

Prepared by	Faculty Name Mrs. H. SASIKALA	Signature
Verified by	Hod Dr. S.V. MANEMARAN	Signature

U20MABT03 - TBVP.

Internal Assessment - III.

1) $py - qx = 0$.

2). $z = \int p\,dx + \int q\,dy$.

$z = k\,[e^{x} + e^{y}] + c$.

3) $Z(n) = \frac{z}{(z-1)^2}$

4) $Z(1/n) = \log\left(\frac{z}{z-1}\right)$

5) a) $z = \frac{k}{1-k}\left(\frac{x^2}{2}\right) - k\left(\frac{y^2}{2}\right) + a$

b) $C.F = f_1(y+x) + x f_2(y+x)$

$P.I = \frac{-1}{16}\cos(x-3y)$.

$z = f_1(y+x) + x f_2(y+x) - \frac{1}{16}\cos(x-3y)$

6) a) $Z(1) = \frac{z}{z-1}$, $Z(a^n) = \frac{z}{z-a}$

b) $\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$, $lx + my + nz = C_2$

7) a) $C.F = f_1(y+2x) + f_2(y-x) + \frac{5x^3}{6} + \frac{3x^2y}{2} - \frac{1}{35}e^{3x+4y}$

b) $A = -1$, $B = 1$, $x(n) = (-2)^n - (-3)^n$.

S.No	Reg. No	Name	U20MABT03
1	U21BR001	ADITH BALACHANDRAN	43
2	U21BR002	AKOIJAM CHARULATA DEVI	44
3	U21BR004	ARUNACHALAM A	49
4	U21BR005	DADAM TEJASREE	46
5	U21BR006	HARINI PRIYA A	45
6	U21BR007	JAYAKUMAR M	44
7	U21BR008	JENIFER D	35
8	U21BR009	JHANANI K G	46
9	U21BR010	JHOSHITHA D	46
10	U21BR011	JOSHITHA A	46
11	U21BR012	KHUSHI PANWAR	42
12	U21BR013	KISLAY PRASAD	20
13	U21BR014	MAGESH L	39
14	U21BR015	MANJULA B	45
15	U21BR016	PADMINI D	45
16	U21BR017	POOJA PILLAI	46
17	U21BR018	PRAPANJAN K K	49
18	U21BR019	PRATHIKSHA S	46
19	U21BR020	PULI SRI LAKSHMI	40
20	U21BR021	PUNITHA S	39
21	U21BR022	RAKSHAA D	45
22	U21BR023	SAMYUKTHA O K	38
23	U21BR024	SANMATHIPRIYA T	45
24	U21BR025	SRIKAKULAPU KESAVA CHANDRA	44
25	U21BR026	SUMMANA SREE G	45
26	U21BR027	SYED NAFEER	36
27	U21BR028	TEJASREE R	40
28	U21BR030	DIVYADHARSHINI P	49

29	U21BR031	GADDAM SUJITHA	43
30	U21BR032	LAKSHMI M	39
31	U21BR033	PRIYADHARSHINI S	45
32	U21BR034	PRIYAVARSHINI U	43
33	U21BR036	VINSLIN JOE A J	46
34	U21BR037	VISHAL V	34
35	U21BR038	HARINI K	44
36	U21BR039	LEELASHREE P	32
37	U21BR040	SARANYA S	36
38	U21BR041	SHIFA RAIHAAN SIDDIQUE	47
39	U21BR042	UPPELLI HEMA MALINI	41
40	U21BR043	SELVA FINA T	46
41	U21BR044	SASIDHARAN S	41
42	U21BR045	KODIMUDI DEEPTHI	41
43	U21BR046	SHANA M	44
44	U21BR047	PRAVEEN U	41
45	U21BR048	NIVETHA E	40
46	U21BR049	DARUNGANESH B.N	41
47	U21BR050	ABDUL KALAM M	44
48	U21BR051	MOHAMED ISMAIL S	30

S.No	Roll. No	Name	U20MABT03
1	U21AC001	AJAYARAVINDU G	30
2	U21AC002	AKSHAYA SHREE P	49
3	U21AC003	ANBARASAN S	37
4	U21AC004	ANUGU MADHAVA REDDY	25
5	U21AC005	ARISH K	38
6	U21AC006	CHENGALA REKHA RANI	49
7	U21AC007	DANYASI SUSHMITHA	49
8	U21AC008	DARAM SAIRAM REDDY	40
9	U21AC009	DHARAVATH SAICHANDUNAIK	38
10	U21AC010	DURAIMURUGAN A	25
11	U21AC011	JAYAPRATHEEP M	32
12	U21AC012	KAMINI SREE HARSHA VARDHAN RED	35
13	U21AC013	MAGESH KUMAR E V	39
14	U21AC014	MANDAVA HARI KRISHNA SRI	15
15	U21AC018	SACHIN C	35
16	U21AC019	SEETHAMANI MAJHI B	46
17	U21AC020	SHOBITHA SHREEMAYI M	46
18	U21AC021	THIVITHKUMAR R K	45
19	U21AC023	VISHAL A G	44
20	U21AC024	VUDUTHURU CHANDRA MOULISWARA R	45
21	U21AC026	PITHANI VVSS LAKSHMI PRAVEEN	41
22	U21AC027	SAMINENI ANILKUMAR	38
23	U21AC028	SHAIK KHASIM SURAJ	40
24	U21AC029	ULLAMPARTI JAYA KIRAN	40
25	U21AC030	VAISHNAVI K	46
26	U21AC031	DASARI NAGENDRA PRASAD	38
27	U21AC032	DHAKSHAN P V	48
28	U21AC033	MEKALA RAVI SHANKAR	40
29	U21AC034	NIKHILA S	38
30	U21AC035	KOPPALA MEGHANA	45
31	U21AC036	CHINNAKOTALA LOKESH	45
32	U21AC037	ADORNA J	46
33	U21AC038	AVULA MADHAV	45
34	U21AC039	THOMAS DANIEL EMMETT	38
35	U21AC041	VASEEKARAN D	40
36	U21AC042	TAMILSELVAN A	45
37	U21AC044	RUBASRI T	45
38	U21AC045	ADAPA ABHINAY	40
39	U21AC046	CHILUKURI SRAVYA	43

40	U21AC047	PAVITHRA G	44
41	U21AC048	SETHUPATHY S	38
42	U21AC049	KEERTHANA S	44
43	U21AC050	PAVITHRA B	44
44	U21AC051	RAJESH P	45

S.No	Roll. No	Name	U20MABT03
1	U21BT001	BASHABOINA VINAY	40
2	U21BT002	CHAKRALA THANUJA	41
3	U21BT003	JALADI DIVYA	40
4	U21BT004	KALAISELVAN K	40
5	U21BT005	KAMALESH A	47
6	U21BT006	KAUSHIK M N	40
7	U21BT007	KODUMUNURI AKSHAYA	45
8	U21BT008	KRUTHIKVARSHA S	44
9	U21BT010	PONNEKANTI ADITHYA CHAKRAVARTHI	34
10	U21BT011	RAYI LALITH	48
11	U21BT012	SHIBIN B	44
12	U21BT013	VANGA DHARMA REJA	40
13	U21BT014	VISHAL V S	32
14	U21BT015	ANURAG SURESHBABU	48
15	U21BT017	T P KARTHIKEYAN	25
16	U21BT018	PALAKONDU SREYA REDDY	42
17	U21BT019	PRANEET PATEL	40
18	U21BT020	ARCHANA N	42
19	U21BT021	HEMAMALINI G	42
20	U21BT022	MITTAPALLI CHAITHANYA PRIYA	42
21	U21BT023	GUGULOTH NIKHIL KUMAR	40
22	U21BT024	JATTOTH SHASHANK	42
23	U21BT025	PUTTUR ANUSHA	46
24	U21BT026	MANI SRIDHAR LILY EMERALD	40
25	U21BT027	KONDREEDI GARI KISHOR KUMAR REDDY	45
26	U21BT028	MOKA DHANASREE	41
27	U21BT029	DUGGIRALA NANDINI	41
28	U21BT030	GOSALA SRAVAN KUMAR	10

INTERNAL ASSESSMENT -1

U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 12.10.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 60 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8)Answer all questions			
1	Write down the Fourier series formula	2	CO1	2
2	Find a_0 and a_n, if $f(x) = x, in -\pi < x < \pi$.	2	CO1	2
3	State Dirichlet conditions.	2	CO1	2
4	Write down the Parseval's identity formula.	2	CO1	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) Find a_0 and a_n the fourier series for the function $f(x) = x^2, in\ (0,2l)$.	6	CO1	2
	(Or)			
	(b) Find a_0 and a_n the fourier series for the function $f(x) = x(2\pi - x), in\ (0,2\pi)$.	6	CO1	2
6	(a) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0,l)$.	6	CO1	2
	(Or)			
	(b) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0,\pi)$.	6	CO1	2
	PART – C (1X10 =10) Answer either or question			
7	(a) Find the Fourier series for the function in f(x) = x+x2 in (π,-π) and deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$.	10	CO1	2
	(Or)	10	CO1	2
	(b)Find the fourier series expansion of period 2π for the function y= f(x) which is defined in (0,2π) by means of the table of the values given below. Find the series up to to third harmonic.	10	CO1	2

X	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

CO	Weightage
CO1	30
CO2	

CO3	
CO4	
CO5	
CO6	
Total	50

Prepared by	Staff Name H.SASIKALA	Signature
Verified by	HoD Dr.S.V. Manemaran	Signature

U20MABT03 - Transform & Boundary Value Problems

Internal Assessment - I :

A)

1) $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx)$

2). $a_0 = 0$, $a_n = 0$.

3). $f(x)$ is periodic, continuous,

4) $\frac{1}{2\pi} \int_0^{2\pi} [f(x)]^2 dx = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

B) 5) a) $a_0 = \frac{8l^2}{3}$, $a_n = -\frac{4}{n^2\pi^2}$.

b) $a_0 = \frac{4l^2}{3}$, $a_n = -\frac{4l^2}{n^2\pi^2}$.

6) a) $a_0 = l$, $a_n = \frac{2l}{n^2\pi^2} [(-1)^n - 1]$

b) $a_0 = \pi$, $a_n = \frac{2}{\pi n^2} [(-1)^n - 1]$

c) 7) a) $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{n^2}(-1)^{n+1}$, $b_n = \frac{2}{n}(-1)^{n+1}$

b) $f(x) = 1.45 + (-0.36 \cos x + 0.173 \sin x) + (-0.1 \cos 2x - 0.057 \sin 2x) + (0.033 \cos 3x)$

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Electrical and Electronics Engineering

INTERNAL ASSESSMENT -II
U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 21.11.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 90 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8) Answer all questions			
1	Classify the differential equation $3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}-u=0.$	2	CO3	2
2	State the Fourier integral theorem.	2	CO4	2
3	If $F\{f(x)\}=F(s)$, then $F\{f(x)\cos ax\}=$.............	2	CO4	2
4	What are the various solutions of one dimensional wave equation?	2	CO3	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $k(lx-x)$, then show that $y(x,t)=\frac{8kl^3}{a\pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{4}\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$. (OR)	6	CO3	2
	(b) Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & \lvert x\rvert>1.\end{cases}$		CO4	2
6	(a) State and prove shifting theorem. (OR)	6	CO4	2
	(b) State and prove Modulation theorem.		CO4	2
	PART – C (1X110 =10) Answer either or question			
7	(a) A metal bar 30cm has its end A and B 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature distribution function u(x,t) taking x=0 at A .		CO3	2
	(b) Show that the Fourier transform of $f(x)=\begin{cases}1-x^2 & \lvert x\rvert<1\\ 0 & \lvert x\rvert>1>0\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s-s\cos s}{s^3}\right)$. Hence deduce that $\int_0^\infty \frac{\sin s-s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$.	10	CO4	2

CO	Weightage
CO1	
CO2	
CO3	10
CO4	20
CO5	
CO6	
Total	30

Prepared by	Staff Name H.Sasikala	Signature
Verified by	HoD Dr.S.V. Manemaran	Signature

TBVP_ U20MABT03.

CLA – II.

1) $B^2-4AC=-56$, Ellipse.

2) $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{i(x-t)s}\, dt\, ds$.

3) $F\{f(x)\cos ax\}=\frac{1}{2}\left[f(s+a)+f(s-a)\right]$

4) $y(x,t)=(c_1e^{px}+c_2e^{-px})(c_3e^{pat}+ce^{-pat})$.

$y(x,t)=(c_1\cos px+c_2\sin px)(c_3\cos pat+c_4\sin pat)$

$y(x,t)=(c_1x+c_2)(c_3t+c_4)$.

5) b) $f(x)=\sqrt{\frac{2}{\pi}}\cdot\left(\frac{\sin s}{s}\right)$.

6) a) Shifting thm

$F[f(x)]=F(s)$ then

$F[f(x-a)]=e^{isx}F(s)$

b) Modulation thm

$F[f(x)]=F(s)$ then

$F[f(x)\cos ax]=\frac{1}{2}[F(s+a)+F(s-a)]$.

7) a) $c_2=20$, $c_1=\frac{60}{l}$ $p=\frac{n\pi}{r}$, $b_n=\frac{40}{n\pi}[1+4(-1)^n]$

b) $F(s)=2\sqrt{\frac{2}{\pi}}\left[\frac{\sin s-s\cos s}{s^3}\right]$

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

CONTINUOUS LEARNING ASSESSMENT – III

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date	: 26.12.2022
Academic Year / Semester	:2022-2023/ODD
Duration	:1 hour 15 mins
Instructions	: Part A- Answer all questions
	Part B - Answer either A or B for the questions 5and 6
	Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (5X2=10)			
1	Form the partial differential equation by eliminating the arbitrary function in Z = f(x^2+y^2).	2	CO2	***R***
2	Solve pe^y=qe^x.	2	CO 2	***R***
3	Prove that Z[n] = $\frac{z}{(z-1)^2}$.	2	CO 5	***U***
4	Find Z[$\frac{1}{n}$].	2	CO 5	***U***
	PART B (2x6=12)			
5	(a) Solve Z = px + qy + $p^2 - q^2$ **(OR)** (b) Solve the equation $\left(D^2-2DD' + D'^2\right)z=\cos(x-3y)$..	6	CO2	***U***
6	(a)Find the Z – transform of the following, i) Z [1] ii) Z [a^n] **(OR)** (b) Solve $(mz-ny)p+(nx-lz)q=ly-mx$	6	CO 5	***U***
	PART C (1x10=10)			
7	a)Solve $(D^2-DD' + 2D'^2)z=2x+3y+e^{3x+4y}$	10	CO 2	***U***
	b)Find $Z^{-1}[\frac{Z}{Z^2+5Z+6}]$			

CO	Weightage
CO1	
CO2	20
CO3	
CO4	
CO5	10
CO6	-
Total	30

Prepared by	Faculty Name Mrs. H. SASIKALA	Signature
Verified by	Hod Dr. S.V. MANEMARAN	Signature

U20MABT03 – TBVP.

Internal Assessment – III.

1) $py - qx = 0.$

2). $z = \int p\,dx + \int q\,dy.$

$z = k\,[e^{x} + e^{y}] + c.$

3) $Z(n) = \frac{z}{(z-1)^2}$

4) $Z(1/n) = \log\left(\frac{z}{z-1}\right)$

5) a) $z = \frac{k}{1-k}\left(\frac{x^2}{2}\right) - k\left(\frac{y^2}{2}\right) + a$

b) $C.F = f_1(y+x) + x f_2(y+x)$

$P.I = -\frac{1}{16}\cos(x-3y).$

$z = f_1(y+x) + x f_2(y+x) - \frac{1}{16}\cos(x-3y)$

6) a) $Z(1) = \frac{z}{z-1}$, $Z(a^n) = \frac{z}{z-a}$

b) $\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$, $lx + my + nz = C_2$

7) a) $C.F = f_1(y+2x) + f_2(y-x) + \frac{5x^3}{6} + \frac{3x^2y}{2} - \frac{1}{35}e^{3x+4y}$

b) $A = -1$, $B = 1$, $x(n) = (-2)^n - (-3)^n.$

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARC

Department of Electrical and Electronics Engineering

II YEAR 2021 BATCH INTERNAL MARKS

S.No	Roll No.	Student Name	U20MABT03 TB&V
1	U21EE001	KARTHIK P	60
2	U21EE002	KOTHAKOTA NARENDRABABU	90
3	U21EE003	MEDISHETTI CHAITANYA	94
4	U21EE004	NAGULA VENKANNA BABU	94
5	U21EE005	GUDIKADI VAMSHI KRISHNA	93
6	U21EE006	GUNTI KOTESHWAR RAO	85
7	U21EE007	CHINTHALAPALL IPANI KUMAR REDDY	80
8	U21EE008	KANCHARLAPALLI SAI DHANUSH	82
9	U21EE009	SIDDAVATAM BALA SAINATH REDDY	85
10	U21EE010	YERUVA SIVA REDDY	79
11	U21EE011	A GOVARDHAN REDDY	75
12	U21EE012	SIDDANNA VENKATA KARUNAKAR	50
13	U21EE013	BIJINIVEMULA NAVEEN KUMAR REDDY	83
14	U21EE014	JUTURU AJAY KUMAR REDDY	98
15	U21EE015	BOLLAE JAGADEESHWARA	80
16	U21EE016	ADABALA AYYAPPA SWAMI	73
17	U21EE017	THATIGUTLA LOKESHWAR REDDY	83
18	U21EE018	ADAKA VENKATESWARLU	85
19	U21EE019	SEETHARU ESWAR REDDY	67
20	U21EE020	JOHN VESLY M	85
21	U21EE021	ADAVI MADHUSUDHAN	85
22	U21EE022	KOTTE CHANDRA SEKHAR	84
23	U21EE023	PURNENDU KUMAR YADAV	0
24	U21EE024	MOLAKA GOVARDHAN REDDY	80
25	U21EE025	M BHARATH KALYAN	70
26	U21EE026	VIJAYA KUMAR A	91
27	U21EE701	Aakash Vijay Doss.V	82
28	U21EE702	Akkinapalli Vijaya Prakash	75
29	U21EE703	Balakrishnan.B.N	65
30	U21EE704	Barige Venkata Srinu	74
31	U21EE705	BHUKYA KESHAVARDHAN NAYAK	75
32	U21EE706	Dasari Vinesh Kumar	70
33	U21EE707	Eeda Tharun	79
34	U21EE708	GAJULA MADHU CHARAN	83
35	U21EE709	Honi Tatam	85
36	U21EE710	Jayasurya K	77
37	U21EE711	Kambala Poorna Kumar	85
38	U21EE712	Kancheti Venkat	88
39	U21EE713	Linto David.K	88
40	U21EE714	Marikanti Jeswanth	95
41	U21EE715	Munagala Sai Surendra	96
42	U21EE716	NARIMETI VIJAY	86
43	U21EE717	Naveen Kumar.K	88
44	U21EE718	Osan.S	77
45	U21EE719	PAVITRALAKSHMI N	0
46	U21EE720	Ragavan,A	78
47	U21EE721	Raven Glenn.R	78
48	U21EE722	Gaddam Rohith	82
49	U21EE723	Sobhanapu Shivaji	92
50	U21EE724	Sridhar.E	88
51	U21EE725	Vignesh.V	85

SET - B

Ques	Type
Question	Type
What is the suitable expansion for the Fourier series?	MCQ
Find the Fourier constants b_n for xsinx in (-π , π)	MCQ
Find the value of a_n from Fourier series f(x) = x^3 in(-π , π)	MCQ
Find the half range cosine series for the function f(x) = xsinx value a_n	MCQ
Form the partial differential equation by eliminating the arbitrary constants from z= ax+by+a^2+b^2	MCQ
What is the form of equation in Clairaut's type ?	MCQ
Which of the following is Lagrange's Method	MCQ
When the R.H.S of a given PDE is in exponential , then to find particular integral, we will substitute	MCQ
Which of the following represents a^2 in the wave equation?	MCQ
Which one of the following is the most suitable solution of one dimensional wave equation?	MCQ
Classify the equation if B^2-4AC=0 ?	MCQ
When the ends A and B of a rod length 10 cm have their temperature 20^0 C and 70^0 C. Find the steady state temperature on the rod?	MCQ
What is the Fourier Cosine Transform F_C [f(ax)] = ?	MCQ
What is the value of F[f(x-a)] ? If F(s) is the Fourier transform of f(x).	MCQ
Find the Fourier cosine transform of e^-x	MCQ
Find the Fourier sine transform of e^-x	MCQ
Find Z(n) = ?	MCQ
When z(sinnθ) = Zsinθ/(Z^2-2zcosθ+1) then sin(nπ/2) =?	MCQ
Find Z[a^n f(n)]=	MCQ
Form the difference equation of y_n=a+(b 3^n)	MCQ

Option1	Option1M	Option2	Option2M
Answer 1	Answer	Answer 2	Answer 2
aperiodic function	0	periodic function	1
0	1	2 π	0
π	0	2 π	0
(π^2)/3	1	(2π^2)/3	0
z= ax+by+a^2+b^2	0	z= px+qy+a^2+b^2	0
Z=px+qy+f(p,q)	1	Z=px+qy	0
By elimination of arbitrary constants	0	By elimination of arbitrary functions	0
D=a. D'=b	1	D=1/a. D'=1/b	0
Tension/ Mass per unit length	1	Mass/Tension per unit length	0
y(x,t) = (Acospx-Bsinpx)(Ccospat-Dsinpat)	0	y(x,t) = (Acospx-Bsinpx)(Ccospat+Dsinpat)	0
Parabolic	1	Hyperbolic	0
u(x) =5x+20	1	u(x) = 5x+2	0
1/a F_C [s]	0	F_C [s/a]	0
e^ias F(s)	1	e^ias F(s/a)	0
√(2/π) [1/(-1-s^2)]	0	√(2/π) [s/(-1-s^2)]	0
√(2/π) [s/(1+s^2)]	1	√(2/π) [s/(-1-s^2)]	0
Z/(Z-1)	0	Z/(Z)^2	0
Z/(1- Z^2)	0	Z/(Z^2+1)	1
F[z/a]	1	F[z]	0
2y_(n+2)- 〖4y〗_(n+1)+ 〖3y〗_n=0	0	y_(n+2)- 〖4y〗_(n+1)+ 〖3y〗_n=0	1

Option3	Option3M	Option4
Answer 3	Answer 3	Answer 4
series function	0	both periodic and aperiodic
π/2	0	1
0	1	π/2
(π^2)/9	0	(π^2)/6
z= ax+by+p^2+q^2	0	z= px+qy+p^2+q^2
Z=f(p,q)	0	Z=px+qy+f(a,b)
Methods of multiplier	1	By Method of Division and Multiplication
D= 2a. D'=2 b	0	D= -a. D'= -b
Tension/ Force per unit length	0	Tension/ Force per unit length
y(x,t) = (Acospx+Bsinpx)(Ccospat-Dsinpat)	0	y(x,t) = (Acospx+Bsinpx)(Ccospat+Dsinpat)
Elliptic	0	Rectangular Hyperbola.
u(x) = 15x+20	0	u(x) = 6x+20
1/a F_C [s/a]	1	1/a F_C [1/a]
e^-ias F(s)	0	e^ias F(-s)
√(2/π) [1/(1+s^2)]	1	none
√(2/π) [s/(1-s^2)]	0	√(2/π) [s/(-1+s^2)]
Z/(Z-1)	0	Z/(Z-1)^2
Z/(Z^2-1)	0	Z/(Z^2)
F[az]	0	F[a]
y_(n+2)- 〖y〗 _(n+1)+ 〖3y〗 _n=0	0	y_(n+2)- 〖4y〗 _(n+1)+ 〖y〗 _n=0

ption4Mar	CorrectA	WrongAr	DescAns	Marks	GroupID
swer 4 Ma	Right Answer (1 to 5)				GroupID
0	Option2			1	Part A
0	Option1			1	Part A
0	Option3			1	Part A
0	Option1			1	Part A
1	Option4			1	Part A
0	Option1			1	Part A
0	Option3			1	Part A
0	Option1			1	Part A
0	Option1			1	Part A
1	Option4			1	Part A
0	Option1			1	Part A
0	Option1			1	Part A
0	Option3			1	Part A
0	Option1			1	Part A
0	Option3			1	Part A
0	Option1			1	Part A
1	Option4			1	Part A
0	Option2			1	Part A
0	Option1			1	Part A
0	Option2			1	Part A

SET-A

Ques	Type
Question	Type
Find the value of b_n for the Fourier series for f(x)=x^2 in (-π,π)?	MCQ
The function f(x)=cosx is	MCQ
Find a_0 for the function f(x)=1/2(π-x) in (0,2π)	MCQ
Which of the following function is periodic?	MCQ
What is the form of the equation √p+√q=1 is?	MCQ
Which of the following represented in Lagrange's Method	MCQ
Which of the linear partial differential equation of the first order is known as clairaut's form	MCQ
When the R.H.S of a given PDE is in exponential, then to find particular integral, we will substitute	MCQ
Classify the equation if B^2-4AC=0 ?	MCQ
When the ends A and B of a rod length 10 cm have their temperature 20^0 C and 70^0 C, Find the steady state temperature on the rod?	MCQ
What is the name of equation (∂^2 y)/(∂t^2)=a^2 (∂^2 y)/(∂x^2)	MCQ
Which one of the following is the most suitable solution of one dimensional wave equation?	MCQ
What is the value of F[e^iax f(x)] ,when F(s) is the Fourier transform of f(x).	MCQ
Find the Fourier cosine transform of e^-x	MCQ
Find the Fourier sine transform of e^-x	MCQ
Find the value of Z(n).	MCQ
When z(sinnθ) = Zsinθ/(Z^2-2zcosθ+1) then sin(nπ/2) =?	MCQ
Find Z[a^n f(n)].	MCQ
Form the difference equation of y_n=a+(b 3^n)	MCQ
What is the simple pole at the point for the function F(z)= z/((z-1) 〖(z-2)〗^2)?	MCQ

Option1	ption1Mar	Option2	Option2Mark
Answer 1	**swer 1 Ma**	**Answer 2**	**nswer 2 Mar**
0	1	1	0
Even	1	Odd	0
3	0	1	0
log x	0	Exponential	0
f(p,q,z)=0	0	f(p,q,y)=0	0
By elimination of arbitrary constants.	0	Methods of grouping.	1
z= px+qy+f(p,q)	1	Px+Qy=R	0
D=a. D'=b	1	D=1/a. D'=1/b	0
Parabolic	1	Hyperbolic	0
u(x) =5x+20	1	u(x) = 5x+2	0
Laplace Equation	0	Wave equation	1
y(x,t) = (Acospx-Bsinpx)(Ccospat-Dsinpat)	0	y(x,t) = (Acospx-Bsinpx)(Ccospat+	0
F(s-a)	0	F(s)	0
√(2/π) [1/(-1-s^2)]	0	√(2/π) [s/(-1-s^2)]	0
√(2/π) [s/(1+s^2)]	1	√(2/π) [s/(-1-s^2)]	0
Z/(Z-1)	0	Z/(Z)^2	0
Z/(1- Z^2)	0	Z/(Z^2+1)	1
F[z/a]	1	F[z]	0
2y_(n+2)- 〖4y〗 _(n+1)+ 〖3y〗 _n=0	0	y_(n+2)- 〖4y〗 _(n+1)+ 〖3y〗 _n=	1
z=0	0	z=1	1

Option3	Option3Mark	Option4	Option4Mark
Answer 3	nswer 3 Mar	Answer 4	Answer 4 Mark
2	0	3	0
Neither even nor odd	0	Indefinite	0
2	0	0	1
sin x	1	x	0
f(p,q)=0	1	f(p,q,x)=0	0
By elimination of arbitrary functi	0	By Method of Division and Multiplication	0
dx/P=dy/Q=dz/R	0	u=a and v=b	0
D= 2a. D'=2 b	0	D= -a. D'= -b	0
Elliptic	0	None	0
u(x) = 15x+20	0	u(x) = 6x+20	0
Heat equation	0	Difference equation	0
y(x,t) = (Acospx+Bsinpx)(Ccosp	0	y(x,t) = (Acospx+Bsinpx)(Ccospat+Dsin	1
F(s+a)	1	F(a)	0
√(2/π) [1/(1+s^2)]	1	none	0
√(2/π) [s/(1-s^2)]	0	√(2/π) [s/(-1+s^2)]	0
Z/(Z-1)	0	Z/(Z-1)^2	1
Z/(Z^2-1)	0	Z/(Z^2)	0
F[az]	0	F[a]	0
y_(n+2)- 〖y〗 _(n+1)+ 〖3y〗 _n	0	y_(n+2)- 〖4y〗 _(n+1)+ 〖y〗 _n=0	0
z=2	0	z=3	0

CorrectAnswer	WrongAnswer	DescAns	Marks	GroupID
Right Answer (1 to 5)				GroupID
Option1			1	Part A
Option1			1	Part A
Option4			1	Part A
Option3			1	Part A
Option3			1	Part A
Option2			1	Part A
Option1			1	Part A
Option1			1	Part A
Option1			1	Part A
Option1			1	Part A
Option2			1	Part A
Option4			1	Part A
Option3			1	Part A
Option3			1	Part A
Option1			1	Part A
Option4			1	Part A
Option2			1	Part A
Option1			1	Part A
Option2			1	Part A
Option2			1	Part A

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH
Declared as Deemed to be University under section 3 of UGC act 1956
173, Agaram Main Road, Selaiyur, Chennai – 600 073, Tamil Nadu

End SemesterExaminations – Nov-2021
Regulation – 2020

Programme(s)	Year	Semester	Course Code(s)	Course Title
B. TECH	II Year	III	U20MABT03	Transforms and Boundary value problems

Time: Three Hours **Max Marks: 100**
No. of Pages:02

	Part – B: (5 X 6 = 30 Marks) Answer either (a) or (b)	Marks	BT	CO
21a (or)	Find the half range Fourier cosine series of $f(x)=x$ in $(0,l)$	6	U	1
21b	Obtain the Fourier series to represent the function $f(x)=\|x\|, -\pi<x<\pi$	6	U	1
22a (or)	Form the P.D.E by eliminating the arbitrary function from $z=f\left(\frac{y}{x}\right)$	6	A	2
22b	Solve $(D^2+2DD'+D'^2)z=e^{x-y}$	6	A	2
23a (or)	A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position. Find the displacement $y(x,t)$ at any distance x from one end at any time t.	6	A	3
23b	A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are kept at $0°C$ and kept so. Find the temperature distribution.	6	A	3
24a (or)	Show that the function $e^{\frac{-x^2}{2}}$ is self-reciprocal under Fourier transform.	6	A	4
24b	Find the Fourier sine and cosine transform $f(x)=e^{-ax}$	6	A	4
25a (or)	Find $Z(r^n\cos n\theta)$ and $Z(r^n\sin n\theta)$ also find $Z(\cos n\theta)$ and $Z(\sin n\theta)$	6	A	5
25b	Find $Z^{-1}\left(\frac{z}{(z-1)(z-2)}\right)$ using Residue theorem.	6	A	5

Part – C: (5 X 10 = 50 Marks) Answer either (a) or (b)	Marks	BT	CO

26a (or)	Express as a Fourier series of a period in the interval . Hence deduce that the sum of the series	10	U	1
26b	Find the Fourier series for $f(x)=x^2$ in $-\pi<x<\pi$. Hence show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+...=\frac{\pi^4}{90}$	10	U	1
27a (or)	Solve $(mz-ny)p+(nx-lz)q=ly-mx$	10	A	2
27b	Solve $(D^2-DD'-2D'^2)z=2x+3y+e^{3x+y}$	10	A	2
28a (or)	A string is stretched and fastened to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string into the form $y=k(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time t	10	A	3
28b	A rod of length l has its ends A and B kept at $0°C$ and $150°C$ respectively, until steady state conditions prevail. If the temperature at B is reduced to $0°C$ and kept so, while that of A is maintained so, find the temperature $u(x,t)$ at a distance x form A and at time t.	10	A	3
29a (or)	Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & in\ \lvert x\rvert\le 1\\ 0, & in\ \lvert x\rvert>1\end{cases}$. Hence prove that $\int_0^\infty\left(\frac{\sin s-s\cos s}{s^3}\right)\cos\frac{s}{2}ds=\frac{3\pi}{16}$	10	A	4
29b	Find $F_s(e^{-ax})\ \&\ F_c(e^{-ax})$ and hence deduce that (i) $\int_0^\infty\frac{x^2dx}{(x^2+a^2)^2}$, (ii) $\int_0^\infty\frac{dx}{(x^2+a^2)^2}$ $a>0$	10	A	4
30a (or)	Find $Z^{-1}\left(\frac{z^3+3z}{(z-1)^2(z^2+1)}\right)$ by using Partial fraction method.	10	A	5
30b	Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ by using Convolution theorem.	10	A	5

Assessment Summary:

COs	Remember	Understand	Apply	Analyze	Evaluate	Create	Total
CO1		16					16
CO2			16				16
CO3			16				16
CO4			16				16
CO5			16				16
CO6							

Name :- Tangala priya Reg no :- U

[i] fourier series :-

If f(x) is a period function and di

then it can be represented by an infinite

let f(x) be a periodic function defined

series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$

Here a_0, b_n, a_n are called fourier series

[2] Dirichlet's Conditions :-

If a function f(x) is defined in $c \leq x \leq c$

expanded as a fourier series of the for

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi} \int f(x) dx$, $a_n = \frac{1}{\pi} \int f(x) \cos nx d$

provided the following dirichlet's condition

[i] f(x) is single valued & finite in (c, c+

[ii] f(x) is continous in (c, c+, 2π)

(iii) f(x) is has a finite number of maxim

* $\sin x$, $\cos x$, $\operatorname{cosec} x$, $\sec x$ have period 2π
* $\tan x$ and $\cot x$ have period π

Eg :- $f(x) = \sin x = \sin(x+2\pi) = \sin(x+4\pi) + \ldots$

[4] $f(x) = \pi - x$ in $(0 \le x \le \pi)$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \pi - x\,dx$$

$$= \frac{1}{\pi}\left[\frac{(\pi - x)^2}{2}\right]_0^{\pi}$$

$$= \frac{(\pi-\pi)^2 - (\pi-0)^2}{-2\pi}$$

$$= \frac{-\pi^2}{2\pi}$$

$$\boxed{a_0 = \pi/2}$$

[7] Step 1 :-

(a) the fourier series of $f(x)$ in $(0,2)$ is

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n x + \sum_{n=1}^{\infty} b_n \sin n x} \rightarrow (1)$$

Step 2 :-

To find a_0 :-

$$\boxed{a_0 = \int_0^2 f(x)\,dx}$$

$$= \int_0^2 (2x - x^2)\,dx$$

Step 2 :-

To find a_0 :-

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{2}\right]$$

$$a_0 = \pi \rightarrow (2)$$

Step 3 :-

To find a_n :-

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\frac{\sin nx}{n} + (1)\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(\frac{\pi\sin n\pi}{n} + \frac{\cos n\pi}{n^2}\right] - \left[0 + \frac{\cos(0)}{n^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\cos n\pi - \cos(0)\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} \frac{-4}{n^2\pi} & \text{if 'n' is odd} \end{cases} \rightarrow (3)$$

Step 4 :-

To find b_n :-

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\left[\int_0^{l} f(x)\sin\frac{n\pi x}{l}\,dx + \int_l^{2l} f(x)\sin\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{1}{l}\left[\int_0^{l} kx\sin\frac{n\pi x}{l}\,dx + 0\right]$$

$$= \frac{1}{l}\left[-kx\frac{\cos\frac{n\pi x}{l}}{n\pi/l} + k\frac{\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right]_0^l$$

$$= \frac{1}{l}\left[\left(-kl\frac{\cos n\pi}{n\pi/l} - k\frac{\sin \pi n}{\frac{n^2\pi^2}{\pi^2}}\right) - (0+0)\right]$$

$$= \frac{1}{l}\left[\frac{-kl}{\frac{n\pi}{l}}[\cos n\pi]\right]$$

$$= \frac{-kl}{n\pi}[\cos n\pi]$$

$$b_n = \frac{-kl}{n\pi}[(-1)^n] \rightarrow (4)$$

Step 5 :-

the required fourier series substituting (2)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$= \frac{kl}{4} + \sum_{n=1}^{\infty}\frac{-2kl}{n^2\pi^2}\cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty}\frac{-kl}{n\pi}[(-1)$$

[8] Step 1 :-

[b] the fourier series for $f(x)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \rightarrow (1)$$

step 2 :-

To find a_0 :-

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{1}{l}\left[\int_0^{l} kx\,dx + \int_0^{2l} 0\,dx\right]$$

$$= \frac{1}{l}\left[\frac{kx^2}{2}\right]_0^{l}$$

$$= \frac{1}{l}\left[\frac{kl^2}{2}\right]$$

$$a_0 = \frac{kl}{2} \rightarrow (2)$$

step 3 :-

to find a_n :-

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{1}{l}\left[\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx + \int_l^{2l} f(x)\cos\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{1}{l}\left[\int_0^{l} kx\cos\frac{n\pi x}{l}\,dx + 0\right]$$

$$= \frac{1}{l}\left[\int_0^{l} \frac{kx\sin\frac{n\pi x}{l}}{n\pi/l} + \frac{k\cos\frac{n\pi x}{l}}{n^2\pi^2}\right|^{l}$$

$= \int_0^2 (2x - x^2)\cos n\pi x\,dx$

$= \left[(2x-x^2)\frac{\sin n\pi x}{n\pi} + (2-2x)\frac{\cos n\pi x}{n^2\pi^2} - 2\frac{\sin n\pi}{n^3\pi}\right.$

$= \left[\left(0 - \frac{2}{n^2\pi^2} - 0\right) - \left(0 + \frac{2}{n^2\pi^2} - 0\right)\right]$

$= \left[\frac{-2}{n^2\pi^2} - \frac{2}{n^2\pi^2}\right] \Rightarrow \frac{-4}{n^2\pi^2}$

$a_n = \frac{-4}{n^2\pi^2} \longrightarrow (3)$

Step 4 :-

To find b_n :-

$b_n = \int_0^2 (2x - x^2)\sin n\pi x\,dx$

$= \left[(2x-x)^2 - \frac{\cos n\pi x}{n\pi} + (2-2x)\frac{\sin n\pi x}{n^2\pi^2} - 2\frac{\cos}{n}\right.$

$= \left[\left(0 + 0 - \frac{2}{n^3\pi^3}\right) - \left(-\frac{2}{n^3\pi^3}\right)\right] = 0$

substituting (2) (3) & (4) in (1) we get

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$

$= \frac{2}{3} + \sum_{n=1}^{\infty} \frac{-4}{n^2\pi^2} \cos n\pi x + 0$

(5)

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

Step 5 :-

deduction

put $x=0$ in (4)

LHS. put (4) implies

$f(x)=x$

$f(0)=0$

R.H.S put (4) implies

$$=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\cdots\right]$$

$$\frac{\pi}{2}=\frac{4}{\pi}\left[\frac{\cos(0)}{1^2}+\frac{\cos(0)}{3^2}+\frac{\cos(0)}{5^2}+\cdots\right)$$

$$\frac{\pi^2}{2}\cdot\frac{\pi^2}{4}=\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

$$\frac{\pi^2}{8}=\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right]$$

[11]
[b] Step 1 :-

the half range cosine series of $f(x)$ in $(0,\pi)$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx \longrightarrow (1)$$

Step 2 :-

To find a_0 :-

$$a_0=\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$=\frac{2}{\pi}\int_0^{\pi} x\,dx$$

$$=\frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[x \frac{\sin nx}{n} + (1)\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(0 + \frac{\cos n\pi}{n^2}\right) - \left(0 + \frac{1}{n^2}\right)\right]$$

$$= \frac{2}{\pi}\left(\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right)$$

$$= \frac{2}{n^2\pi}\left[\cos n\pi - 1\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} 0 & \text{where } n \text{ is even} \\ \frac{2}{n^2\pi}\left[(-1)^n - 1\right] & \text{when } n \text{ is odd} \end{cases} \rightarrow (3)$$

Step 4 :-

the required cosine series

Substituting (2) & (3) in (1) we get

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} -\frac{4}{n^2\pi} \cos nx$$

Step 5 :-

the parsevalis identity for fourier cosine se

$$q^2 = \frac{2}{\pi}\int_0^{\pi}\left[f(x)\right]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 \qquad a_0 =$$

$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{\pi^2}{2} + \sum_{n=1,3}^{\infty} \frac{16}{n^4\pi^2}$$

[6] Bernoulli's formula :-

Bernoulli's form of integration by parts

$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \cdots$

[u] - differentiation

[v] - integration

[9] $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

To find a_n :-

$$a_n = \frac{2}{l} + \int_0^l f(x) \sin \frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l} \int_0^l x \sin \frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[x\left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right.$$

$$= \frac{2}{l}\left[\frac{-l \cos n\pi}{n\pi/l}\right]$$

$$a_n = \frac{2l}{n\pi}(-1)^{n+1}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2l}{n\pi}(-1)^{n+1} \sin \frac{n\pi x}{l}$$

$$f(x) = \frac{2l}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{l}$$

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* Tan x and cot x have period π

4) sol- Given $f(x) = \pi - x \quad 0 \le x \le \pi$

$$a_0 = 1/\pi \int_0^\pi f(x)\,dx$$
$$= 1/\pi \int_0^\pi (\pi - x)\,dx$$
$$= 1/\pi \left(-\frac{(\pi-x)^2}{2} \right)_0^\pi$$
$$= 1/\pi \left(-0 + \frac{\pi^2}{2} \right)$$
$$= 1/\pi \, (\pi^2/2)$$
$$\boxed{a_0 = \pi/2}$$

5) sol.- $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{l} + \sum_{n=1}^{\infty} b_n \frac{\sin n\pi x}{l}$ in the interval $(0, 2l)$

then $a_0 = 1/l \int_0^{2l} f(x)\,dx$

$$a_n = \frac{1}{l}\int_0^{2l} f(x) \frac{\cos n\pi x}{l}\,dx, \; n \ge 0 \text{ and}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x) \frac{\sin n\pi x}{l}\,dx, \; n \ge 1$$

6) sol:- Bernoulli's formula

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 \ldots$

where u and v are functions of x

$u' = \frac{du}{dx}$ $\quad v_1 = \int v\,dx$

$u'' = \frac{d^2u}{dx^2}$ $\quad v_2 = \int v_1\,dx$

$u''' = \frac{d^3u}{dx^3}$ $\quad v_3 = \int v_2\,dx$

Maths cla-1

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01

Part-A

1) ans:- Fourier series

let $f(x)$ be a periodic function define on $(-\pi, \pi)$ fourier series of $f(x)$ is defined as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx.$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx.$$

here a_0, a_n, b_n are called fourier series

2) ans:- Dirichlet's conditions

i) If $f(x)$ is defined and single valued except possibly at a finite number of points in $(0, 2\pi)$ (or) $(-\pi, \pi)$

ii) If $f(x)$ is periodic with period $(0, 2\pi)$

iii) $f(x)$ & $f'(x)$ are piecewise

3) ans:- periodic function:-

let $f(x)$ be a real valued function and if there exist a least positive constant T such that $f(x+T) = f(x)$. Then $f(x)$ is said to be periodic function with period T.

Result:-

* All trigonometric function are periodic functions
* $\sin x, \cos x, \operatorname{cosec} x, \sec x$, have period

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03

Part - B

3b) sol. The sine series of $f(x)$ in $(0,l)$ is

$$f(x) = \Sigma b_n \sin \frac{n\pi x}{l}$$

To find b_n

$$b_n = \frac{2}{l}\int_0^l f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l}\int_0^l x \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l}\left[x\left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^l$$

$$= \frac{2}{l}\left[\frac{-x\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l\cos \frac{n\pi l}{l}}{\frac{n\pi}{l}} - 0\right]$$

$$= \frac{-2}{l} l (-1)^n \frac{l}{n\pi}$$

$$\boxed{b_n = \frac{2l(-1)^{n+1}}{n\pi}}$$

$$f(x) = \Sigma \frac{2l}{n\pi}(-1)^{n+1} \sin \frac{n\pi x}{l}$$

$$= \frac{2l}{\pi}\left[\sin \frac{\pi x}{l} - \frac{\sin \frac{2\pi x}{l}}{2} + \cdots\right]$$

Since it is a Half range sine series By Root Mean square value (R.M.S)

$$\boxed{\bar{y}^2 = \frac{1}{2}\Sigma b_n^2}$$

$$\frac{2}{l}\int_0^l [f(x)]^2 dx = \Sigma \frac{4l^2}{n^2\pi^2}(-1)^{2n+2}$$

$$\frac{2}{l}\int_0^l x^2 dx = \Sigma \frac{4l^2}{n^2\pi^2}(-1)^{2n+2} \quad \frac{4l^2}{\pi^2}\Sigma \frac{(-1)^{2n+1}}{n^2}$$

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04

$\frac{2}{1}\left[\frac{x^3}{3}\right]_0^l = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\dots\right]$

$\frac{2l^2}{3} \times \frac{\pi^2}{4l^2} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots$

$$\boxed{\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots}$$

5.a)
Sol:- The half rang cosine series function $f(x)$ is given by

$$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx$$

To find a_0

$$a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx$$
$$= \frac{2}{\pi}\int_0^\pi [x^2]\,dx$$
$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^\pi = \frac{2}{\pi}\left[\frac{\pi^3}{3}-0\right]$$
$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

To find a_n

$$a_n = \frac{2}{\pi}\int_0^\pi f(x)\cos nx\,dx$$

$u = x^2$ $\quad dv = \cos nx$
$u' = 2x$ $\quad v = \frac{\sin nx}{n}$
$u'' = 2$ $\quad v_1 = \frac{-\cos nx}{n^2}$
$v_2 = -\frac{\sin nx}{n^3}$

$$a_n = \frac{2}{\pi}\int_0^\pi x^2\cos nx\,dx$$
$$= \frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^\pi$$
$$= \frac{2}{\pi}\left[2x\frac{\cos nx}{n^2}\right]_0^\pi$$
$$= \frac{2}{\pi}\left[2\pi\frac{\cos n\pi}{n^2}\right]$$
$$\boxed{a_n = \frac{4}{n^2}(-1)^n}$$

$\therefore$ The required fourier series is

$$f(x) = \frac{2\pi^2}{3\times 2}+\sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\cos nx$$

$$\boxed{f(x) = \frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}}$$

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05

9) Sol:- $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

To find a_n:

$a_n = \frac{2}{l}\int_{0}^{l} f(x) \sin\frac{n\pi x}{l} dx$

$= \frac{2}{l}\int_0^l x \sin\frac{n\pi x}{l} dx$

$= \frac{2}{l}\left[x\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin\frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^l$

$= \frac{2}{l}\left[\frac{-l\cos n\pi}{n\pi/l}\right]$

$a_n = \frac{2l}{n\pi}(-1)^{n+1}$

$f(x) = \sum_{n=1}^{\infty} \frac{2l}{n\pi}(-1)^{n+1} \sin\frac{n\pi x}{l}$

$$\boxed{f(x) = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi x}{l}}$$

Part-C

10)

Sol:- The fourier series given by

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ — ① $f(-x) = -(-x)$
$= x = f(x)$

To find a_0

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\, dx$

$= \frac{2}{\pi}\int_0^{\pi} x\, dx$

$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\frac{\pi^2}{2}\right]$

$= \pi$ — ①

8

To find a_n

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{x\sin nx}{n} + (1)\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(\frac{\pi \sin n\pi}{n^2} + \frac{1\cos n\pi}{n^2}\right) - \left(0 + \frac{\cos(0)}{n^2}\right)\right]$$

$$= \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{2}{n^2\pi}\left[\cos n\pi - \cos(0)\right]$$

$$= \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_n = \begin{cases} \frac{-4}{n^2\pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases} \quad - (3)$$

The required fourier series substituting (2) and (3) in (1) we get

$$f(x) = \pi/2 - \sum_{n=odd} \frac{4}{n^2\pi}\cos nx$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=odd}^{\infty} \frac{\cos nx}{n^2}$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5} + \cdots\right] - (4)$$

Deduction

Put $x=0$ in 4

LHS part (4) implies

$f(x) = x$

$f(0) = 0$

RHS part implies

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$= \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right]$

$\frac{\pi}{2} = \frac{4}{\pi}\left[\frac{\cos(0)}{1^2} + \frac{\cos(0)}{3^2} + \frac{\cos(0)}{5^2} + \dots\right]$

$\frac{\pi}{2} \cdot \frac{\pi}{4} = \quad \frac{\pi}{2} \cdot \frac{\pi}{4} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$

$\frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$

1(b)

Sol:- The half range cosine series of $f(x)$ is $(0,\pi)$ is

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ — ①

To find a_0

$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$= \frac{2}{\pi}\int_0^{\pi} x\,dx$

$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\frac{\pi^2}{2}\right]$

$a_0 = \pi$ — ②

To find a_n

$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx.$

$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$

$= \frac{2}{\pi}\left[\frac{x\sin nx}{n} + (1)\frac{\cos nx}{n^2}\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\left(0 + \frac{\cos n\pi}{n^2}\right) - \left(0 + \frac{1}{n^2}\right)\right]$

$= \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right]$

$= \frac{2}{n^2\pi}\left[\cos n\pi - 1\right]$

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$= \frac{2}{n^2\pi}[(-1)^n-1]$

$a_n = \begin{cases} 0 & \text{when } n \text{ is even} \\ \frac{2}{n^2\pi} & \text{when } n \text{ is odd} \end{cases}$ ③

The required cosine series subsituting ② and ③ in ① we get

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$= \frac{\pi}{2} + \sum_{n=1}^{\infty} -\frac{4}{n^2\pi} \cos nx$

The Parseval's Identity for fourier cosine series in $(0,\pi)$ is

$\pi^2 = \frac{2}{\pi}\int_0^\pi [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$

$\frac{2}{\pi}\int_0^\pi x^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$ $\quad a_0 = \pi,\ a_n = \frac{-4}{n^2\pi}$ n-odd

$\frac{2\pi^2}{3} = \frac{\pi^2}{2} + \frac{16}{\pi^2}\sum_{n=odd} \frac{1}{n^4}$

$\frac{2\pi^2}{3} - \frac{\pi^2}{2} = \frac{16}{\pi^2}\sum_{n=odd} 1/n^4$

$\frac{4\pi^2 - 3\pi^2}{6} = \frac{16}{\pi^2}\sum_{n=odd} \frac{1}{n^4}$

$\frac{\pi^2}{6} \cdot \frac{\pi^2}{16} = \sum_{n=odd} \frac{1}{n^4}$

$\frac{\pi^4}{96} = \left[\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right]$

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Time: Three Hours [redacted]

Date: 23.01.2023 / FN

Part A – (10 x 2 = 20 Marks)
(Answer All Questions)

Q.No	Question	BL	CO
1	Write down the Fourier series formula.	R	CO1
2	Find a_0 and a_n, if $f(x) = x, \quad -l < x < l$.	U	CO1
3	Find the complete solution of $pq = 1$.	U	CO2
4	Solve $(D^2 - 7DD' + 6D'^2)z = 0$.	U	CO2
5	Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$.	U	CO3
6	State Fourier law of heat conduction.	U	CO3
7	If F(s) is the Fourier transform of $f(x)$, find the Fourier transform of F (ax), where a>0.	U	CO4
8	State Parseval's identity on Fourier transform.	R	CO4
9	Prove that $Z\left[\frac{1}{n!}\right] = e^{1/z}$.	Ap	CO5
10	Form a difference equation from $y_n = a.3^n$.	U	CO5

Part B – (5 x 4 = 20 Marks)
(Answer All Questions)

Q.No	Question	BL	CO
11	Find the Fourier series for the function $f(x) = x, in\ (-l, l)$.	Ap	CO1
12	Form the PDE by eliminating the arbitrary function Ø from $\phi(x^2 + y^2 + z^2,\ ax + by + cz) = 0$.	Ap	CO2
13	Classify the partial differential equation $2U_{xx} + 5U_{xy} + U_{yy} + 2U_x - 3U_y = 0$.	Ap	CO3
14	Show that the Fourier transform of $e^{\frac{-x^2}{2}}$ is $e^{\frac{-s^2}{2}}$	Ap	CO4
15	State and prove convolution theorem.	Ap	CO5

Part C – (5 x 12 = 60 Marks)
(Answer either (a) or (b) of each questions)

Q.No	Question	BL	CO
16(a)	Find the half range cosine series for $f(x) = x^2$ in $-\pi < x < \pi$. $0 < x < \pi$	Ap	CO1
	OR		
16(b)	Find the Fourier series as far as the second harmonic to represent the function given in the following data.	Ap	CO1

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

Q.No	Question	BL	CO
17(a)	Solve $(3z - 4y)p + (4x - 2z)q = 2y - 3x$.	Ap	
	OR		

$(D^2 - 2DD' + D'^2)z \rightarrow -2\frac{\partial^3 z}{\partial x^3 \partial y}$?

17(b)	Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = e^{x+2y} + \sin(2x - 3y)$.	Ap	CO2
18(a)	A tightly stretched string with fixed end points $x = 0$ and $x = l$ initially in a position given by $y(x,0) = y_0 \sin^3(\frac{\pi x}{l})$. It has released from rest from this position, find the displacement y at any time and at any distance from the end $x = 0$.	Ap	CO3
	OR		
18(b)	A metal bar 30cm long has its ends A and B kept at $20^0 C$ and $80^0 C$ respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to $0^0 C$ and kept so. Find the resulting temperature distribution function $u(x,t)$ taking $x = 0$ at A.	Ap	CO3
19(a)	Find the Fourier Transform of $f(x) = \begin{cases} 1 - \lvert x \rvert & if \lvert x \rvert < 1 \\ 0 & if \lvert x \rvert > 1 \end{cases}$ Hence deduce that $\int_0^\infty (\frac{\sin t}{t})^2 dt = \frac{\pi}{2}$, $\int_0^\infty (\frac{\sin t}{t})^4 dt = \frac{\pi}{3}$	Ap	CO4
	OR		
19(b)	(i). Find the Fourier cosine transform of $\frac{1}{1+x^2}$ (ii). Find the Fourier sine transform of $\frac{x}{1+x^2}$	Ap	CO4
20(a)	Find the Z transform of (i) $r^n \sin n\theta$ (ii) $r^n \cos n\theta$	Ap	CO5
	OR		
20(b)	Solve the difference equation $y(k+2) - 5y(k+1) + 6y(k) = 4^n$ given $y(0) = 0, y(1) = 1$.	Ap	CO5

BIO-MEDICAL

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CONTINUOUS LEARNING ASSESSMENT – I

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date :18-10-2022

Academic Year / Semester :2022-2023/ODD

Duration :1 hour 15 mins

Instructions : Part A- Answer all questions

Part B - Answer either A or B for the questions 5and 6

Part C- Answer either A or B for the question 7

No	Questions	Weightage	CO	Bloom's Level
	PART A (4X2=8)			
1	What are the various solutions of one dimensional wave equation?	2	CO3	1
2	State the Fourier integral theorem	2	CO4	1
3	Find the Fourier sine transform of (x) = e^{-ax},a>0	2	CO4	2
4	State the Fourier Transform pair	2	CO4	1
	PART B (2x6=12)			
5 (A)	Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & otherwise\end{cases}$ Hence deduce that	6	CO4	3
5 (B)	Show that is a self reciprocal with respect to Fourier transform			
6 (A)	Find the Fourier sine transform of $f(x)$ =	6	C04	3
6 (B)	Find the Fourier cosine transform of $f(x)$ =			
	PART C (1X10=10)			
7 (A)	A metal bar 30cm has its ends A and B kept 20^0C and 80^0C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature distribution function $u(x,t)$ taking $x=0$ at A	10	CO3	3
7 (B)	A string is stretched and fastened to two point's at a distance l apart. Motion is started by displacing the string into the form and then released it from this position at time t=0. Find the displacement y at anyDistance x from one end at any time t			

CO	Weightage
CO1	
CO2	

CO4	18
CO5	-
CO6	-
Total	30

Department of mechatronics

Continuous Learning Assessment - I

U20MABT03 - Transform and Boundary Value problems (Set I)

Answer key:

PART - A.

(1) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$.

(i) f(x) is defined and single valued except possibly at a finite number of points in $(c, c+2l)$

ii) f(x) is periodic in $(c, c+2l)$. f(x) and f'(x) are piecewise continuous in $(c, c+2l)$

iii) f(x) has finite number of maxima or minima in $(0, 2\pi)$.

(2) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$, $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx\, dx$

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx\, dx$.

(3) $f(x) = x \Rightarrow f(-x) = -x \Rightarrow -f(x)$ is an odd function

$\therefore a_0 = 0 \quad a_n = 0$.

(4) $f(x) = x^2 \Rightarrow f(-x) = (-x)^2 = x^2 = f(x)$ is an even function

$\therefore b_n = 0$.

5(a) $f(x) = |x|$ is an even function $\therefore b_n = 0$.

$a_0 = \pi \quad a_n = \frac{-4}{n^2\pi}$ if n is odd.

$\therefore f(x) = \frac{\pi}{2} + \sum_{n\,odd}^{\infty} \frac{-4}{n^2\pi} \cos nx$.

5(b) Parseval's thm: $\frac{1}{\pi}\int_0^{\pi} [f(x)]^2 dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1} a_n^2$

6(a) $a_0 = l \qquad a_n = \begin{cases} \frac{-4l}{n^2\pi^2} & \text{if } n \text{ is odd.} \\ 0 & \text{if } n \text{ is even,} \end{cases}$

$$f(x) = \frac{l}{2} - \sum_{n=1}^{\infty} \frac{4l}{n^2\pi^2} \cos\frac{n\pi x}{l}.$$

6(B) $a_0 = \frac{2l^2}{3}$ $a_n = \frac{4l^2(-1)^n}{n^2\pi^2}$

$$f(x) = \frac{l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos\frac{n\pi x}{l}.$$

7(a) $a_0 = 2.9$ $a_1 = 0.37$ $a_2 = -0.1$ $b_1 = 0.17$ $b_2 = -0.06$

$$y = 1.45 - 0.37\cos x + 0.17\sin x - 0.1\cos 2x - 0.06\sin 2x.$$

7(b) $a_0 = \frac{4}{3}l^2$ $a_n = \frac{-4l^2}{n^2\pi^2}$ $b_n = 0.$

$$f(x) = \frac{l^2}{3} - \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\frac{n\pi x}{l}.$$

Answer Key

U20MABT03 – Transforms and Boundary Value problems

(CSE, AI, CC, IB)

Part – A

1. The Fourier series for a function is given by numerical values is known as harmonic analysis. In harmonic analysis the Fourier Coefficients a_0, a_n, b_n of the fun $y=f(x)$ in $(0,2\pi)$ are given by

$a_0 = 2$ [mean value of y in $(0,2\pi)$]
$a_n = 2$ [mean value of $y\cos nx$ in $(0,2\pi)$]
$b_n = 2$ [mean value of $y\sin nx$ in $(0,2\pi)$]

2. $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx, \quad f(x)=x^2$ in $(-\pi,\pi)$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\sin nx\,dx = 0$ (It is called odd fun)

$\boxed{b_n = 0}$ [$\because$ even x odd = odd]

3. A partial diff. eqn is an eqn involving a function of two or more variables and some of its partial derivatives.

Notations $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}, r=\frac{\partial^2 z}{\partial x^2}, s=\frac{\partial^2 z}{\partial x\partial y}, t=\frac{\partial^2 z}{\partial y^2}$

4. $P.I = \frac{1}{9}\cos(3x+2y)$

5. $A=4, B=4, C=1, \Rightarrow B^2-4AC = 16-4(4)(1) = 0$
The given eqn is parabolic eqn.

6. $u_t = a^2 u_{xy}$,
$a^2 = \frac{k}{s\rho}$ is known as diffusivity of the material of the bar.

7. Fourier sine tranform of $f(x) = e^{-ax}, a>0$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$$

$$F_s[f(x)] = \sqrt{2/\pi} \left[\frac{s}{s^2+a^2}\right] \qquad \left[\because \int_0^\infty e^{-ax} \sin bx \, dx = \frac{b}{a^2+b^2}\right]$$

8. <u>Convolution theorem</u>

The Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transforms.

ie, $F[f(x) * g(x)] = F(s)\, G(s) = F[f(x)]\, F[g(x)]$

9. Find the Z-transform of $(\frac{1}{2})^{n-1}$

Ans $F(z) = \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n}$

10. Find $Z\left[\frac{a^n}{n!}\right]$

WKT $Z[a^n f(n)] = [F(z)]_{z \to \frac{z}{a}}$

$Z\left[a^n \frac{1}{n!}\right] = e^{a/z}$

<u>Part - B</u>

11). Find a_0, a_n, $f(x) = x(2\pi - x)$ in $(0, 2\pi)$.

$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) \, dx = \frac{4\pi^2}{3}$ ——— (2m)

$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx \, dx = -\frac{4}{n^2}$ ——— (2m)

12). Solve $(D^2 + 3DD' + 2D'^2) z = x + y$

$z = C.F + P.I$

$z = \phi_1(y - x) + \phi_2(y - 2x) + \frac{x^2}{6}(3y - 2x)$

put $D = m$, $D' = 1$, $m^2 + 3m + 2 = 0$

$m = -1, m = -2$

13) Classify the diff eqn $f_{xx} - f_{yy} = 0$

Here $A = 1, B = 0, C = -1$

$\therefore B^2 - 4AC = 0 - 4(1)(-1)$

$B^2 - 4AC = 4 > 0$ [Hyperbolic eqn]

14) Find the F.T of $f(x) = \begin{cases} 1, & \text{if } |x| < a \\ 0, & \text{if } |x| \geq a \end{cases}$

$$F[f(x)] = F[s] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$F[s] = \sqrt{2/\pi} \left(\frac{\sin as}{s} \right)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

$$f(x) = \frac{2}{\pi} \int_{0}^{\infty} \left(\frac{\sin as}{s} \right) \cos sx \, ds$$

$$\therefore \int_{0}^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$$

15) Find $Z^{-1}\left[\frac{z}{z^2+5z+6} \right]$

$$\frac{F(z)}{z} = \frac{A}{(z+2)} + \frac{B}{(z+3)}$$

$$\frac{F(z)}{z} = \frac{1}{(z+2)} + \frac{(-1)}{(z+3)}$$

$$F(z) = \frac{z}{z+2} - \frac{z}{(z+3)}$$

$$\boxed{x(n) = (-2)^n - (-3)^n}$$

Part - C

16)
a) $y = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + b_1 \sin x + b_2 \sin 2x$

$$a_0 = 2 \frac{\Sigma y}{n} = 2\left[\frac{8.7}{6} \right] = 2.9$$

$$a_1 = 2 \frac{\Sigma y \cos x}{n} = 2\left[\frac{-1.1}{6} \right] = -0.37$$

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$$a_2 = 2\frac{\Sigma y \cos 2x}{n} = 2\left[\frac{0.3}{6}\right] = -0.1$$

$$b_1 = 2\frac{\Sigma y \sin x}{n} = 2\left[\frac{0.5156}{6}\right] = 0.17$$

$$b_2 = 2\frac{\Sigma y \sin 2x}{n} = 2\left[\frac{-0.1732}{6}\right] = -0.06$$

16b) $f(x) = x(2l-x)$ in $(0, 2l)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx = \frac{4l^2}{3}$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx = -\frac{4l^2}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx = 0$$

$$f(x) = \frac{2}{3}l^2 + \sum_{n=1}^{\infty}\left(-\frac{4l^2}{n^2\pi^2}\right)\cos\left(\frac{n\pi x}{l}\right)$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$ //

17) a). Solve $(D^2 - DD' - 2D'^2)z = (2x+3y) + e^{3x+4y}$

$C.F = \phi_1(y-x) + \phi_2(y+2x)$

$$PI = \frac{5}{6}x^3 + \frac{3}{2}x^2y - \frac{1}{35}e^{3x+4y}$$

$$PI_1 = \frac{5}{6}x^3 + \frac{3}{2}x^2y, \quad PI_2 = -\frac{1}{35}e^{3x+4y}$$

17b). Solve $(D^2 - 2DD')z = x^3y + e^{2x}$

$C.F = \phi_1(y+0x) + \phi_2(y+x)$

$C.F = \phi_1(y) + \phi_2(y+x)$

$$PI_1 = \frac{x^5y}{20} + \frac{x^6}{60} + \frac{e^{2x}}{4} \leftarrow PI_2$$

$$PI_2 = \frac{e^{2x}}{4}$$

18a). The heat flow eqn is

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

Steady state conditions becomes

$$\frac{\partial^2 u}{\partial x^2} = 0$$

$\therefore u(x) = ax+b$, given $l = 30$

$$u(x,t) = \frac{2x}{3} + 40 - \frac{40}{\pi} \sum_{1}^{\infty} \sin\frac{n\pi x}{15} e^{-\frac{\alpha^2 n^2 \pi^2 t}{225}}$$

18b). Given $y = 6(2x-x^2)$, $t=0$

Boundary conditions

The wave eqn is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

i). $y(0,t) = 0, \forall t>0$

ii) $y(l,t) = 0, \forall t>0$

iii) $\left(\frac{\partial y}{\partial t}\right)_{(x,0)} = 0, \quad 0<x<l$

(iv). $y(x,0) = 6(2x-x^2), \quad 0<x<l$

Suitable solution is

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos pat + C_4 \sin pat)$$

19a). F T $f(x) = \begin{cases} 1-x^2, & \text{if } |x| \le 1 \\ 0, & \text{if } |x| > 1 \end{cases}$

$$F[s] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx = \frac{4}{\sqrt{2\pi}} \left(\frac{\sin s - s\cos s}{s^3} \right)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds = \frac{4}{\pi} \int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos sx \, ds$$

put $x = 1/2$, $\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos s/2 \, ds = \frac{3\pi}{16}$.

19b). $F[s] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$, $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$

$F[x e^{-a|x|}] = i\sqrt{2/\pi} \frac{2as}{(s^2+a^2)^2}$, (ii) $\int_0^{\infty} \frac{\cos xt}{a^2+t^2} dt = \frac{\pi}{2a} e^{-a|x|}$

20a). Find $z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$ by using convolution thm.

$$z^{-1}\left[\frac{z^2}{(z-a)^2}\right] = z^{-1}\left[\frac{z}{z-a} \cdot \frac{z}{z-a}\right]$$

$$= z^{-1}\left[\frac{z}{z-a}\right] * z^{-1}\left[\frac{z}{z-a}\right]$$

$$= a^n * a^n$$

$$= \sum_{k=0}^{n} a^{n-k} a^k$$

$$= (n+1) a^n$$

20b). Find $z^{-1}\left[\frac{z^3 - 20z}{(z-2)^3(z-4)}\right]$

$$\frac{F(z)}{z} = \frac{z^2 - 20}{(z-2)^3(z-4)}$$

$$\frac{z^2 - 20}{(z-2)^3(z-4)} = \frac{A}{(z-2)} + \frac{B}{(z-2)^2} + \frac{C}{(z-2)^3} + \frac{D}{(z-4)}$$

$A = 1/2$, $B = 2$, $C = 8$, $D = -1/2$

$$x(n) = \frac{1}{2}\left(2^n + 2n^2 2^n - 4^n\right)$$

DEPARTMENT OF MATHEMATICS

COURSE FILE

U20MABT03 /TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed-to-be University under section 3 of UGC Act, 1956)

A GRADE
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BIHER INDIA

DEPARTMENT OF BIO-MEDICAL
CONTINUOUS LEARNING ASSESSMENT – II
U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date	:18-10-2022
Academic Year / Semester	:2022-2023/ODD
Duration	:1 hour 15 mins
Instructions	: Part A- Answer all questions
	Part B - Answer either A or B for the questions 5and 6
	Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (4X2=8)			
1	What are the various solutions of one dimensional wave equation?	2	CO3	1
2	State the Fourier integral theorem	2	CO4	1
3	Find the Fourier sine transform of (x) = e^{-ax},a>0	2	CO4	2
4	State the Fourier Transform pair	2	CO4	1
	PART B (2x6=12)			
5 (A)	Find the Fourier transform of $f(x)$ if $f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & otherwise\end{cases}$ Hence deduce that $\int_0^{\infty}\frac{\sin x}{x}dx=\frac{\pi}{2}$	6	CO4	3
5 (B)	Showthat $e^{\frac{-x^2}{2}}$ is a selfreciprocalwithrespecttoFourier transform			
6 (A)	Find the Fourier sine transform of $f(x)=\begin{cases} sinx, & 0<x<a\\ 0, & x>a\end{cases}$	6	C04	3
6 (B)	Find the Fourier cosine transform of $f(x) = \begin{cases} cosx, & 0<x<a\\ 0, & x>a\end{cases}$			
	PART C (1X10=10)			
7 (A)	A metal bar 30cm has its ends A and B kept 20^0C and 80^0C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature distribution function $u(x,t)$ taking $x=0$ at A	10	CO3	3
7 (B)	A string is stretched and fastened to two point's at a distance l apart. Motion is started by displacing the string into the form $y(x,0)=y_0 sin^3\left(\frac{\pi x}{l}\right)$. and then released it from this position at time t=0. Find the displacement y at anyDistance x from one end at any time t			

CO	Weightage
CO1	
CO2	
CO3	12
CO4	18
CO5	-
CO6	-
Total	30

Department of Mechatronics.

Continuous Learning Assessment – I.

U20MABT03 – Transform and Boundary Value Problem (Set-2)

Answer key:

1. $\bar{y} = \frac{2\pi}{\sqrt{3}}$

2. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$.

$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$, $a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$.

$b_n = \frac{1}{l}\int_0^{2l} f(x)\cdot\sin\frac{n\pi x}{l}\,dx$.

3. $f(-x) = (-x)^2 = x^2 = f(x)$.

$\therefore f(x)$ is an even function.

$\therefore b_n = 0$.

4. i) $f(x)$ is defined and single valued except possibly at a finite number of points in $(c, c+2l)$

ii) $f(x)$ and $f'(x)$ are piecewise continuous in $(c, c+2l)$

iii) $f(x)$ has finite number of maxima or minima in $(0, 2\pi)$

5(A) $b_n = \begin{cases} \frac{4a}{n\pi} & \text{if } n \text{ is odd.} \\ 0 & \text{if } n \text{ is even.} \end{cases}$

$\therefore a = \sum_{n=1}^{\infty} \frac{4a}{n\pi}\sin\frac{n\pi x}{l}$.

5(B) $f(x) = |x|$ is an even function $\therefore b_n = 0$,

$a_0 = \pi$ $a_n = -\frac{4}{n^2\pi}$ if n is odd.

$\therefore f(x) = \frac{\pi}{2} + \sum_{n=odd}^{\infty} \frac{-4}{n^2\pi}\cos nx$.

6(A) $a_0 = \pi \quad a_n = \begin{cases} -\dfrac{4}{n^2\pi} & \text{if } n \text{ is odd.} \\ 0 & \text{if } n \text{ is even.} \end{cases}$

$$f(x) = \frac{\pi}{2} - \frac{4}{n^2\pi} \sum_{n=odd}^{\infty} \cos nx.$$

6(B) $a_0 = \dfrac{2l^2}{3} \quad a_n = \dfrac{4l^2(-1)^n}{n^2\pi^2}$

$$f(x) = \frac{l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi x}{l}.$$

7(a) $a_0 = 41.67 \quad a_1 = -8.33 \quad b_1 = -1.16 \quad a_2 = -2.33$

$b_2 = 0.00013.$

$$y = 20.84 - 8.33 \cos \frac{\pi x}{3} - 1.16 \sin \frac{\pi x}{3} - 2.33 \cos\left(\frac{2\pi x}{3}\right) + 0.00013 \sin\left(\frac{2\pi x}{3}\right)$$

7(b) $a_0 = -\dfrac{\pi}{2} \quad a_n = \begin{cases} \dfrac{2}{\pi n^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases} \quad b_n = \dfrac{1}{n}\left[1-2(-1)^n\right]$

$$f(x) = -\frac{\pi}{4} + \sum_{n=odd}^{\infty} \frac{2}{\pi n^2} \cos nx + \sum_{n=1}^{\infty} \frac{1}{n}\left[1-2(-1)^n\right] \sin nx$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \cdots$$

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DEPARTMENT OF BIO-MEDICAL

CONTINUOUS LEARNING ASSESSMENT – III

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date : 2.01.2023

Academic Year / Semester :2022-2023/ODD

Duration :1 hour 15 mins

Instructions : Part A- Answer all questions

Part B - Answer either A or B for the questions 5and 6

Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (4X2=8)			
1	Find the general solution of $\frac{\partial^2 z}{\partial x \partial y} = \sin x.$	2	CO2	2
2	Solve (.	2	CO2	2
3	Prove that Z[n] = .	2	CO5	2
4	Define inverse z transform	2	CO5	1
	PART B (2x6=12)			
5 (A)	Solve	6	CO2	3
5 (B)	Solve			
6 (A)	Solve (D^2- 2DD') z = x^3y +	6	CO2	3
6 (B)	Find the Complete integral of the PDE .			
	PART C (1X10=10)			
7 (A)	Find [by using convolution method	10	CO5	3
7 (B)	Solve by Z transform $U_{n+2}-2U_{N+1}+U_n = 2^n$ with $u_0 = 2$ and $U_1 = 1$			

CO	Weightage
CO2	16
CO5	14
Total	30

Prepared by	Staff Name A.HEMA	Signature
Verified by	H.O.D Dr. S.V. MANEMARAN	Signature

U20MABT03 - Transform and Boundary Value problems

Department of Bio - Medical.

Answer key:

Part - A.

(1) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, $a^2 = \frac{T}{m} = \frac{\text{Tension}}{\text{Mass per unit of the string}}$

(2) $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(t) e^{i(x-t)s} dt \cdot ds.$

(3) $F_S(S) = \sqrt{\frac{2}{\pi}} \left[\frac{S}{S^2+a^2}\right]$

(4) $F(S) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx.$

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(S) e^{-isx} ds.$

Part - B.

5(a) $F(S) = \sqrt{\frac{2}{\pi}} \left[\frac{\sin sa}{s}\right] \Rightarrow \int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

5(b) $e^{-s^2/2}$

6(a) $F_S(S) = \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(S-1)a}{S-1} - \frac{\sin(S+1)a}{S+1}\right]$

6(b) $F_C(S) = \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(S+1)a}{S+1} + \frac{\sin(S-1)a}{S-1}\right]$

Part - C.

7(a) $u(x,t) = \frac{40}{n\pi} \left[1 - 4(-1)^n\right] \sin\frac{n\pi x}{20} e^{-\frac{\alpha^2 n^2 \pi^2 t}{400}}$

7(b) $y(x,t) = \frac{3y_0}{4} \sin\frac{\pi x}{l} \sin\frac{\pi at}{l} - \frac{y_0}{4} \sin\frac{3\pi x}{l} \sin\frac{3\pi at}{l}.$

DEPARTMENT OF MATHEMATICS

COURSE FILE – ACADEMIC YEAR – 2022-2023

SEMESTER / TERM / YEAR	:	ODD / I / II
COURSE CODE	:	U20MABT03
COURSE NAME	:	TRANSFORMS AND BOUNDARY VALUE PROBLEMS

ECE

Unit - 3

Assignment.

① A tightly stretched flexible string has its ends fixed at $x=0$ and $x=l$. At time $t=0$ the string is given a shape defined by $f(x) = \mu x(l-x)$ where μ is a constant and then released. Find the displacement of the string at any time t.

② If the string of length l is initially at rest in equilibrium position and each of its points is given the velocity $V_0 \sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{\pi x}{l}\right)$ where $0<x<l$ at $t=0$. determine the displacement function $y(x,t)$.

③ A rod 30cm long has its ends A and B kept at 20°C and 80° respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function $u(x,t)$ taking $x=0$ at A.

ASSIGNMENT – I

TRANSFORMS AND BOUNDARY VALUE PROBLEMS

BIO-MEDICAL

Sub :- TBVP

1) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| \ge 1 \end{cases}$ as a fourier integral. Hence evalute $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} dx$ and find the value of $\int_0^\infty \frac{\sin\lambda}{\lambda} dx$.

Ans $F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

$f(x) = 1 \qquad -1 < x < 1.$

$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} 1 (\cos sx + i \sin sx) dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} \cos sx\, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} \sin sx\, dx$

$\int_{-1}^{1} \cos sx$ is an even function

$\int_{-1}^{1} \sin sx$ is an odd function.

$= \frac{2}{\sqrt{2\pi}} \int_0^1 \cos sx\, dx$

$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin sx}{s} \right]_0^1$

$F(s) = \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s}{s} \right]$

Inverse fourier transform

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s}{s} \right] (\cos sx - i \sin sx) dx$

$= \frac{2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx\, dx - \frac{2i}{2\pi} \int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx\, dx$

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx$ is an even function

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx$ is an odd function.

$$= \frac{2}{\pi}\int_0^\infty \frac{\sin s - \cos sx}{s}\, ds$$

Put $s = \lambda$

$ds = d\lambda$

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda$$

Put $x = 1$ $\quad f(x) = 1$

$$1 = \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda}{\lambda} \cos\lambda(1)\, d\lambda$$

$$\pi/2 = \int_0^\infty \frac{\sin\lambda}{\lambda}\, d\lambda$$

Hence proved

2) Find the fourier transform of $e^{-a^2x^2}, a>0$. Hence show th'- $e^{-x^2/2}$ is a self reciprocal under the fourier transform.

$f(x) = e^{-a^2x^2}$

Ans:- $F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty f(x) e^{isx}\, dx$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-a^2x^2} e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-a^2x^2}(\cos sx + i\sin sx)\, dx.$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-a^2x^2}\cos sx\, dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-a^2x^2}\sin sx\, dx$$

$\int_{-\infty}^\infty e^{-(ax)^2}\cos sx$ is an even function

$\int_{-\infty}^\infty e^{-(ax)^2}\sin sx$ is an odd function.

$$= \frac{2}{\sqrt{2\pi}}\int_0^\infty e^{-(ax)^2}\cos sx\, dx$$

$$\therefore F(s) = \frac{2}{\sqrt{2\pi}}\left[\frac{a}{a^2+b^2}\right]^2 \qquad \left(\because \int_0^\infty e^{-ax}\cos sx\, dx = \frac{a}{a^2+b^2}\right)$$

(ii) $e^{-x^2/2}$ is a self reciprocal Fourier transform

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx} . e^{s^2/2} e^{-s^2/2} dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-x^2/2 + isx + s^2/2} . dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-\left(\frac{x-is}{\sqrt{2}}\right)^2} dx \quad \left[\because \frac{-x^2+2isx+s^2}{2} = -\frac{(x-is)^2}{2}\right]$$

let $t = \frac{x-is}{\sqrt{2}}$, $dt = \frac{dx}{\sqrt{2}}$, $dt\sqrt{2} = dx$

$x \to -\infty \qquad t \to -\infty$

$x \to \infty \qquad t \to \infty$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2}\, dt$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} . 2 \int_{0}^{\infty} e^{-t^2} dt \quad [\because e^{-t^2} \text{ is an even function}]$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} . 2 \left[\frac{\sqrt{\pi}}{2}\right]$$

$\therefore F(s) = e^{-s^2/2}$

3) Find the fourier transform of $f(x) = \begin{cases} a-|x| & \text{for } |x|<a \\ 0, & \text{for } |x|>a>0 \end{cases}$

Ans:

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$f(x) = a - x \qquad -a > x < a$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a-x)(\cos sx + i \sin sx) dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a-x) \cos sx \, dx + \frac{i}{\sqrt{2\pi}} \int_{-a}^{a} \sin sx (a-x) dx$$

$\int_{-a}^{a} (a-x) \cos sx$ is an even function

$\int_{-a}^{a} (a-x) \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{a} (a-x) \cos sx \, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[(a-x) \frac{\sin sx}{s} + (-1) \frac{\cos sx}{s^2} \right]_0^a$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{-\cos sa}{s^2} + \frac{1}{s^2} \right]$$

$$F(s) = \frac{2}{\sqrt{2\pi}} \left[\frac{1-\cos sa}{s^2} \right]$$

Inverse Fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi}} \left(\frac{1-\cos sa}{s^2} \right) (\cos sx - \sin sx) ds$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \left(\frac{1-\cos sa}{s^2} \right) \cos sx \, dx - \frac{i}{\pi} \int_{-\infty}^{\infty} \left(\frac{1-\cos sa}{s^2} \right) \sin sx \, ds.$$

W.K.T

$\Rightarrow \int_{-\infty}^{\infty} \left(\frac{1-\cos sa}{s^2}\right) \cos sx$ is an even function

$\int_{-\infty}^{\infty} \left(\frac{1-\cos sa}{s^2}\right) \sin sx$ is an odd function.

$$= \frac{2}{\pi} \int_0^{\infty} \left(\frac{1-\cos sa}{s^2}\right) \cos sx\, ds \qquad \left[\because 1-\cos a = 2\sin^2 \theta/2\right]$$

$$a - x = \frac{2}{\pi} \int_0^{\infty} \frac{2\sin^2 \frac{sa}{2}}{s^2} \cos sx\, dx$$

Put $x = 0$

$$a = \frac{4}{\pi} \int_0^{\infty} \left(\frac{\sin \frac{sa}{2}}{s}\right)^2 (1)\, ds$$

Put $\frac{sa}{2} = t$, $s = \frac{2t}{a}$, $ds = 1/a \cdot 2 \cdot dt$

$$a = \frac{4}{\pi} \int_0^{\infty} \left(\frac{\sin t}{2 \cdot t / a}\right) \cdot 1/a \cdot 2 \cdot dt$$

$$a = \frac{4a^2}{\pi} \cdot 1/2 \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

$$\frac{\pi a}{2a} = \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

4) Find the fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1. \end{cases}$

Hence show that $\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos \frac{s}{2} \cdot ds = \frac{3\pi}{16}$ Also show that $\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^6}\right)^2 dx = \pi/15$?

Ans: Fourier transform

$$F(s) = 1/\sqrt{2\pi} \int_{-\infty}^{\infty} f(x) e^{isx} dx.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) (\cos sx + \sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \cos sx \, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \sin sx \, dx$$

$\int_{-1}^{1} (1-x^2) \cos sx$ is an even function

$\int_{-1}^{1} (1-x^2) \sin sx$ is an odd function.

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{1} (1-x^2) \cos sx \, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[(1-x^2) \frac{\sin sx}{s} - 2x \frac{\cos sx}{s^2} + \frac{2 \sin sx}{s^3} \right]_0^1$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{-2 \cos s}{s^2} + \frac{2 \sin s}{s^3} \right]$$

$$F(s) = \frac{4}{\sqrt{2\pi}} \left[\frac{\sin s - s \cos s}{s^3} \right]$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{4}{\sqrt{2\pi}} \left[\frac{\sin s - s \cos s}{s^3} \right] (\cos sx - i \sin sx)\, ds.$$

$$= \frac{2}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin s - s \cos s}{s^3} \right) \cos x\, ds - \frac{2i}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin s - s \cos s}{s^3} \right) \sin sx\, ds$$

$\int_{-\infty}^{\infty} \frac{\sin s - s \cos s}{s^3} \cos sx$ is an even function.

$\int_{-\infty}^{\infty} \frac{\sin s - s \cos s}{s^3} \sin sx$ is an odd function

$$1-x^2 = \frac{4}{\pi}\int_0^\infty \frac{\sin s - s\cos}{s^3}\cos sx\, ds.$$

Put $x = 1/2$ is continuous sub eqn ①

$$1-(1/2)^2 = 4/\pi \int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos s/2\, ds$$

$$3/4 \times \pi/4 = \int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos s/2\, ds.$$

$$\frac{3\pi}{16} = \int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos s/2\, ds,,.$$

(ii) using parseval's identity

$$= \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$= \int_{-\infty}^{\infty} (1-x^2)^2 dx = \int_{-\infty}^{\infty}\left(\frac{4}{\sqrt{2\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)\right)^2 ds.$$

$$= \int_0^1 (1-x^2)^2 dx = 2\cdot\frac{16}{2\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds$$

put $s = x$

$ds = dx$

$$= \int_0^1 (1^2+x^4-2x^2)\, dx = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$= \left[x + \frac{x^5}{5} - 2\frac{x^3}{3}\right]_0^1 \times \pi/16 = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{16}{15} \times \pi/16 = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx.$$

$$\pi/15 = \int_0^\infty \frac{(\sin x - x\cos x)^2}{x^6} dx$$

$$\pi/15 = \int_0^\infty \frac{-(x\cos x - \sin x)^2}{x^6} dx,,.$$

5) Find the fourier sine transform of $1/x$?

Ans: $F_s(s) = \sqrt{2/\pi} \int_0^\infty f(x) \sin sx \, dx$

$= \sqrt{2/\pi} \int_0^\infty \frac{1}{x} \sin sx \, dx$

Put $sx = y$, $x = y/s$, $dx = \frac{dy}{s}$

$= \sqrt{2/\pi} \int_0^\infty \frac{\sin y}{y/s} \cdot \frac{dy}{s}$

$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin y}{y} \, dy$

$= \sqrt{2/\pi} \left[\pi/2 \right]$

$F_s(s) = \pi/2$ or $\sqrt{\pi}/\sqrt{2}$

$\therefore \int_0^\infty \frac{\sin x}{x} \, dx = \pi/2$

ASSIGNMENT - II

TRANSFORMS AND BOUNDARY VALUE PROBLEMS

BIO - MEDICAL

⑤. Find the fourier sine transform of $1/x$?

Ans

$$F_s(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{1}{x}\sin sx\,dx$$

put $sx = y$, $x = \frac{y}{s}$, $dx = \frac{dy}{s}$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin y}{y/s}\cdot\frac{dy}{s}$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin y}{y}\cdot dy$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{\pi}{2}\right]$$

$$F(s) = \frac{\pi}{2} \text{ or } \frac{\sqrt{\pi}}{\sqrt{2}}$$

$$\therefore \boxed{\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}}$$

①. Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a fourier integral. Hence evalute $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\,dx$ and find the value of $\int_0^\infty \frac{\sin\lambda}{\lambda}\,dx$

Ans

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}\,dx$$

$f(x) = 1$ $\quad -1 < x < 1$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} 1\,(\cos sx + i\sin sx)\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{1}\cos sx\,dx + \frac{i}{\sqrt{2\pi}}\int_{-1}^{1}\sin sx\,dx$$

$\int_{-1}^{1}\cos sx$ is an even function

$\int_{-1}^{1}\sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 \cos sx \, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin sx}{s} \right]_0^1$$

$$F(s) = \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s}{s} \right]$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \, e^{-isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s}{s} \right] (\cos sx - i \sin sx) \, dx$$

$$= \frac{2}{2\pi} \int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx \, dx - \frac{2i}{2\pi} \int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx \, dx$$

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx$ is an even function

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx$ is an odd function

$$= \frac{2}{\pi} \int_0^{\infty} \frac{\sin s \cos sx}{s} \, ds$$

put $s = \lambda$

$ds = d\lambda$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{\sin\lambda \cos\lambda x}{\lambda} \, d\lambda$$

put $x = 1$ $\quad f(x) = 1$

$$1 = \frac{2}{\pi} \int_0^{\infty} \frac{\sin\lambda}{\lambda} \cos\lambda(1) \, d\lambda$$

$$\frac{\pi}{2} = \int_0^{\infty} \frac{\sin\lambda}{\lambda} \, d\lambda$$

Hence proved

(2). Find the fourier transform of $e^{-a^2x^2}$, $a>0$. Hence show that $e^{-x^2/2}$ is a self reciprocal under the fourier transform.

Ans

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx} dx \qquad f(x) = e^{-a^2x^2}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} (\cos sx + i \sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} \cos sx\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} \sin sx\, dx$$

$\int_{-\infty}^{\infty} e^{-(ax)^2} \cos sx$ is an even function

$\int_{-\infty}^{\infty} e^{-(ax)^2} \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-(ax)^2} \cos sx\, dx$$

$$\therefore F(s) = \frac{2}{\sqrt{2\pi}} \left[\frac{a}{a^2+b^2}\right]^2 \qquad \left(\because \int_0^{\infty} e^{-ax} \cos sx\, dx = \frac{a}{a^2+b^2}\right)$$

(ii) $e^{-x^2/2}$ is a self reciprocal

Fourier trans for

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx} \cdot e^{s^2/2} e^{-s^2/2}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-x^2/2 + isx + s^2/2} \cdot dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-\left(\frac{x-is}{\sqrt{2}}\right)^2} dx \qquad \left[\because \frac{-x^2+2isx+s^2}{2} = -\frac{(x-is)^2}{2}\right]$$

let $t = \frac{x - is}{\sqrt{2}}$ $dt = \frac{dx}{\sqrt{2}}$ $dt\sqrt{2} = dx$

$x \to -\infty$ $t \to -\infty$

$x \to \infty$ $t \to \infty$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2}\, dt$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} \cdot 2\int_{0}^{\infty} e^{-t^2} dt \qquad \left[\because e^{-t^2} \text{ is an even function}\right]$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} \cdot 2 \left[\frac{\sqrt{\pi}}{2}\right]$$

$$\therefore F(S) = e^{-s^2/2}$$

③. Find the fourier transform of $f(x) = \begin{cases} a - |x| & \text{for } |x| < a \\ 0 & \text{, for } |x| > a > 0 \end{cases}$

Ans

$$F(S) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$-a < x < a$

$f(x) = a - x$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a - x)(\cos sx + i \sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a - x) \cos sx\, dx + \frac{i}{\sqrt{2\pi}} \int_{-a}^{a} \sin sx (a - x)\, dx$$

$\int_{-a}^{a} (a - x) \cos sx$ is an even function

$\int_{-a}^{a} (a - x) \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{a} (a - x) \cos sx\, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[(a - x) \frac{\sin sx}{s} + (-1) \frac{\cos sx}{s^2}\right]_0^a$$

$$= \frac{2}{\sqrt{2\pi}}\left[\frac{-\cos sa}{s^2} + \frac{1}{s^2}\right]$$

$$F(s) = \frac{2}{\sqrt{2\pi}}\left[\frac{1-\cos sa}{s^2}\right]$$

Inverse Fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi}}\left(\frac{1-\cos sa}{s^2}\right)(\cos sx - \sin sx)\, ds$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\cos sx\, dx - \frac{i}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\sin sx\, ds$$

WKT $\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\cos sx$ is an even function

$\Rightarrow \int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\sin sx$ is an odd function

$$= \frac{2}{\pi}\int_{0}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\cos sx\, ds \qquad \left[\because 1-\cos a = 2\sin^2\theta/2\right]$$

$$a - x = \frac{2}{\pi}\int_{0}^{\infty}\frac{2\sin^2 \frac{sa}{2}}{s^2}\cos sx\, dx$$

put $x = 0$

$$a = \frac{4}{\pi}\int_{0}^{\infty}\left(\frac{\sin \frac{sa}{2}}{s}\right)^2 (1)\, ds$$

put $\frac{sa}{2} = t$, $s = \frac{2t}{a}$ $\qquad ds = \frac{1}{a}\cdot 2\cdot dt$

$$a = \frac{4}{\pi}\int_{0}^{\infty}\left(\frac{\sin t}{\frac{2\cdot 2t}{2\cdot a}}\right)\cdot\frac{1}{a}\cdot 2\cdot dt$$

$$a = \frac{\cancel{4}^{2} a}{\pi} \cdot \frac{1}{\cancel{2}_1} \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

$$\frac{\pi a}{2a} = \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

$$\frac{\pi}{2} = \int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt$$

(4). Find the fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1. \end{cases}$

Hence show that $\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2} \cdot ds = \frac{3\pi}{16}$

Also show that $\int_0^{\infty} \left(\frac{x\cos x - \sin x}{x^6}\right)^2 dx = \frac{\pi}{15}$?

Ans (i) Fourier transform

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)(\cos sx + i\sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)\cos sx\, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \sin sx\, dx$$

$\int_{-1}^{1} (1-x^2)\cos sx$ is an even function

$\int_{-1}^{1} (1-x^2)\sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 (1-x^2)\cos sx\, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[(1-x^2)\frac{\sin sx}{s} - 2x\frac{\cos sx}{s^2} + 2\frac{\sin sx}{s^3}\right]_0^1$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{-2\cos s}{s^2} + \frac{2\sin s}{s^3}\right]$$

$$F(s) = \frac{4}{\sqrt{2\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{4}{\sqrt{2\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right](\cos sx - i\sin sx)\, ds$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)\cos sx\, ds - \frac{2i}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)\sin sx\, ds$$

$\int_{-\infty}^{\infty}\frac{\sin s - s\cos s}{s^3}\cos sx$ is an even function

$\int_{-\infty}^{\infty}\frac{\sin s - s\cos s}{s^3}\sin sx$ is an odd function

$$1-x^2 = \frac{4}{\pi}\int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos sx\, ds$$

put $x = \frac{1}{2}$ is continuous sub for equ^n ①

$$1-\left(\frac{1}{2}\right)^2 = \frac{4}{\pi}\int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\frac{3}{4}\times\frac{\pi}{4} = \int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\frac{3\pi}{16} = \int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

(ii) using parseval's identity

$$\int_{-\infty}^{\infty}|f(x)|^2 dx = \int_{-\infty}^{\infty}|F(s)|^2 ds$$

$$\int_{-1}^{1}(1-x^2)^2 dx = \int_{-\infty}^{\infty}\left(\frac{4}{\sqrt{2\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)\right)^2 ds$$

$$= \int_0^1 (1-x^2)^2 dx = \frac{2\cdot 16}{2\pi} \int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds$$

put $s = x$

$ds = dx$

$$= \int_0^1 (1^2 + x^4 - 2x^2) dx = \frac{16}{\pi} \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$= \left[x + \frac{x^5}{5} - 2\frac{x^3}{3}\right]_0^1 \times \frac{\pi}{16} = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{\not{16}}{15} \times \frac{\pi}{\not{16}} = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{\pi}{15} = \int_0^\infty \frac{(\sin x - x\cos x)^2}{x^6} dx$$

$$\frac{\pi}{15} = \int_0^\infty \frac{-(x\cos x - \sin x)^2}{x^6} dx$$

ıarath Institute of Science and Technology (CBCS

Department of Biomedical Engineering

II Year Internal Marks (Out of 50)

S.No	REG.NO	NAME	TBVP
1	U21BM002	ARELLA MEGHANA	45
2	U21BM004	DHILIPAN S	48
3	U21BM005	GANDI ABHINAV	40
4	U21BM006	GOKUL C	46
5	U21BM008	JANANI D	45
6	U21BM010	KADIRI MANU SATHWIK NAIDU	44
7	U21BM011	KALAIYARASI H	49
8	U21BM012	KARUNYA E	44
9	U21BM013	KAVYA S	46
10	U21BM014	KOTRA SANGEETHA	47
11	U21BM015	KRISHNA PRIYA	42
12	U21BM016	MADDELA SUJAY REDDY	40
13	U21BM017	MADESH RAJ M	41
14	U21BM018	MOHAN RAJ C	40
15	U21BM019	NILAA S	46
16	U21BM020	PRIYADARSHAN K	48
17	U21BM021	SAI AKASH V	47
18	U21BM022	SAMPREETH V	35
19	U21BM024	SANJUVIGASINI K R	42
20	U21BM025	SEETHARAMAN S	40
21	U21BM026	SHAIK KARISHMA	41
22	U21BM027	SHRUTI VIJAY KUMAR SHARMA	44
23	U21BM028	SOWMIYA G	45
24	U21BM029	CHAITANYA GURRAM	46
25	U21BM031	LINGAREDY VEERA SIVA REDDY	38
26	U21BM032	MAMILLAPALLI SOUMYA	49
27	U21BM034	PANDI SARANYA R	45
28	U21BM035	SHAIK SAMEERA	46
29	U21BM036	BODDAPATI ASHA KIRANI	42
30	U21BM037	GS MADHAN KUMAR	38
31	U21BM038	NELLUTLA JOSHNA SRI	36
32	U21BM039	PODILA ASHRITH	40
33	U21BM041	SHANIGARAPU KEERTHANA	38
34	U21BM042	SPURGEON JAYAKARAN F	42
35	U21BM043	VALLISH KARTHIKEYA BANGARU	39
36	U21BM044	PITTAMALLA ANANTHARAJU	44
37	U21BM045	SHAIK FIROZ	47
38	U21BM046	BRACELIN L	40
39	U21BM047	ABINASH T	36
40	U21BM048	MULLA UMAR FAROOQ	34
41	U21BM049	KIRAN ZEHRA	46
42	U21BM050	HRISHIKESH HARIDAS	42
43	U21BM051	PUTTU MUNEESWARI	46
44	U21BM052	PULAGANI NANDHINI	47
45	U21BM053	CHILLAKURU SANJANA REDDY	40
46	U21BM054	SANGA SRAVANI	46
47	U21BM055	SRIGADE TEJASWINI	44

48	U21BM056	CHINTALAPALLI PREETHI	45
49	U21BM057	TUPILI SAI NIKHIL	44
50	U21BM058	ROHIT KUMAR SHARMA	47
51	U21BM059	VENKATA KRISHNA REDDY	46
52	U21BM060	MADIGA HARIVARDHAN	42
53	U21BM061	PODUGU UDAY KIRAN	45
54	U21BM062	ANNU ABHIRAM	38
55	U21BM064	NANDIGAM BOBBY	41
56	U21BM065	VETTRIVEL V	38
57	U21BM701	SRINIVASALU REDDY	45

CONTINUOUS LEARNING ASSESSMENT - I

U20MABT03– TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 24.01.2022

Academic Year / Semester : 2021-2022 / ODD

Duration : 1.5 Hours (90 minutes)

Instructions : Descriptive Type Questions

Q No	Questions	Weightage	CO	Bloom's Level
	PART A (6x2=12 Marks)			
1	Define Fourier Series.	2	CO1	R
2	Define Dirichlet's condition.	2	CO1	R
3	Define Periodic Function.	2	CO1	R
4	Find a_0 for the function $f(x) = \pi - x$ in $0 \le x \le \pi$	2	CO1	U
5	Write down the formula for the Fourier series in $(0,2l)$	2	CO1	R
6	Write down the Bernoulli's formula.	2	CO1	R
	PART B (3x6=18 Marks)			
7	(a) Find the Fourier series for $f(x) = 2x - x^2$ in $0 < x < 2$ **(OR)** (b) Find the Fourier sine series for $f(x) = x$ in $0 < x < l$. Show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$	6	CO1	A
8	(a) Obtain half range cosine series for $f(x) = x^2$ in $(0,\pi)$ **(OR)** (b) Find the Fourier series for the function $f(x)=\begin{cases} kx, & 0<x<l \\ 0, & l<x<2l \end{cases}$	6	CO1	A
9	(a) Find the Fourier series of $f(x) = x$ in $(0,l)$ **(OR)** (b) Find the Fourier series for $f(x) = x^3$ in $(-\pi,\pi)$	6	CO1	A
	PART C (2x10=20 Marks)			
10	(a) Obtain the Fourier series for the function $f(x)=\begin{cases} -x, & -\pi<x<0 \\ x, & 0<x<\pi \end{cases}$ and hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$ **(OR)** (b) Find the first two harmonic of the Fourier series of $f(x)$ given by the following table (see below)	10	CO1	A
11	(a) Find the Fourier series for $f(x) = x + x^2$ in $-\pi < x < \pi$. Hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots=\frac{\pi^2}{6}$ **(OR)** (b) Find the half range cosine series for $f(x) = x$ in $(0,\pi)$. Deduce that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots=\frac{\pi^4}{96}$	10	CO1	A

Table for Q10 (b):

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1

U20MABT03 - Transform And Boundary Value problems

Internal Assessment - I

Answer Key

① $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

② $a_0 = 0$ and $a_n = 0$

③ $b_n = 0$.

④. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$.

⑤. $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$.

⑥ Parseval's Identity:

$$\frac{1}{2\pi} \int_0^{2\pi} [f(x)]^2 dx = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

Part - B.

(7) (a) $a_0 = \frac{8l^2}{3}$, $a_n = \frac{-4}{n^2\pi^2}$.

(or)

(b) $a_0 = \frac{4l^2}{3}$, $a_n = \frac{-4l^2}{n^2\pi^2}$.

(8) (a) $a_0 = a_n = 0$, $b_n = \frac{-2l}{n\pi}(-1)^n$.

(or)

(b) $a_0 = \frac{2l^2}{3}$, $a_n = \frac{4l^2}{n^2\pi^2}(-1)^n$

$b_n = 0$

(9) (a) $a_0 = l$, $a_n = \frac{2l}{\pi^2 n^2}[(-1)^n - 1]$

(or)

(b) $a_0 = \pi$, $a_n = \frac{2}{\pi n^2}[(-1)^n - 1]$

Part - C.

(10) (a) $a_0 = \frac{4l^2}{3}$, $a_n = \frac{-4l^2}{n^2\pi^2}$, $b_n = 0$.

(or)

(b) $a_0 = \frac{2K+\pi}{2}$, $a_n = \frac{1}{n^2\pi}[(-1)^n - 1]$

$b_n = \frac{K}{n\pi}[(-1)^n - 1]$.

(11) (a) $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{\pi n^2}((-1)^n - 1)$, $b_n = 0$

(or)

(b) $a_0 = \frac{-\pi}{2}$, $a_n = \frac{1}{\pi n^2}((-1)^n - 1)$

$b_n = \frac{1}{n}[1 - 2(-1)^n]$.

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Electronics and Communication Engineering

INTERNAL ASSESSMENT -II
U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date & Time : 23.11.2022 & (2.20 PM – 03.50 PM)
Academic Year / Semester : 2022-2023/ III
Duration : 90 min

Q.No	PART – A (2X6 = 12) Answer all questions	Weightage	CO	Revised Bloom's Level
1	Classify the differential equation $$3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}-u=0.$$	02	CO3	U
2	Classify the differential equation $4\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$.	02	CO3	U
3	What does α^2 represent in one dimensional heat flow equation $u_t = \alpha^2 U_{xy}$?	02	CO3	U
4	State the Fourier integral theorem.	02	CO4	U
5	If $F\{f(x)\}=F(s)$, then $F\{f(x)\cos ax\}=$	02	CO4	U
6	Write down Parseval's Identity formula in the Fourier transforms.	02	CO4	U
	PART – B (6X3 = 18) Answer either-or question			
7 (Or)	(a) What are the various solutions of one dimensional wave equation?	6	CO3	U
	(b) Write all the three possible solutions one dimensional heat equation.	6	CO3	U
8 (Or)	(a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $k(lx-x)$, then show that $$y(x,t)=\frac{8kl^3}{a\pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{4}\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$$	6	CO3	U
	(b) Find the Fourier transform of $f(x)$ if $$f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & \lvert x\rvert>1.\end{cases}$$	6	CO4	U
9 (Or)	(a) State and prove shifting theorem.	6	CO4	U
	(b) State and prove Modulation theorem.	6	CO4	U
	PART – C (10X2 =20) Answer either or question			
10 (Or)	(a) A metal bar 30cm has its ends A and B kept 20^0C and 80^0C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0^0C and kept so. Find the resulting temperature distribution function $u(x,t)$ taking $x=0$ at A?	10	CO3	A
	(b) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y=k(lx-x^2)$ If it is released from rest from its position, find the displacement y at any time and any distance from the end $x=0$.	10	CO3	A
11 (Or)	(a) Show that the Fourier transform of $$f(x)=\begin{cases}1-x^2 & \lvert x\rvert<1\\ 0 & \lvert x\rvert>1>0\end{cases}$$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s-s\cos s}{s^3}\right)$. Hence deduce that $\int_0^{\infty}\frac{\sin s-s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$. Using Parseval's identity show that $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)dx=\frac{\pi}{15}$.	10	CO4	A

	(b) Find the Fourier transform of $f(x)$ if $$f(x)=\begin{cases}1, & \lvert x\rvert<1\\0, & otherwise\end{cases}$$ Hence deduce that $\int_0^\infty \frac{\sin x}{x}dx=\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$	10	CO4	A

ASSESSMENT SUMMARY

COs	Remember	Understand	Apply	Analyse	Evaluate	Create	Total (50)	Total (30)
CO1								
CO2							22	13
CO3		12	10				28	27
CO4		18	10					
CO5								
CO6								

Prepared by	Staff Name Dr.V.VETRIVEL	Signature
Verified by	HoD Dr.H.UMMA HABIBA	Signature

U20MABT03 - Transforms and Boundary Value problems.

Internal Assessment - II

Answer Key

(1) -56 Ellips.

(2) $B^2-4AC=0$ $A=4, B=0, C=0$

$=0$.

(3) α^2 represent $\Rightarrow k/\rho c$

$\alpha^2 = k/\rho c$

k - thermal conductivity

ρ - density

(4) $$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{i(x-t)s}\, dt\, ds.$$

(5) $$F\{f(x)\cos ax\} = \frac{1}{2}\left[F(s+a) + F(s-a)\right].$$

(6) $$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

Part - B

7. (a) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$

$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$,

$y(x,t) = (c_1 x + c_2)(c_3 t + c_4)$.

(b) (or)

$u(x,t) = (A \cos px + B \sin px) e^{-\alpha^2 p^2 t}$.

8. (a)

(or)

(b) $f(x) = \sqrt{2/\pi} \left(\frac{\sin s}{s}\right)$

(9) (a) Shifting theorem

$F[f(x)] = F(s)$

$F[f(x-a)] = e^{isa} F(s)$.

(or)

(b) Modulation theorem.

$F[f(x)] = F(s)$ then

$F[f(x) \cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$.

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

CONTINUOUS LEARNING ASSESSMENT – III

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date	: 27.12.2022
Academic Year / Semester	:2022-2023/ODD
Duration	:1 hour 15 mins
Instructions	: Part A- Answer all questions
	Part B - Answer either A or B for the questions 5and 6
	Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (6X2=12)			
1	Form the partial differential equation by eliminating the arbitrary function in Z = f(x^2+y^2)	2	CO2	*R*
2	Solve p+q=1	2	CO2	*U*
3	Solve pe^y=qe^x	2	CO 2	*R*
4	Prove that Z[n] = $\frac{z}{(z-1)^2}$	2	CO 5	*U*
5	Find Z[$\frac{1}{n}$]	2	CO 5	*U*
6	State Initial and Final value theorem	2	CO5	*U*
	PART B (3x6=18)			
7	(a) Solve Z = px + qy + p^2-q^2 **(OR)** (b)State and prove convolution theorem.	6	CO2	*A*
8	(a)Find the Z – transform of the following, i) Z[1]ii) Z[a^n] **(OR)** (b) Solve $(mz-ny)p+(nx-lz)q=ly-mx$	6	CO 5	*A*
9	Solve $(D^2-7DD'+6D'^2)z=0$. **(OR)** Solve $(D^3-7DD'^2-6D'^3)z=0$	6	CO2	*A*
	PART C (2x10=20)			
10	Solve $(D^2-DD'+2D'^2)z=2x+3y+e^{3x+4y}$	10	CO 2	*A*
11	Find $Z^{-1}[\frac{z}{z^2+5z+6}]$			
12	Solve $(D^3+D^2D'+4DD'^2+4D'^3)z=\cos(2x+y)$.	10	CO2	*A*

CO	Weightage
CO1	
CO2	28
CO3	
CO4	
CO5	22
CO6	-
Total	50

Prepared by	Faculty Name Mrs.H.SASIKALA	Signature
Verified by	Hod Dr.S.V.MANEMARAN	Signature

V20MABT03 - Transforms and Boundary Value problems

Internal Assessment - III

Answer Key.

(1) $py - qx = 0$

(2) $z = ax + (1-a)y + c$

(3) $\log a - \log b = x - y$.

(4) $\frac{z}{(z-1)^2}$

(5) $Z\left(\frac{1}{n}\right) = -\log\left(\frac{z}{z-1}\right)$

(6) Initial value theorem

If $Z[f(n)] = f(z)$ then

$$f(0) = \lim_{z \to \infty} f(z)$$

Final value theorem

If $Z[f(n)] = f(n)$ then

$$\lim_{n \to \infty} f(n) = \lim_{z \to 1} (z-1) \cdot f(z).$$

Part - B.

7. (a) $z = ax + by + a^2 - b^2$

~~$z = ...$~~ $f_2(y + bx)$

(b) Convolution theorem

8. (a) (i) $Z(1) = \frac{z}{z-1}$, $|z| > 1$,

(ii) $Z\{a^n\} = \frac{z}{z-a}$, $|z| > a$)

(b) $Q\left(\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}\right)$, $(lx + my + nz) = c$

9 (a) $m = 1, 6$; $z = f_1(y + m_1 x) + f_2(y + m_2 x)$

(or)

(b) The roots are $-1, -2, 3$

$C.F = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x)$

Part - C

10.

$m = \pm 2i$

$C.F = f_1(y - 1) + f_2(y - 2i) + f_3(y + 2i)$

$P.I = -\frac{\sin(2x + y)}{24}$

(11) $z^{-1}\left[\frac{z}{z^2 + 5z + 6}\right]$. $B = -1$, $A = 1$

(12) $x(n) = (-2)^n - (-3)^n$

$m = -1, +2i, -2i$

$P.I = \frac{1}{24}\left[-2\sin(2x + y) + \sin(2x + y)\right]$

Part - C.

10. (a) $c_2 = 20$, $c_1 = \frac{60}{l}$, $p = \frac{n\pi}{l}$, $b_n = \frac{40}{n\pi}\left[1 + 4(-1)^{n+1}\right]$

(or)

(b)

11. (a) $F(s) = 2\sqrt{\frac{2}{\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$

(or)

(b) $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cdot \cos s/2 \, ds = \frac{3\pi}{16}$.

$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) dx = \pi/15$.

ECE

TRANSFORMS AND BOUNDARY VALUE PROBLEMS

ASSIGNMENT

NAME: S. Lavanya

Rg. NO: U21EC354

section: F section

⇒ A metal bar 30cm has its ends A and B kept 20°c and 80°c respectively, until steady state conditions prevail. the temperature at each end is then suddenly reduced to 0°c and kept so. find the resulting temperature distribution function $u(x,t)$ taking $x=0$ at A?

Solution: we know that

The heat eqn is $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \longrightarrow (1)$

when steady state cond exist the heat flow eqn becomes

$$\frac{\partial^2 u}{\partial x^2} = 0 \rightarrow (2)$$

Solving (2), we get $u(x) = c_1 x + c_2 \rightarrow (3)$

From the given problem we having the flg boundary cond

i) $u(0) = 20$

ii) $u(l) = 80$ ∴ $l = 30$ cm given in Problem

Applying the B.C (i) in eq (3) we get $x=0$

$u(0) = c_1(0) + c_2 = 20 \Rightarrow c_2 = 20$

Sub in eq (3)

$u(x) = c_1 x + 20 \longrightarrow (4)$

i.e $x = l$

$u(l) = C_1 l + 20 = 80$

$C_1 l = 80 - 20 = 60$

$C_1 l = 60$

$C_1 = \frac{60}{l}$ sub in eq ④

$u(x) = \frac{60}{l} x + 20$

The temp distribution reached at the steady state become -s initial temparature distribution for the unsteady state.

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \rightarrow ⑤$$

The new boundary Cond are

i) $u(0,t) = 0$, $t>0$

ii) $u(l,t) = 0$, $t>0$

iii) $u(x,0) = \frac{60}{l} x + 20$ $0 < x < l$

The given sol. is

$$u(x,t) = (A \cos px + B \sin px) e^{-\alpha^2 p^2 t} \rightarrow ①$$

Applying B.c (i) in eq ①

i.e $x=0$ we get

$u(0,t) = A e^{-\alpha^2 p^2 t} = 0$

Let $A = 0$ sub in eq ①

$u(x,t) = B \sin px \, (e^{-\alpha^2 p^2 t}) \rightarrow ②$

Applying the B.c (ii) in eq ②

i.e $x = l$

i.e $x = l$

$$u(l,t) = B \sin pl \,(e^{-\alpha^2 p^2 t}) = 0$$

$$\sin pl = 0 = \sin n\pi$$

$$pl = n\pi$$

$$p = \frac{n\pi}{l} \quad \text{sub in eq } 2$$

$$u(x,t) = B \sin \frac{n\pi}{l} x \left[e^{-\alpha^2 \frac{n^2 \pi^2}{l^2} t} \right] \rightarrow ③$$

Applying the B.C (iii) in eq ③

i.e $t = 0$

$$u(x,0) = B \sin \frac{n\pi x}{l} = \frac{60x}{l} + 20$$

$$u(x,t) = B_n \sin \frac{n\pi x}{l} \cdot e^{-\alpha^2 p^2 t} \rightarrow ④$$

$$b_n = B_n = \frac{2}{l} \int_0^l f(x) \cdot \sin \frac{n\pi}{l} dx$$

$$= \frac{2}{l} \int_0^l \left(\frac{60x}{l} + 20 \right) \cdot \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l} \int_0^l \left[\frac{60x + 20l}{l} \right] \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l^2} \int_0^l \underbrace{60x + 20l}_{u} \; \underbrace{\sin \frac{n\pi}{l} x}_{v} \; dx$$

$$= \frac{2}{l^2} \left[(60x + 20l) \left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right) - (60) \left(\frac{-\sin \frac{n\pi x}{l}}{\left(\frac{n\pi}{l} \right)^2} \right) \right]_0^l$$

$$= \frac{2}{l^2} \left[(-80l) \frac{(-1)^n}{\frac{n\pi}{l}} + 0 \right) - \left(-20l \right) \frac{1}{\frac{n\pi}{l}} \bigg) \right]$$

$$= \frac{2}{l^2} \left[-80l \, (-1)^n / n\pi/l + 20l / \frac{n\pi}{l} \right]$$

$$= \frac{2(20l)}{l^2 \left(\frac{n\pi}{l}\right)} \left[-4(-1)^n + 1\right]$$

$$= \frac{40l^2}{l^2 n\pi} \left[(-1)4 (-1)^n + 1\right]$$

$$\therefore b_n = \frac{40}{n\pi} \left[1 + 4(-1)^{n+1}\right]$$

$b_n = B_n$ in eq ⑤

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{l} \, e^{-\alpha^2 p^2 t}$$

$$= \sum_{n=1}^{\infty} \frac{40}{n\pi} \left[1 + 4(-1)^{n+1}\right] . \sin\frac{n\pi x}{l} . e^{-\alpha^2 p^2 t}$$

$$u(x,t) = 40/\pi \sum_{n=1}^{\infty} \frac{1}{n} \left[1 + 4(-1)^{n+1}\right] \sin\frac{n\pi x}{l} . e^{-\alpha^2 p^2 t}$$

⇒ A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = k(lx - x^2)$ If it is released from rest from its position, find the displacement y at any time and any distance from the end $x=0$

Solution:

The motion of the string is given

$$\frac{\partial^2 y}{\partial x^2} = a^2 \frac{d^2 y}{\partial y^2} \longrightarrow ②$$

Boundary Condition are

(i) $y(0,t)=0$, $t>0$

[ii] $y[l,t]=0$, $t>0$

(iii) $\frac{\partial y}{\partial t}(x,0)=0$, $0<x<t$

(iv) $y(x,0)=k(lx-x^2)$, $0<x<l$

The general sol of (1) is given by

$y(x,t)=[c_1\cos px+c_2\sin px][c_3\cos pat+c_4\sin pat] \rightarrow (1)$

$y(0,t)=[c_1\cos 0+c_2\sin 0][c_3\cos pat+c_4\sin pat]$

$=c_1(c_3\cos pat+c_4\sin pat]=0$

let $c_1=0$ sub in (1)

$y(x,t)=c_2\sin px[c_3\cos pat+c_4\sin pat] \rightarrow (2)$

Applying Boundary Condition (ii) in eq (1)

$y(l,t)=c_2\sin pl[c_3\cos pat+c_4\sin pat]=0$

$\sin pl=0=\sin n\pi$

$pl=n\pi$

$p=\frac{n\pi}{l} \rightarrow$ sub in eq (2)

$y(x,t)=c_2\sin\frac{n\pi}{l}x[c_3\cos\frac{n\pi}{l}at+c_4\sin\frac{n\pi}{l}at] \rightarrow (3)$

diff (3) partially w.r.to 't'

$\frac{\partial y}{\partial t}(x,t)=c_2\sin\frac{n\pi x}{l}[c_3(-\sin\frac{n\pi}{l}at)\frac{n\pi a}{l}+c_4\cdot\cos\frac{n\pi at}{l}\cdot\frac{n\pi a}{l}]$

Applying boundary Condition (iii) in (3)

Let $c_4=0$

$y(x,t)=c_2\sin\left(\frac{n\pi}{l}\right)x\cdot c_3\cos\left(\frac{n\pi}{l}\right)at$

$$= C_2 C_3 \sin \frac{n\pi x}{l} \cos \frac{n\pi}{l} at$$

$$= C_n \sin \frac{n\pi x}{l} \cdot \cos \frac{n\pi a t}{l}$$

In general

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{l} \cdot \cos \frac{n\pi a t}{l} \rightarrow (5)$$

Apply boundary Condition (iv) in (5)

$$y(x,0) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{l} = k(lx - x^2)$$

The above eqn half range sine series

$$\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} = f(x)$$

To calculate C_n we use b_n formula

$$b_n = C_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{l} \int_0^l k(lx - x^2) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2k}{l} \int_0^l (lx - x^2) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2k}{l} \left[(lx - x^2) \left[\frac{-\cos \frac{n\pi}{l} x}{\frac{n\pi}{l}} \right] - (l - 2x) \left(\frac{-\sin \frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2} \right) + (-2) \left[\frac{\cos \frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^3} \right] \right]_0^l$$

$$= \frac{2k}{l} \left[(l^2 - l^2) \left[\frac{-\cos (n\pi/l) l}{\frac{n\pi}{l}} \right] + (l - 2l) \left(\frac{\sin (n\pi/l) l}{\left(\frac{n\pi}{l}\right)^2} \right) - 2 \frac{\cos \frac{n\pi}{l} l}{\left(\frac{n\pi}{l}\right)^3} - 0 \right]$$

$$= \frac{2k}{l} \left[0 - \frac{2(-1)^n}{\left(\frac{n\pi}{l}\right)^3} - 0 - 0 + 2 \frac{1}{\left(\frac{n\pi}{l}\right)^3} \right]$$

$$= \frac{2k}{l} \left[\frac{-2(-1)^n}{\left(\frac{n\pi}{l}\right)^3} + \frac{2}{\left(\frac{n\pi}{l}\right)^3} \right]$$

$$= \frac{4\ell}{\ell}\left[\frac{1-(-1)^n}{\left(\frac{n\pi}{\ell}\right)^3}\right]$$

$$= \frac{4k}{\frac{n^3\pi^3}{\ell^3}\ell}\left[1-(-1)^n\right]$$

$$= \frac{4k\ell^2}{n^3\pi^3}\left[1-(-1)^n\right]$$

When n is odd — when n is even

$$b_n = \frac{8k\ell^2}{n^3\pi^3} \qquad b_n = 0$$

sub in in (5)

$$y(x,t) = \sum_{n=1,3,5}^{\infty} \frac{8k\ell^2}{n^3\pi^3} \cdot \sin\frac{n\pi x}{\ell} \cdot \cos\frac{n\pi a t}{\ell}$$

⇒ Show that the fourier transform of

$$f(x) = \begin{cases} 1-x^2 & |x|<1 \\ 0 & |x|>1>0 \end{cases}$$ is $2\sqrt{\frac{2}{\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$

Solution:

Given $f(x) = \begin{cases} 1-x^2 & |x|<1 \\ 0 & |x|>1>0 \end{cases}$

We know that fourier transform is

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1}^{1}(1-x^2)\,(\cos sx + i\sin sx)\,dx$$

$$= \frac{2}{\sqrt{2\pi}}\int_{0}^{1}(1-x^2)\cos sx\,dx + i\int_{0}^{0}(1-x^2)\sin sx\,dx$$

We take

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 (1-x^2) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^1 \underset{u}{(1-x^2)} \quad \underset{v}{\cos sx \, dx}$$

$$= \sqrt{\frac{2}{\pi}} \left[(1-x^2) \left(\frac{\sin sx}{s}\right) - (-2x)\left(\frac{-\cos sx}{s^2}\right) + (-2)\left(\frac{-\sin sx}{s^3}\right) \right]_0^1$$

$$= \sqrt{\frac{2}{\pi}} \left[(1-x^2)\left(\frac{\sin sx}{s}\right) - 2x\left(\frac{\cos sx}{s^2}\right) + 2\frac{\sin sx}{s^3} \right]_0^1$$

$$= \sqrt{\frac{2}{\pi}} \left[0 - 2\frac{\cos s}{s^2} + \frac{2\sin s}{s^3} \right] - 0$$

$$= 2\sqrt{\frac{2}{\pi}} \left[\frac{-s\cos s + \sin s}{s^3} \right]$$

$$\therefore \quad F(s) = 2\sqrt{\frac{2}{\pi}} \left[\frac{\sin s - s\cos s}{s^3} \right]$$

⇒ Hence deduce that $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2} \, ds = \frac{3\pi}{16}$. Using parseval's identity show that $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) dx = \frac{\pi}{15}$

Solution:

Invers fourier transform is

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \, e^{-isx} \, dx$$

$$1 - x^2 = \frac{2}{\sqrt{2\pi}} \int_0^\infty 2\sqrt{\frac{2}{\pi}} \left(\frac{\sin s - s\cos s}{s^3}\right) e^{-isx} \, ds$$

$$= \frac{2}{\sqrt{2\pi}} \times 2\sqrt{\frac{2}{\pi}} \int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right) (\cos sx - i\sin sx) \, ds$$

$$= \frac{4}{\pi}\left[\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos sx\, ds - i\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\sin sx\, ds\right.$$

Equating Real part

$$1-x^2 = \frac{4}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos sx\, ds$$

put $x = 1/2$ $t = s$ $ds = dt$

$$1-(1/2)^2 = \frac{4}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos s/2\, ds$$

$$1-1/4 = \frac{4}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)\cos s/2\, ds$$

$$\frac{3\pi}{4}\times 4 = \int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right).\cos s/2\, ds$$

$$\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right).\cos s/2\, ds = \frac{3\pi}{16}$$ Hence proved.

→ parseral's Identity.

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$\int_{-1}^{1} |(1-x^2)|^2 dx = \int_{-\infty}^{\infty} \left|2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)\right|^2 ds$$

$$2\int_0^1 (1-2x^2+x^4)\,dx = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2$$

$$2\left[x - \frac{2x^3}{3} + \frac{x^5}{5}\right]_0^1 = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds$$

$$2\left[\frac{3-2}{3} + \frac{1}{5}\right] = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds$$

$$2\left[\frac{1}{3}+\frac{1}{5}\right] = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$2\left[\frac{5+3}{15}\right] = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{16}{15} = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\Rightarrow \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx = \frac{\pi}{15}$$

TRANSFORM AND BOUNDARY VALUE PROBLEMS

ASSIGNMENT

Name: Kajal kumari

Rg. No: U21EC339

Section: F section

TRANSFORMS AND BOUNDARY VALUE PROBLEMS U20MABT03

CIA - 2

KAJAL KUMARI U21EC339 ECE IInd year

Part - C

10 a) - - -

WKT, the heat equation is

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad — ①$$

when steady state condn exist, the heat flow equation becomes

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad — ②$$

Solving ②, we get $u(x) = c_1 x + c_2$ — ③

from the given problem we have the following boundary condn

i) $u(0) = 20$

ii) $u(l) = 80$

$l = 30\text{cm}$

Apply the (i) in ③, we get i.e $x = 0$

$u(0) = c_1(0) + c_2 = 20$

$\boxed{c_2 = 20}$ sub in eq ③

$u(x) = c_1 x + 20$ — ④

Applying the B.C (ii) in ④

(ie) $x = l$

$u(l) = c_1 l + 20 = 80$

$c_1 l = 80 - 20$

$\boxed{c_1 = \frac{60}{l}}$ sub in eq ④

$$\boxed{u(x) = \frac{60}{l}x + 20}$$

The temperature distribution reached at the steady state becomes "initial temperature distribution for the unsteady state

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad -(5)$$

The new boundary condn are

i) $u(0,t) = 0, \quad t > 0$

ii) $u(l,0) = 0, \quad t > 0$

iii) $u(x,0) = \frac{60x}{l} + 20; \quad 0 < x < l$

The general soln is

$$u(x,t) = (A\cos px + B\sin px)e^{-a^2p^2t} \quad -(1)$$

Applying the Boundary condn in (1) (i.e) $x=0$, we get

$$u(0,t) = (A)e^{-a^2p^2t} = 0$$

let $A = 0$ sub in eq (1)

$$u(x,t) = B\sin px\,(e^{-a^2p^2t}) \quad -(2)$$

Applying the B.C (ii) in (2) i.e $x = l$

$$u(l,t) = B\sin pl\,(e^{-a^2p^2t}) = 0$$

$$\sin pl = \sin n\pi$$

$$pl = n\pi$$

$$\boxed{p = \frac{n\pi}{l}} \text{ in eq (2)}$$

$$u(x,t) = B\sin\frac{n\pi x}{l}\left(e^{-\frac{a^2n^2\pi^2}{l^2}t}\right) - (3)$$

Applying the B.C (iii) in (3) i.e $t = 0$

$$u(x,0) = B\sin\frac{n\pi x}{l} = \frac{60x}{l} + 20$$

$$u(x,t) = B_n\sin\frac{n\pi x}{l}\cdot e^{-a^2p^2t}f(x) \rightarrow (4)$$

$$b_n = B_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l \left(\frac{60x}{l} + 20\right)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l^2}\int_0^l (60x+20l).\sin\frac{n\pi x}{l}\,dx$$

$$\int uv\,du = uv_1 + u'v_2 + u''v_3$$

$$= \frac{2}{l^2}\left[(60x+20l)\left(\frac{-\cos\frac{n\pi x}{l}}{\frac{n\pi}{l}}\right) - (60)\frac{-\sin\frac{n\pi x}{l}}{\left(\frac{n\pi}{l}\right)^2}\right]_0^l$$

$$= \frac{2}{l^2}\left[(-20l)\left(\frac{(-1)^n}{\frac{n\pi}{l}}+0\right) - (20l)\frac{1}{\left(\frac{n\pi}{l}\right)}\right]$$

$$= \frac{2}{l^2}\left[-80l-\frac{(-1)^n}{n\pi/l} + \frac{20l}{\left(\frac{n\pi}{l}\right)}\right]$$

$$= \frac{2.20l}{l^2\left(\frac{n\pi}{l}\right)}\left[-4(-1)^n+1\right]$$

$$= \frac{40l^2}{l^2 n\pi}\left[-1(4)(-1)^n+1\right]$$

$$b_n = \frac{40}{n\pi}\left[1+4(-1)^{n+1}\right]$$

Sub b_n in eq (4)

$$u(x,t) = \sum_{n=1}^{\infty}\frac{40}{n\pi}\left[1+4(-1)^{n+1}\right].\sin\frac{n\pi x}{l}.e^{-\alpha^2 p^2 t}$$

$$u(x,t) = \frac{40}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[1+4(-1)^{n+1}\right].\sin\frac{n\pi x}{l}.e^{-\alpha^2 p^2 t}$$

11.b —

The boundary Cond^n are

i) $y(0,t)=0, \quad t>0$

$y(0,t)=0, \quad t>0$

iii) $\frac{\partial y}{\partial t}(x,0)=0\ \forall x$

(iii) ~~$x(n) = \cos(n/4)$~~

~~$2\pi f = \frac{1}{4}$~~

~~$f = \frac{1}{2\pi} = k/N$ it is irrational.~~

~~$\therefore$ It is non-periodic signal.~~

iv) $y(x,0) = y_0 \sin^3 \frac{n\pi x}{l}$, $0<x<l$

The soln for y satisfying the boundary condn are

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{l} \cdot \cos \frac{n\pi at}{l} \quad -(1)$$

Applying (iv) in (1)

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{l} \cdot \cos 0$$

$$= \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{l} = y_0 \sin^3 \frac{\pi x}{l}$$

we know that

$$\sin^3 \frac{\pi x}{l} = +\frac{1}{4}\left(3\sin \frac{\pi x}{l} - \sin \frac{3\pi x}{l}\right) \quad -(2)$$

sub (2) in above equation

$$C_1 \sin \frac{\pi x}{l} + C_2 \sin \frac{2\pi x}{l} + C_3 \sin \frac{3\pi x}{l} = \frac{1}{4}\left(3\sin \frac{\pi x}{l} - \frac{\sin 3\pi x}{l}\right)$$

equating co-efficients on both sides

$$C_1 = \frac{3y_0}{4}; \quad C_3 = \frac{-y_0}{4}, \quad C_2 = C_4 = 0$$

sub these value in eq (1)

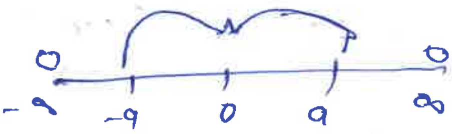
$$y(x,t) = \frac{3y_0}{4} \sin \frac{\pi x}{l} \cos \frac{\pi at}{l} - \frac{y_0}{4} \sin \frac{3\pi x}{l} \cos \frac{3\pi at}{l}.$$

11.a ...

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx.$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{-a} f(x) e^{isx} dx + \int_{-a}^{+a} f(x) e^{isx} dx + \int_{a}^{\infty} f(x)^{isx} dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + \int_{-a}^{a} (a^2 - x^2) e^{isx} dx \right]$$



$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a^2 - x^2) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a^2 - x^2) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a^2 - x^2)(\cos sx + i \sin sx \, dx)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} \left[(a^2 - x^2) \cos sx \, dx + (a^2 - x^2)(i \sin sx \, dx) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-a}^{a} (a^2 - x^2) \cos sx \, dx + 0 \right]$$

$[\because (a^2 - x^2) \sin x \rightarrow$ odd function $= 0$

$$= \frac{1}{\sqrt{2\pi}} 2 \int_{0}^{a} (a^2 - x^2) \cos sx \, dx$$

$((a^2 - x^2) \cos sx \rightarrow$ even function

$$\int_{-a}^{a} (a^2 - x^2) \cos sx \, dx = 2 \int_{0}^{a} (a^2 - x^2) \cos sx \, dx$$

$\therefore$ By Bernoulli's

$\int dv = \cos sx$

$u = a^2 - x^2$ $v = \frac{\sin sx}{s}$

$u' = -2x$ $v_1 = \frac{-\cos sx}{s^2}$

$u'' = -2 \qquad v_2 = -\frac{\sin sx}{s^3}$

$$= \sqrt{\frac{2}{\pi}}\left[(a^2-x^2)\left(\frac{\sin sx}{s}\right) - (-2x)\left(\frac{-\cos sx}{s^2}\right) - 2\left(\frac{-\sin sx}{s^3}\right)\right]_0^a$$

$$= \sqrt{\frac{2}{\pi}}\left[(a^2-x^2)\left(\frac{\sin sx}{s}\right) - 2x\left(\frac{\cos sx}{s^2}\right) + \frac{2\sin sx}{s^3}\right]$$

$$= 2\sqrt{\frac{2}{\pi}}\left[\frac{\sin as - as\cos as}{s^3}\right]$$

$$\therefore \quad F(s) = 2\sqrt{\frac{2}{\pi}}\left[\frac{\sin as - as\cos as}{s^3}\right]$$

Put $a=1$, $F(s) = 2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)$ — (A)

using inverse fourier tranform for (A) we get

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}\,dx$$

$$= \frac{2}{\sqrt{2\pi}}\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty}\frac{1}{s^3}\left[\sin s - s\cos sx\right]e^{-isx}\,dx$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)\cos sx\,ds - \frac{2i}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)\sin sx\,ds$$

$$f(x) = \frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\sin s - s\cos s}{s^3}\cos sx\,dx$$

$\therefore \frac{\sin s - s\cos s}{s^3}\cos sx$ - even

$$= 2\left[x + \frac{x^5}{5} - 2\cdot\frac{x^3}{3}\right]_0^1$$

$$= 2\left[\left(1+\frac{1}{5}-\frac{2}{3}\right) - (0+0-0)\right]$$

$$= 2\left(\frac{15+3-10}{15}\right)$$

$$= 2\left(\frac{8}{15}\right)$$

$$= \frac{16}{15}\ \frac{\pi}{\pi}\int_0^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds = \frac{\pi}{15}$$

$$= \int_0^{\infty}\left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$$

$$\therefore \frac{4}{\pi}\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos sx\, ds$$

$$\therefore \int_{-\infty}^{\infty} \frac{\sin s - s\cos s}{s^3} \cos sx\, ds = 2\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos sx\, ds$$

$$\frac{\sin s - s\cos s}{s^3} \int_{-\infty}^{\infty} \sin sx\, ds \rightarrow \text{odd} = 0$$

$$f(x) = \frac{4}{\pi}\int_0^{\infty} \frac{\sin t - t\cos t}{t^3} \cos tx\, dt \quad \rightarrow (B)$$

Putting $x = 0$ in (2)

$$= \frac{4}{\pi}\int_0^{\infty} \frac{\sin t - t\cos t}{t^3}\, dt = f(0) = 1$$

$$\left[\therefore f(x) = 1 - x^2 \right.$$
$$\left. f(0) = 1 - 0 = 1 \right.$$

$$= \int_0^{\infty} \frac{\sin t - t\cos t}{t^3}\, dx = \frac{\pi}{4}$$

11.b Using parsevals identity

$$\int_{-\infty}^{\infty} F(s)^2\, dx = \int_{-a}^{a} |f(x)|^2\, dx$$

$$\frac{8}{\pi}\int_{-\infty}^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right) ds = 2\int_0^1 (1-x)^2\, dx$$

$$= \frac{16}{\pi}\int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right)^2 dx = 2\int_0^1 (1 + x^4 - 2x^2)\, dx$$

$$= 2\left[x + \frac{x^5}{5} - \frac{2 \cdot x^3}{3}\right]_0^1$$

CIVIL

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Electronics and Communication Engineering

CONTINUOUS LEARNING ASSESSMENT - I

U20MABT03– TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 19.10.2022

Academic Year / Semester : 2022-2023 / ODD

Duration : 1.5 Hours (90 minutes)

Instructions : Descriptive Type Questions

Q. No	Questions	Weightage	CO	Bloom's Level
	PART A (6x2=12 Marks)			
1	Define Fourier Series.	2	CO1	U
2	Write down the Bernoulli's formula	2	CO1	U
3	Find a_0 for the function $f(x) = x^2$ in $(0,2l)$.	2	CO1	
4	Write down the formula for the Fourier series in $(0,2l)$.	2	CO1	U
5	Find b_n , if $f(x) = x^2$ in $(-l < x < l)$.	2	CO1	U
6	Find the Root Mean Square value of $f(x) = x$ in $(0,2\pi)$.	2	CO1	U
	PART B (3x6=18 Marks)			
7	(a) Find the Fourier series for the function $f(x) = x^2 + x$ in $(0,2l)$. **(OR)** (b) Find the Fourier series for $f(x) = x(2\pi - x)$ in $(0,2\pi)$.	6	CO1	U
8	(a) Obtain half range cosine series for $f(x) = x(l - x)$ in $(0, l)$. **(OR)** (b) Find the half-range cosine and half- range sine series for the function $f(x) = x$ in $(0, l)$.	6	CO1	U
9	(a) Find the Fourier series for the function $f(x) = \begin{cases} k, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ Hence find sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. **(OR)** (b) Find the Fourier series for $f(x) = \lvert x \rvert$ in $(-\pi, \pi)$	6	CO1	U
	PART C (2x10=20 Marks)			
10	(a) Obtain the Fourier series for the function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$ **(OR)** (b) Find the first two harmonic of the Fourier series of $f(x)$ given by the following table x: 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$, 2π $f(x)$: 1.0, 1.4, 1.9, 1.7, 1.5, 1.2, 1	10	CO1	U
11	(a) Find the Fourier series for $f(x) = (\pi - x)^2$ in $(0,2\pi)$. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}$ **(OR)** (b) Find the half range cosine series for $f(x) = x$ in$(0, \pi)$. Deduce that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{96}$	10	CO1	U

U20MABT03 – TBVP

Internal Assessment – III

1) $py - qx = 0$

2) $z = \int p\,dx + \int q\,dy$

$z = k\,[e^{x} + e^{y}] + c$

3) $Z(n) = \frac{z}{(z-1)^2}$

4) $Z(1/n) = \log\left(\frac{z}{z-1}\right)$

5) a) $z = \frac{k}{1-k}\left(\frac{x^2}{2}\right) - k\left(\frac{y^2}{2}\right) + a$

b) $C.F = f_1(y+x) + x f_2(y+x)$

$P.I = -\frac{1}{16}\cos(x-3y)$

$z = f_1(y+x) + x f_2(y+x) - \frac{1}{16}\cos(x-3y)$

6) a) $Z(1) = \frac{z}{z-1}$, $Z(a^n) = \frac{z}{z-a}$

b) $\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = C_1$, $lx + my + nz = C_2$

7) a) $C.F = f_1(y+2x) + f_2(y-x) + \frac{5x^3}{6} + \frac{3x^2y}{2} - \frac{1}{35}e^{3x+4y}$

b) $A = -1$, $B = 1$, $x(n) = (-2)^n - (-3)^n$.

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Civil Engineering

INTERNAL ASSESSMENT -1
U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 12.10.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 60 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8)Answer all questions			
1	Write down the Fourier series formula.	2	CO1	2
2	Find a_0 and a_n, if $f(x) = x, in -\pi < x < \pi$.	2	CO1	2
3	State Dirichlet conditions.	2	CO1	2
4	Write down the Parseval's identity formula.	2	CO1	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) Find a_0 and a_n the fourier series for the function $f(x) = x^2, in\ (0, 2l)$.	6	CO1	2
	(Or)			
	(b) Find a_0 and a_n the fourier series for the function $f(x) = x(2\pi - x), in\ (0, 2\pi)$.	6	CO1	2
6	(a) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0, l)$.	6	CO1	2
	(Or)			
	(b) Find the Half range cosine series and Half range sine series for the function $f(x) = x, in\ (0, \pi)$.	6	CO1	2
	PART – C (1X10 =10) Answer either or question			
7	(a) Find the Fourier series for the function in f(x) = x+x2 in (π,-π) and deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.	10	CO1	2
	(Or)	10	CO1	2
	(b)Find the fourier series expansion of period 2π for the function y= [see table below] f(x) which is defined in (0,2π) by means of the table of the values given below. Find the series up to to third harmonic.	10	CO1	2

X	0	π/3	2π/3	π	4π/3	5π/3	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

CO	Weightage
CO1	30
CO2	

U20MABT03 - Transform & Boundary Value Problems

Internal Assessment - I

A)

1) $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx)$

2). $a_0 = 0$, $a_n = 0$.

3). f(x) is periodic, continuous,

4) $\frac{1}{2\pi} \int_0^{2\pi} [f(x)]^2 dx = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$

B) 5) a) $a_0 = \frac{8l^2}{3}$, $a_n = \frac{-4}{n^2\pi^2}$.

b) $a_0 = \frac{4l^2}{3}$, $a_n = -\frac{4l^2}{n^2\pi^2}$.

6) a) $a_0 = l$, $a_n = \frac{2l}{n^2\pi^2} [(-1)^n - 1]$

b) $a_0 = \pi$, $a_n = \frac{2}{\pi n^2} [(-1)^n - 1]$

c) 7) a) $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{n^2}(-1)^{n+1}$, $b_n = \frac{2}{n}(-1)^{n+1}$

b) $f(x) = 1.45 + (-0.36 \cos x + 0.173 \sin x) + (-0.1 \cos 2x - 0.057 \sin 2x) + (0.033 \cos 3x$

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Civil Engineering

INTERNAL ASSESSMENT -I1
U20MABT03 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 21.11.2022
Academic Year / Semester : 2022-2023/ODD
Duration : 90 min

Q.No	Question	Weightage	CO	Bloom's Level
	PART – A (4X2 = 8) Answer all questions			
1	Classify the differential equation $$3\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+6\frac{\partial^2 u}{\partial y^2}-2\frac{\partial u}{\partial y}-u=0.$$	2	CO3	2
2	State the Fourier integral theorem.	2	CO4	2
3	If $F\{f(x)\}=F(s)$, then $F\{f(x)\cos ax\}=$.............	2	CO4	2
4	What are the various solutions of one dimensional wave equation?	2	CO3	2
	PART – B (2X6 = 12) Answer either-or question			
5	(a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $k(lx-x)$, then show that $y(x,t)=\frac{8kl^3}{a\pi^4}\sum_{n=1,3,5}^{\infty}\frac{1}{4}\sin\frac{n\pi x}{l}\sin\frac{n\pi at}{l}$. (OR)	6	CO3	2
	(b) Find the Fourier transform of $f(x)$ if $$f(x)=\begin{cases}1, & \lvert x\rvert<1\\ 0, & \lvert x\rvert>1.\end{cases}$$		CO4	2
6	(a) State and prove shifting theorem. (OR)	6	CO4	2
	(b) State and prove Modulation theorem.		CO4	2
	PART – C (1X110 =10) Answer either or question			
7	(a) A metal bar 30cm has its end A and B 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature distribution function u(x,t) taking x=0 at A .		CO3	2
	(b) Show that the Fourier transform of $$f(x)=\begin{cases}1-x^2 & \lvert x\rvert<1\\ 0 & \lvert x\rvert>1>0\end{cases}$$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin s-s\cos s}{s^3}\right)$. Hence deduce that $\int_0^{\infty}\frac{\sin s-s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$.	10	CO4	2

TBVP_ U20MABT03.

CLA - II.

1) $B^2 - 4AC = -56$, Ellipse.

2) $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)\, e^{i(x-t)s}\, dt\, ds.$

3) $F\{f(x)\cos ax\} = \frac{1}{2}\left[f(s+a) + f(s-a)\right]$

4) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c e^{-pat})$

$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$

$y(x,t) = (c_1 x + c_2)(c_3 t + c_4).$

5) b) $f(x) = \sqrt{\frac{2}{\pi}} \cdot \left(\frac{\sin s}{s}\right).$

6) a) Shifting thm

$F[f(x)] = F(s)$ then

$F[f(x-a)] = e^{isx} F(s)$

b) Modulation thm

$F[f(x)] = F(s)$ then

$F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)].$

7) a) $c_2 = 20$, $c_1 = \frac{60}{l}$ $\quad p = \frac{n\pi}{r}$, $b_n = \frac{40}{n\pi}[1 + 4(-1)^{n+1}]$

b) $F(s) = 2\sqrt{\frac{2}{\pi}}\left[\frac{\sin s - s\cos s}{}\right]$

DEPARTMENT OF CIVIL ENGINEERING

CONTINUOUS LEARNING ASSESSMENT – III

U20MABT03 – TRANSFORM AND BOUNDARY VALUE PROBLEMS

Date	: 26.12.2022
Academic Year / Semester	:2022-2023/ODD
Duration	:1 hour 15 mins
Instructions	: Part A- Answer all questions
	Part B - Answer either A or B for the questions 5and 6
	Part C- Answer either A or B for the question 7

Q.No	Questions	Weightage	CO	Bloom's Level
	PART A (5X2=10)			
1	Form the partial differential equation by eliminating the arbitrary function in Z = f($x^2 + y^2$).	2	CO2	***R***
2	Solve pe^y=qe^x.	2	CO 2	***R***
3	Prove that Z[n] = $\frac{z}{(z-1)^2}$.	2	CO 5	***U***
4	Find Z[$\frac{1}{n}$].	2	CO 5	***U***
	PART B (2x6=12)			
5	(a) Solve Z = px + qy + $p^2 - q^2$ **(OR)** (b) Solve the equation $(D^2 - 2DD' + D'^2)z = \cos(x - 3y)$..	6	CO2	***U***
6	(a)Find the Z – transform of the following, i) Z [1] ii) Z [a^n] **(OR)** (b) Solve $(mz - ny)p + (nx - lz)q = ly - mx$	6	CO 5	***U***
	PART C (1x10=10)			
7	a)Solve $(D^2 - DD' + 2D'^2)z = 2x + 3y + e^{3x+4y}$	10	CO 2	***U***
	b)Find $Z^{-1}[\frac{Z}{Z^2+5Z+6}]$			

ASSIGNMENT

UNIT-III : Applications of Partial differential equation.

1. String is related to one-dimensional wave equation
2. Rod is related to one-dimensional heat equation.
3. Plate is related to two-dimensional heat.

1) One dimensional wave equations. Definition?

$$\frac{\delta^2 y}{\delta t^2} = a^2 \frac{\delta^2 y}{\delta x^2}$$

$$a^2 = T/m = \frac{\text{Tension}}{\text{mass per unit of the string}}$$

2) Write the boundary and initial conditions for one dimensional wave equation?

i) $y(0,t) = 0$

ii) $y(l,t) = 0$

iii) $\left(\frac{\delta y}{\delta t}\right)(x,0) = 0$

iv) $y(x,0) = f(x)$

General solution is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$$

3) A string is stretched and fastened to two points $x=0$ & $x=l$ a post. the motion is started by displaced the string into the form $y = k(ln - x^2)$ from which it is released. A time $t=0$. Find the displacement of any point on the string at a distance of 'x' from one end at time 't'

A) One dimensional wave equation.

$$\frac{\delta^2 y}{\delta t^2} = a^2 \frac{\delta^2 y}{\delta x^2}$$

The boundary and initial conditions

i) $y(0,t) = 0$

ii) $y(l,t) = 0$

iii) $\frac{\delta y}{\delta t}(x,0) = 0$

iv) $y(x,0) = f(x)$

General solution is

$$y(x,y) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat) \longrightarrow (1)$$

- Apply 1st condition in eq (1)

Put $x = 0$ in eq (1)

$$y(0,t) = 0$$

$$y(0,t) = (c_1 \cos p(0) + c_2 \sin p(0))(c_3 \cos pat + c_4 \sin pat).$$

$$0 = c_1(c_3 \cos pat + c_4 \sin pat)$$

$\boxed{c_1 = 0}$ or $c_3 \cos pat + c_4 \sin pat \neq 0$.

$\boxed{c_1 = 0}$ substitute in eq (1)

$$y(x,t) = c_2 \sin px \,(c_3 \cos pat + c_4 \sin pat) \longrightarrow (2)$$

- Apply (ii) condition in eq (2)

put $x = l$, in eq (2).

$$y(l,t) = c_2 \sin pl \,(c_3 \cos pat + c_4 \sin pat)$$

$$0 = c_2 \sin pl \,(c_3 \cos pat + c_4 \sin pat)$$

$c_2 \neq 0$, (or) $\sin pl = 0$ or $c_3 \cos pat + c_4 \sin pat \neq 0$

$$\sin pl = \sin n\pi$$

$$pl = n\pi$$

$$\boxed{p = n\pi/l}$$

$p = \frac{n\pi}{l}$ in substitute in eq ②.

$$y(x,t) = c_2 \sin\frac{n\pi x}{l}\left(c_3 \cos\frac{n\pi at}{l} + c_4 \sin\frac{n\pi at}{l}\right) \longrightarrow ③$$

Apply (iii) condition in eq ③.

$$\frac{\delta y}{\delta t}(x,0) = 0$$

$$\frac{\delta y}{\delta t} = c_2 \sin\frac{n\pi x}{l}\left[-c_3 \sin\frac{n\pi at}{l} \times \frac{n\pi a}{l} + c_4 \cos\frac{n\pi at}{l} \times \frac{n\pi a}{l}\right]$$

$$\left[\frac{\delta y}{\delta t}\right]_{t=0} = c_2 \sin\frac{n\pi x}{l}\left[-c_3 \sin\frac{n\pi a(0)}{l} \times \frac{n\pi a}{l} + c_4 \cos\frac{n\pi a(0)}{l} \times \frac{n\pi}{l}\right]$$

(the term $\sin\frac{n\pi a(0)}{l}$ is struck out, marked $\rightarrow (0)$)

$$= c_2 \sin\frac{n\pi x}{l}\, c_4 \frac{n\pi a}{l}$$

$c_2 \neq 0$, $\sin\frac{n\pi x}{l} \neq 0$, $c_4 = 0$, $\frac{n\pi a}{l} \neq 0$

$c_4 = 0$, Apply for eq ③

$$y(x,t) = c_2 \sin\frac{n\pi x}{l}\, c_3 \cos\frac{n\pi at}{l} \longrightarrow ④$$

Apply (iv) Boundary condition $y(x,0) = f(x)$

From (iv) we get $t=0$ and substitute in eq ④.

$$y(x,0) = c_2 \sin\frac{n\pi x}{l}\, c_3 \cos\frac{n\pi a(0)}{l}$$

$$y(x,0) = c_2 c_3 \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l}$$

$$y(x,0) = c_2 c_3 \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l} \qquad \therefore \text{ Where } c_n = c_2 c_3$$

$$K(lx - x^2) = c_n \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l} \longrightarrow ⑤$$

$c_n = b_n$.

We have to use fourier series $(0, l)$.

$$b_n = \frac{2}{l}\int_0^l f(x)\,\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l k(lx - x^2)\,\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2k}{l}\int_0^l (lx - x^2)\,\sin\frac{n\pi x}{l}\,dx.$$

$u = lx - x^2$ $\qquad\qquad$ $\int dv = \sin\frac{n\pi x}{l}$

$u' = l - 2x$ $\qquad\qquad$ $v = \frac{-l}{n\pi}\cos\frac{n\pi x}{l}$

$u'' = -2$ $\qquad\qquad$ $v_1 = \frac{-l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}$

$u''' = 0$ $\qquad\qquad$ $v_2 = \frac{l^3}{n^3\pi^3}\cos\frac{n\pi x}{l}.$

$$\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \cdots\cdots$$

$$= \frac{2k}{l}\left[-(lx - x^2)\left[\frac{l}{n\pi}\cos\frac{n\pi x}{l}\right] + [l - 2x]\left[\frac{l^2}{n^2\pi^2}\sin\frac{n\pi x}{l^2}\right]\right.$$ (first term $\to (0)$)

$$\left. - 2\left[\frac{l^3}{n^3\pi^3}\cos\frac{n\pi x}{l}\right]\right]_0^l$$

$$= \frac{2k}{l}\left[\frac{-l(x) + l^2(l)}{n\pi} - \frac{2l^3}{n^3\pi^3}\right] + 2\left[\frac{l^3}{n^3\pi^3}\right]$$

$$= \frac{2k}{l}\left[\frac{2(-1)^n}{n^3\pi^3} + \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{-4k}{l}\left[\frac{(-1)^n l^3}{n^3\pi^3} - \frac{l^3}{n^3\pi^3}\right]$$

$$= \frac{-4k}{l}\,\frac{l^3}{n^3\pi^3}\left[(-1)^n - 1\right]$$

$$= \frac{-4kl^2}{n^3\pi^3}\left[(-1)^n - 1\right]$$

$$= \frac{4\cdot kl^2}{n^3\pi^3}\left[1 - (-1)^n\right]$$

* if n = odd numbers

$$b_n = \frac{8kl^2}{n^3\pi^3}$$

$$b_n = c_n = \begin{cases} \dfrac{8kl^2}{n^3\pi^3} & \text{if } n = \text{odd} \\ 0 & \text{if } n = \text{even.} \end{cases}$$

$$y(x,t) = \sum_{n=\text{odd}}^{\infty} \frac{8kl^2}{n^3\pi^3} \sin\frac{n\pi x}{l} \cos\frac{n\pi a t}{l}$$

Problems on vibrating string with non-zero initial velocity.

1) write the Boundary and initial conditions ?

i) $y(0,t) = 0$

ii) $y(l,t) = 0$

iii) $y(x,0) = 0$

iv) $\frac{\delta y}{\delta t}(x,0) = f(x)$

General solution is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat).$$

2) A tightly stretched string fixed points $x=0$ & $x=l$, is initially in a position given by $y(x,0) = y_0 \sin^3\left[\frac{\pi x}{l}\right]$, if it is released from rest. this position, find the displacement y at any distance x from one end at any time t.

A) One dimensional wave equation

$$\frac{\delta^2 y}{\delta t^2} = a^2 \frac{\delta^2 y}{\delta x^2}$$

From the given problem, we get the following boundary and initial conditions.

i) $y(0,t) = 0$

ii) $y(l,t) = 0$

iii) $y(x,0) = 0$

iv) $\frac{\delta y}{\delta t}(x,0) = f(x)$

The General solution is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat) \longrightarrow (1)$$

- Apply (i) boundary condition, we get $y(0,t)=0$.

put $x=0$, in eq (1)

$$y(0,t) = (c_1 \cos p(0) + c_2 \sin p(0))(c_3 \cos pat + c_4 \sin pat).$$

$$0 = c_1 (c_3 \cos pat + c_4 \sin pat).$$

$\boxed{c_1 = 0}$ or $(c_3 \cos pat + c_4 \sin pat) \neq 0 \quad (\forall t)$

$c_1 = 0$, substitute the eq (1)

$$y(x,t) = c_2 \sin px (c_3 \cos pat + c_4 \sin pat) \longrightarrow (2)$$

- Apply (ii) condition in eq (2), we get $y(l,t)=0$

put $x=l$ in eq (2)

$$y(l,t) = c_2 \sin pl (c_3 \cos pat + c_4 \sin pat).$$

$$0 = c_2 \sin pl$$

$c_2 \neq 0$ or $\sin pl = 0$ $\quad c_3 \cos pat \; c_4 \cos pat \neq 0.$

$\sin pl = \sin n\pi$

$$\boxed{p = \frac{n\pi}{l}}$$

$$y(x,t) = c_2 \sin \frac{n\pi x}{l} \left[c_3 \cos \frac{n\pi at}{l} + c_4 \sin \frac{n\pi at}{l} \right] \longrightarrow (3).$$

- Apply (iii) Boundary condition in eq (3), we get.

$y(x,0) = 0.$

Put $t=0$, substitute in eq ③.

$$y(x,0) = c_2 \sin\frac{n\pi x}{l}\left[c_3 \cos\frac{n\pi a(0)}{l} + c_4 \sin\frac{n\pi a(0)}{l}\right]$$

$$0 = c_2 \sin\frac{n\pi x}{l}[c_3 + 0]$$

$$0 = c_2 \sin\frac{n\pi x}{l}[c_3 + 0]$$

$$0 = c_2 \sin\frac{n\pi x}{l} c_3$$

$c_2 \neq 0$, (or) $\sin\frac{n\pi x}{l} \neq 0$ $c_3 = 0$

$\boxed{c_3 = 0}$ substitute in eq ③.

$$y(x,t) = c_2 \sin\frac{n\pi x}{l} \; c_4 \sin\frac{n\pi at}{l}$$

$$= c_2 c_4 \sin\frac{n\pi x}{l} \sin\frac{n\pi at}{l}$$

$$y(x,t) = c_n \sin\frac{n\pi x}{l} \sin\frac{n\pi at}{l} \longrightarrow ④$$

Apply (iv) condition $\left(\frac{\delta y}{\delta t}\right)(x,0) = f(x)$ in eq ④

In eq ④ w.r.t to 't'.

$$\left(\frac{\delta y}{\delta t}\right) = c_n \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l} \times \frac{n\pi a}{l}$$

$$\frac{\delta y}{\delta t} = c_n \sin\frac{n\pi x}{l} \cos\frac{n\pi a(0)}{l} \times \frac{n\pi a}{l}$$

$$\frac{\delta y}{\delta t}(x,0) = c_n \sin\frac{n\pi x}{l} \times \frac{n\pi a}{l}$$

$$y_0 \sin^3\frac{\pi x}{l} = c_n \frac{n\pi a}{l} \sin\frac{n\pi x}{l} \longrightarrow ⑤$$

$$y_0 \sin^3\frac{\pi x}{l} = b_n \sin\frac{n\pi x}{l} \longrightarrow ⑥$$

where, $b_n = c_n \frac{n\pi a}{l}$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$4\sin^3\theta = 3\sin\theta - \sin 3\theta$$

$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$\sin^3\frac{\pi x}{l} = \frac{3\sin\frac{\pi x}{l} - \sin 3\frac{\pi x}{l}}{4}$$

$$\boxed{\sin^3\frac{\pi x}{l} = \frac{3}{4}\sin\frac{\pi x}{l} - \frac{1}{4}\sin 3\frac{\pi x}{l}}$$

Substitute this in eq (6)

$$\frac{3y_0}{4}\sin\frac{n\pi x}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l} = b_1\sin\frac{\pi x}{l} + b_2\sin\frac{2\pi x}{l} + b_3\sin\frac{3\pi x}{l} + b_4\sin 4\frac{\pi x}{l} + \cdots$$

Comparing the co-efficients

$$b_1 = \frac{3y_0}{4}, \quad b_2 = 0, \quad b_3 = \frac{-y_0}{4}, \quad b_4 = 0$$

$$c_1\frac{\pi a}{l} = \frac{3y_0}{4}, \quad c_2\frac{2\pi a}{l} = 0, \quad c_3\frac{3\pi a}{l} = \frac{-y_0}{4}, \cdots$$

Substitute it in (4)

$$y(x,t) = c_n\sin\frac{n\pi x}{l}\sin\frac{n\pi a}{l}$$

$$= c_1\sin\frac{\pi x}{l}\sin\frac{\pi a}{l} + c_2\sin\frac{2\pi x}{l}\sin\frac{2\pi a}{l} + c_3\sin\frac{3\pi x}{l}\sin\frac{3\pi a}{l} + \cdots$$

$$= \frac{3y_0}{4}\sin\frac{n\pi l}{l}\sin\frac{3\pi a}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l} + \sin\frac{3\pi a}{l} + \cdots$$

$$y(x,t) = \frac{3y_0}{4}\sin\frac{\pi x}{l}\sin\frac{\pi a}{l} - \frac{y_0}{4}\sin\frac{3\pi x}{l}\sin\frac{3\pi a}{l} + \cdots$$

One dimensional heat equation

1) Define one dimensional heat equation

$$\frac{\delta u}{\delta t} = \alpha^2 \frac{\delta^2 y^2}{\delta x^2}$$

$$\alpha^2 = \frac{k}{\rho c}$$

ρ = density.

k = thermal conductivity.

c = specific heat capacity.

2) State fourier law of heat condition.

The rate at which heat follow across a area 'A' at a distance 'x' from one end of a bar is given by

$$Q = -kA\left[\frac{\delta y}{\delta x}\right]x$$

$\left[\frac{\delta y}{\delta x}\right]a$, the temperature gradient at x, and

A = distance.

3) Write down the various possible solution of one dimensional heat equation.

1) $u(x,t) = [Ae^{px} + Be^{px}]\, e^{\alpha^2 . p t^2}$

2) $u(x,t) = [A\cos px + B\sin px]\, e^{-\alpha^2 . p^2 t^2}$

3) $u(x,t) = Ax + B$.

4) In steady state condition divide drive the solution of one dimensional heat follow equation.

$$\frac{\delta u}{\delta t} = \alpha^2 \frac{\delta^2 u}{\delta x^2}.$$

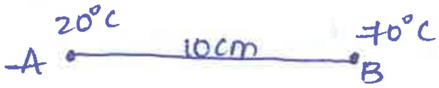
5) The partially differentiable equation of Insteady state condition, the temperature 'u' depends on the 'x' and not the time 't'.

Hence $\frac{\delta u}{dt} = 0$

$$\frac{\alpha^2 \delta^2 u}{\delta x^2} = 0.$$

Integrating with respective 'x' for twice we get general solution is $u = ax + b$, where a, b are orbitary.

6) The end A and B of a rod of length 10cm have their temperature kept at 20°c and 70°c find the steady state temperature distribution on the rod.



$$u(x) = \left[\frac{b-a}{l}\right] x + a$$

$$= \left[\frac{70-20}{10}\right] x + 20$$

$$u(x) = 5x + 20$$

Procedure of one dimensional heat equation

i) $\frac{\delta u}{\delta t} = \alpha^2 \frac{\delta^2 u}{\delta x^2}$

ii) Steady state

$$u(x) = \frac{[b-a]}{l} x + a$$

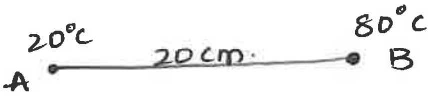
iii) Boundary and initial condition.

$$u(0,t) = 0$$

$$u(l, t) = 0$$

$$u(x, 0) = f(x)$$

7) A rod of length 20cm has its ends A and B kept at temperature 20°c and 80°c respectively until steady state conditions to prevail. If the temp. at each end of rod is then suddenly reduced at 0°c and kept so find the temp. distribution $u(x,t)$ taking $x=0$.



i) One dimensional heat equation

$$\frac{\delta u}{\delta t} = \alpha^2 \frac{\delta^2 u}{\delta x^2} \longrightarrow (A)$$

When the steady state condition.

$$\frac{\delta u}{\delta t} = 0 \quad \text{Hence,}$$

$$\alpha^2 \frac{\delta^2 u}{\delta x^2} = 0.$$

ii) Steady state temp. distribution on the heat

$$u(x) = \left[\frac{b-a}{l}\right] x + a$$

$$= \left[\frac{80-20}{20}\right] x + 20$$

$$= \frac{60}{20} x + 20$$

$$u(x) = 3x + 20 \longrightarrow (B)$$

iii) Boundary and initial conditions

i) $u(0,t) = 0$

ii) $u(l,t) = 0$

iii) $u(x,0) = f(x) = 3x+20$.

Suitable solution.

$$u(x,t) = [A\cos px + B\sin px]\, e^{-\alpha^2 p^2 t} \longrightarrow (1)$$

- Apply (i) boundary condition $u(0,t)=0$

put $x=0$, in eq (1)

$$u(0,t) = [A\cos p(0) + B\sin p(0)]e^{-\alpha^2p^2t}$$

(with $B\sin p(0)$ struck through and marked $\to 0$)

$$0 = Ae^{-\alpha^2p^2t}$$

$A=0$ ∵ $e^{-\alpha^2p^2t} \neq 0$

Substitute $A=0$ in eq (1)

$$u(x,t) = B\sin px\, e^{-\alpha^2p^2t} \longrightarrow (2)$$

- Apply (ii) Boundary condition $u(20,t)=0$

put $x=20$, substitute in eq (2)

$$u(20,t) = B\sin 20p\, e^{-\alpha^2p^2t}$$

$$0 = B\sin 20p\, e^{-\alpha^2p^2t}$$

$B\neq 0$, $\sin 20p = 0$, $e^{-\alpha^2p^2t} \neq 0$

$$\sin 20p = \sin n\pi$$

$$20p = n\pi$$

$$\boxed{p = \frac{n\pi}{20}}$$

Substitute $p=\frac{n\pi}{20}$ in equation (2)

$$u(x,t) = B\sin px\, e^{-\alpha^2p^2t}$$

$$u(x,t) = B\sin\frac{n\pi x}{20}\, e^{-\frac{\alpha^2n^2\pi^2t}{400}}$$

The most general solution

$$u(x,t) = \sum_{n=0}^{\infty} B_n \sin\frac{n\pi x}{20}\, e^{-\frac{\alpha^2n^2\pi^2t}{400}} \longrightarrow (3)$$

- Apply (iii) boundary condition, $u(x,0) = 3x+20$

put $t=0$, substitute in eq (3)

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{20}$$

$$3x+20 = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{20}$$

to find B_n expand to $3x+20$ in a half range sine series in the interval $[0,20]$

$$B_n = \frac{2}{l}\int_0^{20} f(x)\sin nx \, dx$$

$$B_n = \frac{2}{20}\int_0^{20} (3x+20)\sin\frac{n\pi x}{20}\, dx. \qquad \therefore B_n = b_n$$

$$b_n = \frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}\, dx$$

$$b_n = \frac{2}{20}\int_0^{l} 3x+20 \sin\frac{n\pi x}{20}\, dx.$$

$u = 3x+20$ $\qquad \int dv = \sin\frac{n\pi x}{20}$

$u' = 3$ $\qquad v = -\frac{20}{n\pi}\cos\frac{n\pi x}{20}$

$u'' = 0$ $\qquad v_1 = \frac{-400}{n^2\pi^2}\sin\frac{n\pi x}{20}.$

$$= \frac{2}{20}\left[-(3x+20)\left(\frac{20}{n\pi}\left(\cos\frac{n\pi x}{20}\right)\right) + 3\left(\frac{400}{n^2\pi^2}\sin\frac{n\pi x}{20}\right)^{\to(0)}\right]_0^{20}$$

$$= \frac{1}{10}\left[-80\left[\frac{20}{n\pi}(-1)^n\right] - \frac{400}{n\pi}\right]$$

$$= \frac{1}{10}\left(\frac{-1600(-1)^n}{n\pi} + \frac{400}{n\pi}\right)$$

$$= \frac{400}{n\pi\times 10}\left(-4(-1)^n+1\right)$$

$$b_n = \frac{40}{n\pi}\left[1-4(-1)^n\right]$$

$b_n = B_n = \frac{40}{n\pi}\left[1-4(-1)^n\right]$ substitute in eq (3)

$$u(x,t) = \sum_{n=0}^{\infty} \frac{40}{n\pi}\left[1-4(-1)^n\right]\sin\frac{n\pi x}{20}\, e^{-\frac{\alpha^2 n^2\pi^2 t}{400}}.$$

Two dimension heat equation

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta u^2} = 0$$

1) write all three possible solutions for two dimensional heat equations

A) Laplace equation in two dimensional.

$$\nabla^2 u = 0$$

i) $u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$

ii) $u(x,y) = (c_5 \cos px + c_6 \sin px)(c_7 e^{py} + c_8 e^{-py})$

iii) $u(x,y) = (c_9 x + c_{10})(c_{11} y + c_{12})$

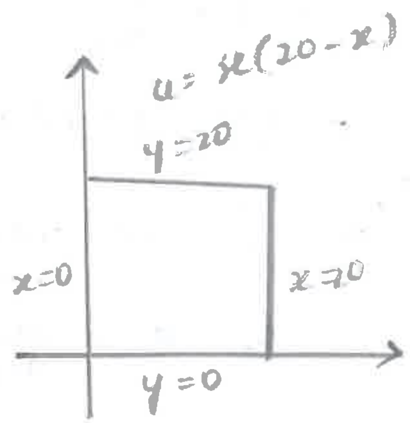
2) A square plate is bounded by the lines $x=0$, $y=0$, $x=20$, $y=20$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,20) = x(20-x)$ $0 < x < 20$, while other two edges are kept at $0°c$. Find the steady temp. distribution in the plate.

A. Let us take the sides of the plates be

$$l = 20$$

let $l(x,y)$ be the temperature at any point of (x,y) then $u(x,y)$ satisfies the Laplace's equation.

$$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = 0$$



Boundary Conditions.

i) $u(0,y) = 0$

ii) $u(20,y) = 0$

iii) $u(x,0) = 0$

iv) $u(x,20) = f(x) = x(20-x)$.

The suitable solution is

$$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py}) \longrightarrow ①$$

Apply (i) Boundary conditions $u(0,y)=0$

Put $x=0$, substitute eq ①

$$u(0,y) = [A\cos p(0) + B\sin p(0)][Ce^{py} + De^{-py}]$$

$$= A[Ce^{py} + De^{-py}]$$

Here $Ce^{py} + De^{-py} \neq 0$ (its defined $\forall y$)

$\therefore A=0$

Substitute $A=0$ in ①, we get

$$u(x,y) = B\sin px\,(Ce^{py} + De^{-py}) \longrightarrow ②$$

Apply (ii) Boundary conditions (2), we get

$$u(20,y) = B\sin 20p\,(Ce^{py} + De^{-py}) = 0$$

Here, $(Ce^{py} + De^{-py}) \neq 0$, $B \neq 0$

$\therefore \sin 20p = 0$

$\sin 20p = \sin n\pi$

$$\boxed{p = \frac{n\pi}{20}}$$

Substitute this eq ②, we get

$$u(x,y) = B\sin\frac{n\pi x}{20}\left[Ce^{\frac{n\pi}{20}y} + De^{-\frac{n\pi}{20}y}\right] \longrightarrow ③$$

Apply (iii) Boundary conditions in eq ③

$$u(x,0) = B\sin\frac{n\pi x}{20}[C+D] = 0$$

Here $\sin\frac{n\pi x}{20} \neq 0$ $[\forall x]$

$B \neq 0$ $\therefore C + D = 0$

$D = -C$

Substitute $D=-C$ in eq ③, we get:

$$u(x,y) = B\sin\frac{n\pi x}{20}\left[Ce^{\frac{n\pi y}{20}} - Ce^{\frac{-n\pi y}{20}}\right]$$

$$= BC\sin\frac{n\pi x}{20}\left[e^{\frac{n\pi y}{20}} - e^{\frac{-n\pi y}{20}}\right]$$

$$= 2BC\sin\frac{n\pi x}{20}\sinh\frac{n\pi y}{20}$$

The most general solution is.

$$u(x,y) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l}\sinh\frac{n\pi y}{l} \longrightarrow (4)$$

Now, we apply condition (iv) in eq (4), we get:

$$u(x,l) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l}\sinh\frac{n\pi l}{l}$$

$$x(l-x) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l}\sinh n\pi \longrightarrow (5)$$

To find C_n expand $x(l-x)$ in a half range fourier sine series in the interval $0<x<l$.

$$x(l-x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \longrightarrow (6)$$

from (5) & (6) we get

$$\sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l}\sinh n\pi = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.$$

Equating like co-efficients of (6), we get:

$$C_n\sinh n\pi = b_n \quad \text{or} \quad C_n = \frac{b_n}{\sinh n\pi} \longrightarrow (7)$$

$$b_n = \frac{2}{l}\int_0^l x(l-x)\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l \left[(xl-x^2)\sin\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{2}{l}\left\{(lx-x^2)\cos\frac{n\pi x}{l}\times\frac{l}{n\pi}\right] + [l-2x]\left[\sin\frac{n\pi x}{l}\times\frac{l^2}{n^2\pi^2}\right]$$

$$-2\left[\cos\frac{n\pi x}{l}\times\frac{l^3}{n^3\pi^3}\right]\Bigg\}_0^l$$

$$= \frac{2}{l}\left[\frac{-2l^3}{n^3\pi^3}\cos n\pi + \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{4l^2}{n^3\pi^3}\left(1-(-1)^n\right]$$

$$b_n = \begin{cases} 0 & \text{when } n \text{ is even} \\ \frac{8l^2}{n^3\pi^3} & \text{when } n \text{ is odd} \end{cases}$$

from (7), we get.

$$C_n = \frac{8l^2}{n^3\pi^3 \sinh n\pi}$$

substitute in (4), we get

$$u(x,y) = \sum_{n=odd}^{\infty} \frac{8l^2}{n^3\pi^3 \sinh n\pi} \sin\frac{n\pi x}{l} \sinh\frac{n\pi y}{l}$$

Replace l by 20, we get

$$u(x,y) = \sum_{n=odd}^{\infty} \frac{3200}{n^3\pi^3 \sinh n\pi} \sin\frac{n\pi x}{20} \sinh\frac{n\pi y}{20}.$$

Sl.NO	Roll. No	Marks (50)	Is Absent
1	U21CE001	32	NO
2	U21CE002	36	NO
3	U21CE003	32	NO
4	U21CE004	35	NO
5	U21CE005	33	NO
6	U21CE006	33	NO
7	U21CE008	39	NO
8	U21CE009	39	NO
9	U21CE010	35	NO
10	U21CE011	34	NO
11	U21CE013	37	NO
12	U21CE014	34	NO
13	U21CE016	35	NO
14	U21CE018	34	NO
15	U21CE020	34	NO
16	U21CE021	32	NO
17	U21CE022	33	NO
18	U21CE023	32	NO
19	U21CE024	32	NO
20	U21CE025	35	NO
21	U21CE701	37	NO
22	U21CE702	32	NO
23	U21CE703	39	NO
24	U21CE704	32	NO
25	U21CE705	32	NO
26	U21CE706	33	NO
27	U21CE707	38	NO
28	U21CE708	32	NO

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH
B.TECH, II YEAR - 2022 - 2023 (SEM - III)
INTERNAL MARKS - CONSOLIDATED MARK STATEMENT
U20MABT03 - TRANSFORMS & BOUNDARY VALUE PROBLEMS - CIVIL

Name of Students	CLA - I	CLA - II	CLA - III
BANDARU BRAHMENDRA	28	30	26
GARAGA LAKSHMI SHANKAR	25	30	39
GIRIDHARAN P	31	26	27
LENISH ADHIKARIMAYUM	36	29	30
MOHAMMAD AZEEM	30	28	30
NALLURI PREM KUMAR	29	28	30
PARTHA MOIRANGTHEM	35	35	38
POLNATI JAGADEESH	36	35	37
SUHAIL RAHMAN M	31	32	30
UCHAVIVID OINAM	26	33	29
ALAKUNTA VAMSHI	30	36	35
B HARISH RAGAVENDRAN	30	28	31
DHANARAJ SAPAM	36	29	30
DEONALD YENGKOKPAM	26	31	32
MEDEMPONG PONGEN B	30	27	32
NUNGSHIWATI.K JAMIR	28	25	28
TEISONEISE SACHU	27	29	29
RAJ BABU RAY	30	25	28
NAVEEN KUMBAM	30	26	27
MAMIDISETTI SHANMUKA BRAHMA	31	30	33
V.BALAJI	36	35	32
BANOTH NIRANJANLAL	26	26	30
JERMY N LYNDDOH	36	35	37
MOHAMMED SHOIAB	30	26	28
NZANTHUNG SHITIRI	34	25	25
OKUTO G JIMO	35	27	26
R.RAHANMOHAMED	35	40	30
TAMIL SELVAN P A	26	25	30

CLA - I	CLA - II	CLA - III	CLA - IV
6	9	8	10
5	9	12	10
6	8	8	10
7	9	9	10
6	8	9	10
6	8	9	10
7	11	11	10
7	11	11	10
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7	11	11	10
6	8	8	10
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7	8	8	10
7	12	9	10
5	8	9	10

INFORMATION TECHNOLOGY

INFORMATION TECHNOLOGY

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Computer Science and Engineering

CONTINUOUS LEARNING ASSESSMENT - I

U20MABT03– TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date	: 24.01.2022
Academic Year / Semester	: 2021-2022 / ODD
Duration	: 1.5 Hours (90 minutes)
Instructions	: Descriptive Type Questions

<table>
<tr><th>Q. No</th><th>Questions</th><th>Weightage</th><th>CO</th><th>Bloom's Level</th></tr>
<tr><td></td><td>PART A (6x2=12 Marks)</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>Define Fourier Series.</td><td>2</td><td>CO1</td><td>U</td></tr>
<tr><td>2</td><td>Define Dirichlet's condition.</td><td>2</td><td>CO1</td><td>U</td></tr>
<tr><td>3</td><td>Define Periodic Function.</td><td>2</td><td>CO1</td><td></td></tr>
<tr><td>4</td><td>Find a_0 for the function $f(x)=\pi - x$ in $0 \le x \le \pi$</td><td>2</td><td>CO1</td><td>U</td></tr>
<tr><td>5</td><td>Write down the formula for the Fourier series in $(0,2l)$</td><td>2</td><td>CO1</td><td>U</td></tr>
<tr><td>6</td><td>Write down the Bernoulli's formula.</td><td>2</td><td>CO1</td><td>U</td></tr>
<tr><td></td><td>PART B (3x6=18 Marks)</td><td></td><td></td><td></td></tr>
<tr><td>7</td><td>(a) Find the Fourier series for $f(x)=2x-x^2$ in $0<x<2$
(OR)
(b) Find the Fourier sine series for $f(x)=x$ in $0<x<l$. Show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots=\frac{\pi^2}{6}$</td><td>6</td><td>CO1</td><td>U</td></tr>
<tr><td>8</td><td>(a) Obtain half range cosine series for $f(x)=x^2$ in $(0,\pi)$
(OR)
(b) Find the Fourier series for the function $f(x)=\begin{cases} kx, & 0<x<l \\ 0, & l<x<2l \end{cases}$</td><td>6</td><td>CO1</td><td>U</td></tr>
<tr><td>9</td><td>(a) Find the Fourier series of $f(x)=x$ in $(0,l)$
(OR)
(b) Find the Fourier series for $f(x)=x^3$ in $(-\pi,\pi)$</td><td>6</td><td>CO1</td><td>U</td></tr>
<tr><td></td><td>PART C (2x10=20 Marks)</td><td></td><td></td><td></td></tr>
<tr><td>10</td><td>(a) Obtain the Fourier series for the function $f(x)=\begin{cases} -x, & -\pi<x<0 \\ x, & 0<x<\pi \end{cases}$
and hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots=\frac{\pi^2}{8}$
(OR)
(b) Find the first two harmonic of the Fourier series of $f(x)$ given by the following table
<table><tr><td>x</td><td>0</td><td>$\frac{\pi}{3}$</td><td>$\frac{2\pi}{3}$</td><td>π</td><td>$\frac{4\pi}{3}$</td><td>$\frac{5\pi}{3}$</td><td>2π</td></tr><tr><td>$f(x)$</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1</td></tr></table></td><td>10</td><td>CO1</td><td>U</td></tr>
<tr><td>11</td><td>(a) Find the Fourier series for $f(x)=x+x^2$ in $-\pi<x<\pi$. Hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\dots=\frac{\pi^2}{6}$
(OR)
(b) Find the half range cosine series for $f(x)=x$ in $(0,\pi)$. Deduce that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\dots=\frac{\pi^4}{96}$</td><td>10</td><td>CO1</td><td>U</td></tr>
</table>

CO	Weightage
CO1	50
CO2	
CO3	
CO4	
CO5	
Total	50

Prepared by	Staff Name Dr. S. Anusuya, AP/MATHEMATICS	Signature
Verified by	HoD	Signature

ANSWERS

1. **Fourier Series** :- Let $f(x)$ be a periodic function define on $(-\pi,\pi)$ fourier series of $f(x)$ is defined as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx.$$

2. **Dirichlet's Condition** :-

(i) If $f(x)$ is defined and single valued except possibly at a finite number of points in $(0,2\pi)$ (or) $(-\pi,\pi)$.

3. **Define periodic function** :-

Let $f(x)$ be a real valued function and if there exist a least positive constant T such that $f(x+T) = f(x)$. Then $f(x)$ is said to be periodic function with period T.

4. Find a_0 for the function

$f(x) = \pi - x$ in $0 \le x \le \pi$ **Ans** $\pi/2$

5. $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

6. $\int u\,dv = uv - u'v_1 + u''v_2 - u'''v_3 + \cdots$

Part - B

7.

a) $l=1$, $a_0 = 4/3$, $a_n = \frac{-4}{n^2\pi^2}$, $b_n = 0$

$$f(x) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos n\pi x.$$

(or)

b) $f(x) = x$, $a_0 = 0$, $a_n = 0$, $b_n = -\frac{2}{n\pi} l(-1)^n$

$$f(x) = \frac{-2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi}{l}.$$

By Parseval's identity,

$$\frac{1}{l}\int_0^l [f(x)]^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$a_0 = 0, \quad a_n = 0, \quad b_n = \frac{-2}{n\pi} l(-1)^n$

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

8) (a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$a_0 = \frac{2\pi^2}{3}, \quad a_n = \frac{4(-1)^n}{n^2}$

$$f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx.$$

(or)

b)

$a_0 = \frac{kl}{2}, \quad a_n = \frac{kl}{n^2\pi^2}\left[(-1)^n - 1\right], \quad b_n = \frac{-kl}{n\pi}(-1)^n$

$$f(x) = \frac{kl}{4} + \sum_{n=1}^{\infty} \left[\frac{kl}{n^2\pi^2}\left[(-1)^n + 1\right] \cos\frac{n\pi x}{l} - \frac{kl}{n\pi}(-1)^n \sin\frac{n\pi x}{l}\right]$$

9) a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}, \quad a_0 = l, \quad a_n = \begin{cases} \frac{-4l}{n^2\pi^2} & \text{if } n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}$

$b_n = \frac{2l}{n^2\pi^2}\left[(-1)^n - 1\right]$

$$f(x) = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{n=1,3,5} \frac{1}{n^2} \cos\frac{n\pi x}{l}.$$

Part – C

10(a) $a_0 = \pi, \quad a_n = \frac{-4}{n^2\pi}; \quad \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

(or)

b) $Y = 1.45 + (-0.37\cos x + 0.17\sin x) + (-0.1\cos 2x + 0.06\sin 2x) + (0.03\cos 3x).$

11 a) $a_0 = 2\pi^2/3, \quad a_n = \frac{4(-1)^n}{n^2}, \quad b_n = \frac{2(-1)^{n+1}}{n}$

$\pi^2/6 = \sum 1/n^2$

(or)

b) $a_0 = \pi, \quad a_n = \frac{2}{n^2\pi}\left[(-1)^n - 1\right],$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right] \cos nx$$

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Computer Science and Engineering

CONTINUOUS LEARNING ASSESSMENT - II

U20MABT03– TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date :22.02.2022

Academic Year / Semester : 2021-2022/ODD

Duration :1.5 Hours (90 minutes)

Instructions :Descriptive Type Questions

Q. No	Questions	Weightage	CO	Bloom's Level
	PART A (6x2=12 Marks)			
1	Form the Partial differential equation by eliminating the arbitrary constants from $z=(x^2+a)(y^2+b)$	2	CO2	U
2	Solve $p+q=z$	2	CO2	U
3	Solve $(D^3-3DD'^2+2D'^3)z=0$	2	CO2	U
4	Classify the Partial differential equation $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+4\frac{\partial^2 z}{\partial y^2}=0$	2	CO3	U
5	Write the possible solutions of one dimensional wave equation.	2	CO3	R
6	Write down the one dimensional wave equation and explain the constant a^2.	2	CO3	R
	PART B (3x6=18 Marks)			
7	(a) Solve $z=px+qy+p^2-q^2$ **(OR)** (b) Write the general integral of $pyz+qzx=xy$	6	CO2	A
8	(a) Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ **(OR)** (b) Solve $(mz-ny)p+(nx-lz)q=(ly-mx)$	6	CO2	A
9	(a) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position. Find the displacement $y(x,t)$ at any distance x from one end at any time t. **(OR)** (b) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are kept at $0°C$ and kept so. Find the temperature distribution.	6	CO3	A
	PART C (2x10=20 Marks)			
10	(a) (i) Solve $(D^4-D'^4)z=0$ (ii) Solve $(D^2-DD'-2D'^2)z=2x+3y+e^{3x+4y}$ **(OR)** (b) (i) Solve $(D^2+2DD'+D'^2)z=e^{x-y}$ (ii) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}=\cos 2x\cos y$	10	CO2	A
11	(a) A string is stretched and fastened to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string into the form $y=k(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time t **(OR)** (b)The ends A and B of a rod 30cm long have their temperature kept at $20°C$ and $80°C$ until steady state condition prevails. The temperature at the end B is then suddenly reduced to $60°C$ and that of A is raised to $40°C$ and maintained so. Find the temperature distribution.	10	CO3	A

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Verified by	HoD	Signature

ANSWER KEY

1) $pq = 4xyz$

2. $(1+a)\log z = x + ay + b$ is complete solution

3. $z = f_1(y+x) + x f_2(y+x) + f_3(y-2x)$

4. $z = f_1(y+2x) + x f_2(y+2x)$

5. $y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$

6. One Dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

Here $a^2 = T/m$ = Tension/mass

7. a) $x + f'(a)y + 2a - 2f(a) \cdot f'(a) = 0$

b) $\phi(x^2 - y^2, y^2 - z^2) = 0.$

8 a) $\phi(x^2/2 + y^2/2 + z^2/2, xyz) = 0$

b) $\phi(x^2/2 + y^2/2 + z^2/2, lx + my + nz) = 0.$

9 a) $y(x,t) = \frac{3y_0}{4} \sin\frac{\pi x}{l} \cos\frac{\pi a t}{l} - \frac{y_0}{4} \sin\frac{3\pi x}{l} \cos\frac{3\pi a t}{l}$

b) $u(x,t) = \frac{440}{\pi} \sum_{n=1,3,5}^{\infty} \frac{1}{n} e^{-\frac{\alpha^2 n^2 \pi^2 t}{l^2}} \sin\frac{n\pi x}{l}.$

10 a) (i) $z = f_1(y+x) + f_2(y-x) + f_3(y+ix) + f_4(y-ix)$

(ii) $z = f_1(y+2x) + f_2(y-x) + \frac{5x^3}{6} + \frac{3}{2}x^2 y - \frac{1}{35} e^{3x+4y}$

b) (i) $f_1(y-x) + x f_2(y-x) + \frac{x^2}{2} e^{x-y}$

(ii) $f_1(y) + f_2[y-x) - \frac{1}{2}\cos(2x+y) - \frac{1}{4}\cos(2x-y)$

11 a) $y(x,t) = \frac{8k l^2}{\pi^3} \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l}$

b) $u(x,t) = \left(\frac{2}{3}x + 40\right) + \sum_{n=2,4,6}^{\infty} -\frac{80}{n\pi} e^{-\frac{\alpha^2 n^2\pi^2 t}{100}} \cdot \sin\frac{n\pi x}{300}$

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
Department of Computer Science and Engineering

CONTINUOUS LEARNING ASSESSMENT - III

U20MABT03– TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Date : 07.03.2022

Academic Year / Semester : 2021-2022/ODD

Duration : 1.5 Hours (90 minutes)

Instructions : Descriptive Type Questions

Q. No	Questions	Weightage	CO	Bloom's Level
	PART A (6x2=12 Marks)			
1	Prove that $F[f(x-a)] = e^{ias}F(s)$	2	CO4	U
2	Define Fourier integral theorem	2	CO4	R
3	Define Parseval's identity for Fourier transform	2	CO4	R
4	Find $Z(n)$	2	CO5	U
5	Find $Z\left(\cos\frac{n\pi}{2}\right)$	2	CO5	U
6	State and prove initial value theorem	2	CO5	R
	PART B (3x6=18 Marks)			
7	(a) Find the Fourier transform of $f(x)=\begin{cases}1, & for\ \lvert x\rvert<1\\ 0, & otherwise\end{cases}$ Hence prove that $\int_0^{\infty}\left(\frac{\sin x}{x}\right)^2 dx=\frac{\pi}{2}$ **(OR)** (b) Show that the function $e^{\frac{-x^2}{2}}$ is self-reciprocal under Fourier transform.	6	CO4	A
8	(a) Find Fourier sine and cosine transform of xe^{-ax} (OR) (b) Find $Z(r^n\cos n\theta)$ and $Z(r^n\sin n\theta)$ also find $Z(\cos n\theta)$ and $Z(\sin n\theta)$	6	CO4, CO5	A
9	(a) Find $Z^{-1}\left(\frac{z^3-20z}{(z-3)^3(z-4)}\right)$ by Partial fraction method.**(OR)** (b) Find $Z^{-1}\left(\frac{z^2-2z}{(z-1)^2(z-3)}\right)$ by Residue method.	6	CO5	A
	PART C (2x10=20 Marks)			
10	(a) Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & in\ \lvert x\rvert\le 1\\ 0, & in\ \lvert x\rvert>1\end{cases}$. Hence prove that (i) $\int_0^{\infty}\left(\frac{\sin s-s\cos s}{s^3}\right)\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$, (ii) $\int_0^{\infty}\left(\frac{\sin s-s\cos s}{s^3}\right)^2 ds=\frac{\pi}{15}$ **(OR)** (b) Find $F_s(e^{-ax})$ & $F_c(e^{-ax})$ and hence deduce that (i) $\int_0^{\infty}\frac{x^2dx}{(x^2+a^2)^2}$ (ii) $\int_0^{\infty}\frac{dx}{(x^2+a^2)^2}$ $a>0$	10	CO4	A
11	(a) (i) Using convolution theorem find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ (ii) Using Partial fraction method $Z^{-1}\left(\frac{z(z+1)}{(z-1)^3}\right)$ **(OR)** (b)Solve $y_{n+2}+4y_{n+1}+3y_n=2^n$ with $y_0=0, y_1=1$ using Z - transform	10	CO5	A

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Verified by	HOD	Signature

ANSWER KEY CLA-3

PART- A

1) $F[f(x-a)] = e^{ias} F(s)$

$$\frac{e^{iax}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y)\, e^{isy}\, dy = \frac{e^{ias}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= e^{ias} F(s)$$

2) Fourier Integral Theorem:-

$$f(x) = \frac{1}{\pi} \int_{0}^{\infty} \int_{-\infty}^{\infty} f(t) \cos\lambda(t-x)\, dx\, d\lambda.$$

3) Parseval's Identity

$$\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |F(s)|^2\, ds$$

4) $Z(n) = \frac{z}{(z-1)^2}$ 5) $Z\left(\cos\frac{n\pi}{2}\right) = \frac{z^2}{z^2+1}$

6) Initial Value theorem

$Z[f(n)] = F(z)$ then $\lim_{z\to\infty} F(z) = f(0) = \lim_{t\to 0} f(t)$

PART-B

7) a) $F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} f(x)\, dx$, $F(s) = \frac{\sin s}{s}$

$$\int_{-\infty}^{\infty} |F(s)|^2\, ds = \int_{-\infty}^{\infty} |f(x)|^2\, dx.$$

b) $f(x) = e^{-x^2/2}$, $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx}\, dx$

$$\int_{0}^{\infty} e^{-y^2}\, dy = \sqrt{\pi}/2$$

8) a) $F_c[xf(x)] = \frac{d}{ds}\{F_s[f(x)]\}$

$$F_c[xe^{-ax}] = \sqrt{\frac{2}{\pi}}\left[\frac{a^2-s^2}{(s^2+a^2)^2}\right]$$

b) $Z[\cos n\theta] = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$

$$Z[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

$$Z[r^n \cos n\theta] = \frac{z(z - r\cos\theta)}{z^2 - 2rz\cos\theta + r^2}$$

$$Z[r^n \sin n\theta] = \frac{rz\sin\theta}{z^2 - 2rz\cos\theta + r^2}$$

9) a) $Z^{-1}\left(\frac{z^3 - 20z}{(z-3)^3(z-4)}\right) = \frac{1}{2}(2^n + 2n^2 2^n - 4^n)$.

b) $Z^{-1}\left(\frac{z^2 - 2z}{(z-1)^3(z-3)}\right) =$

10) a) $f(x) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & |x| \geq 1 \end{cases}$, $\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right) \cos\frac{s}{2}\, ds = \frac{3\pi}{16}$

b) (i) $\int_0^\infty \frac{x^2}{(x^2+a^2)^2} = \pi/4a$ (ii) $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \pi/4a^3$

11) a) (i) $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right) = \left(\frac{1}{2}\right)^n \left[1 + \left(-\frac{1}{2}\right)^1 + \left(-\frac{1}{2}\right)^2 + \cdots \left(\frac{-1}{2}\right)^n\right]$

(ii) $Z^{-1}\left(\frac{z(z+1)}{(z-1)^3}\right) = n(1)^{n-1} + n(n-1)(1)^{n-2}$

b) $y_n = \frac{2^n}{15} + \frac{1}{3}(-1)^n + \frac{2}{5}(-3)^n$, $n \geq 0$

Transforms and Boundary Value Problems:
(U20MABT03).

CLA-II.

Name: M. Poojitha
R.No: U20CN052
Sec: C.

50/50

(1) given,
$z = (x^2+a)(y^2+b) \rightarrow (1)$

Partially differentiating with respect to 'x'

$$\frac{dz}{dx} = 2x(y^2+b)$$

$$p = 2x(y^2+b) \Rightarrow y^2+b = \frac{p}{2x} - (2).$$

(1) Partially differentiating w.r.t 'y'

$$\frac{dz}{dy} = (x^2+a).2y.$$

$$q = (x^2+a).2y \Rightarrow x^2+a = \frac{q}{2y} - (3).$$

Substituting (2),(3) in (1), we get

$$z = \frac{p}{2x} + \frac{q}{2y}$$

$$pq = 4xyz.$$

(2) Solve $p+q = z$.

Given,
$p+q = z.$

this equation of form $f(z,p,q) = 0.$

let, $u = x + ay.$

$$p = \frac{dz}{du}, \quad q = a\frac{dz}{du}$$

$$\frac{dz}{du} + a\frac{dz}{du} = z$$

$$\frac{dz}{du}(1+a) = z \rightarrow (1+a)\frac{dz}{z} = du.$$

Integrating, we get

$$(1+a)\int \frac{dz}{z} = \int du$$

$$(1+a)\log z = u + b.$$

$$(1+a)\log z = x + ay + b.$$

(3) $(D^3 - 3DD'^2 + 2D'^3)z = 0.$

replacing, $D \to m$, $D' \to 1$

auxiliary eqn is $m^3 - 3m + 2 = 0.$

1	1	0	-3	-2
	0		1	-2
1	1	1	-2	0
	0	1	2	
	1	2	0	
-2	0	-2		
	1	0		

$m = 1, 1$ and $-2.$

$$z = f_1(y+x) + xf_2(y+x) + f_3(y-2x).$$

(4) $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0.$

Auxiliary eqn is $m^2 - 4m + 4 = 0.$

replace, $D \to m$, $D' \to 1.$

2	1	-4	4	
	0	2	-4	
2	1	-2	0	
	0	2		
	1	0		

$m = 2, 2$

$$z = f_1(y+2x) + xf_2(y+2x).$$

(5) The possible solutions of one dimensional wave equation are

$$y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{pat} + c_4 e^{-pat})$$

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pa$$

$$y(x,t) = (c_1 x + c_2)(c_3 t + c_4).$$

(6) One-dimensional wave eqn:

one dimensional wave eqn is $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

here, $a^2 = \frac{T}{m}$

where 'T' is Tension & 'm' is mass.

7.(b). General form of $pyz + qzx = xy$.

Given,

$$pyz + qzx = xy.$$

The given eqn is of the form

$$Pp + Qq = R \quad (1)$$

$P = yz, \; Q = zx, \; R = xy.$

The subsidary eqn is $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ — (2)

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy} \quad (3).$$

Consider the pair.

$$\frac{dx}{yz} = \frac{dy}{zx} \Rightarrow x\,dx = y\,dy$$

$$\int x\,dx = \int y\,dy$$

$$\Rightarrow \frac{x^2}{2} = \frac{y^2}{2} + c_1$$

$$c_1 = x^2 - y^2.$$

Consider the pair:

$$\frac{dy}{zx} = \frac{dz}{xy} \Rightarrow y\,dy = z\,dz.$$

$$\int y\,dy = \int z\,dz.$$

$$y^2/2 = z^2/2 + c_2$$

$$c_2 = y^2 - z^2$$

$$\phi(x^2 - y^2, \; y^2 - z^2) = 0$$

(8) (a) given,

$$x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$$

This is in form of

$$Pp + Qq = R.$$

Here, $P = x(y^2-z^2)$, $Q = y(z^2-x^2)$, $R = z(x^2-y^2)$

The substituting eqn's are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}.$$

Each ratio $= \dfrac{x\,dx + y\,dy + z\,dz}{x^2(y^2-z^2) + y^2(z^2-x^2) + z^2(x^2-y^2)}$

$= \dfrac{x\,dx + y\,dy + z\,dz}{x^2y^2 - x^2z^2 + y^2z^2 - y^2x^2 + z^2x^2 - z^2y^2}$

$\Rightarrow x\,dx + y\,dy + z\,dz = 0.$

Integrating on b.s: $\int x\,dx + \int y\,dy + \int z\,dz = \int 0.$

$\Rightarrow \dfrac{x^2}{2} + \dfrac{y^2}{2} + \dfrac{z^2}{2} = C_1$

Each ratio $= \dfrac{dx/x + dy/y + dz/z}{y^2-z^2+z^2-x^2+x^2-y^2}$

Here we take $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$ multipliers.

$$\frac{dx}{x} + dy/y + dz/z = 0.$$

$\dfrac{\frac{1}{x}\cdot dx}{\frac{1}{x}\cdot x(y^2-z^2)} = \dfrac{\frac{1}{y}\cdot dy}{\frac{1}{y}\cdot y(z^2-x^2)} = \dfrac{\frac{1}{z}\cdot dz}{\frac{1}{z}\cdot z(x^2-y^2)}$

integrating on b.s:

$$\log x + \log y + \log z = \log C_1$$

4

$xyz = c_2$ (ans 2).

$\phi\left(\frac{x^2}{2} + y^2/2 + z^2/2,\; xyz\right) = 0$ ϕ(ans 1, ans 2) = 0

9. (a) the wave eqn is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (1)$$

From the given problem we have following boundary conditions:

(i) $y(0,t) = 0$, $t > 0$

(ii) $y(l,t) = 0$, $\forall t > 0$

(iii) $\frac{\partial y}{\partial t}(x,0) = 0$, $\forall x$

(iv) $y(x,0) = y_0 \sin^3 \frac{\pi x}{l}$.

The correct solution of (1) is

$y(x,t) = (c_1 \cos\lambda x + c_2 \sin\lambda x)(c_3 \cos\lambda at + c_4 \sin\lambda at)$ — (2)

Applying condition (i) in (2)

i.e., $x = 0$ in (2).

$y(0,t) = c_1(c_3 \cos\lambda at + c_4 \sin\lambda at) = 0$ (it is defined for all t)

$y(0,t) = 0$

$c_1 = 0$.

Sub $c_1 = 0$ in (2), we get

$y(x,t) = c_2 \sin\lambda x(c_3 \cos\lambda at + c_4 \sin\lambda at)$ — (3)

Applying condition (ii) in (3),

i.e. Put $x = l$ in (3)

$y(l,t) = c_2 \sin\lambda l(c_3 \cos\lambda at + c_4 \sin\lambda at)$ — (3)

Apply conditions in eqn ① we get,

(i) $y(0,t)=0$

∴ put $x=0$ in ①

$y(0,t) = (c_1\cos p(0) + c_2\sin p(0))(c_3\cos pat + c_4\sin pat)$

$= (c_1\cos 0 + c_2\sin 0)(c_3\cos pat + c_4\sin pat)$

$0 = c_1(c_3\cos pat + c_4\sin pat)$

Here $[c_3\cos pat + c_4\sin pat] \neq 0$

∴ we get $\boxed{c_1=0}$

Sub $c_1=0$ in ①, we get

$y(x,t) = c_2\sin px(c_3\cos pat + c_4\sin pat)$ - ②

$y(l,t)=0$

∴ put $x=l$ in ②

$y(l,t) = c_2\sin pl(c_3\cos pat + c_4\sin pat) = 0$

Here, $[c_3\cos pat + c_4\sin pat] \neq 0$

either $c_2=0$ or $\sin pl=0$

Suppose we take $c_2=0$ & already

$c_1=0$.

then we get a trivial solution

∴ we consider $c_2 \neq 0$ &

$\sin pl = 0$

$pl = n\pi$

$\boxed{p = n\pi/l}$

$P.I. = \frac{1}{2}\left(P.I_1 + P.I_2\right)$

$PI_1 = \frac{1}{D^2 - DD'} \cos(2x+y)$

$= \frac{1}{-4-2} \cos(2x+y) \Rightarrow \frac{1}{-6}\cos(2x+y)$

$PI_2 = \frac{1}{D^2+DD'} \cos(2x-y)$

$= \frac{1}{-4+2} \cdot \cos(2x-y)$

$= \frac{-1}{2} \cos(2x-y)$

$m=2, n=-1$
Replace,
$D^2 \to -4$
$DD' \to 2$.

$\therefore z = C.F + \frac{1}{2}\left[P.I_1 + P.I_2\right]$

$= f_1(y) + f_2(y-x) - \frac{1}{12}\cos(2x+y) - \frac{1}{4}\cos(2x-y).$

(11) (a) The one dimensional wave eqn is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \cdot \frac{\partial^2 y}{\partial x^2}$$

The suitable soln is

$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos pat + c_4 \sin pat)$

Since the initial displacement is given the boundary conditions are

(i) ~~Eq~~. $y(0,t) = 0 \quad \forall t$

(ii) $y(l,t) = 0 \quad \forall t$

(iii) $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$

(iv) $y(x,0) = k(lx - x^2)$

10, b, (i) Given,

$(D^2+2DD'+D'^2)z = e^{x-y}$.

Auxiliary eqn is $m^2+2m+1=0$

$(m+1)^2=0 \Rightarrow m=-1,-1$. (equal roots)

$C.F = f_1(y-x) + x f_2(y-x)$.

$P.I = \frac{1}{(D+D')^2} e^{x-y}$

$= \frac{x^2}{2!} \cdot e^{x-y}$

[using $\frac{1}{(bD-aD')^n} F(ax+by) = \frac{x^n}{b^n n!} F(ax+by)$]

Here $a=-1$, $b=1$, $n=2$.

$\therefore$ complete solution is $z = C.F + P.I$

$= f_1(y-x) + x f_2(y-x) + \frac{x^2}{2} e^{x-y}$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = \cos 2x \cos y$.

the given eqn can be written as

$(D^2+DD')z = \frac{1}{2}[\cos(2x+y) + \cos(2x-y)]$

Auxiliary eqn is $m^2+m=0$ (Replace $D \to m$, $D' \to 1$)

$m(m+1)=0$

$m=0,-1$.

$C.F = f_1(y+0x) + f_2(y-x)$.

$$y(x,t) = C_2 \sin\frac{n\pi x}{l}\, C_3 \cos\frac{n\pi at}{l} = C_2 C_3 \sin\frac{n\pi x}{l} \cos\frac{n\pi at}{l}$$

$$y(x,t) = C_n \sin\frac{n\pi x}{l} \cdot \cos\frac{n\pi at}{l} \quad \text{--- (5)}$$

where $C_n = C_2 C_3$

$$y(x,t) = \sum_{n=1}^{\infty} C_n \cdot \sin\frac{n\pi x}{l} \cdot \cos\frac{n\pi at}{l} \quad \text{--- (6)}$$

Applying condition (iv) in (6), we get

$$y(x,0) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{l} = y_0 \sin^3\frac{\pi x}{l} \quad \text{--- (7)}$$

W.K.T,

$$\sin^3 x = \frac{1}{4}[3\sin x - \sin 3x]$$

$$\sin^3\frac{\pi x}{l} = \frac{1}{4}\left[3\sin\frac{\pi x}{l} + \sin\frac{3\pi x}{l}\right] \quad \text{--- (8)}$$

from (7) & (8) we get,

$$\sum_{n=1}^{\infty} C_n \cdot \sin\frac{n\pi x}{l} = \frac{y_0}{4}\left[3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}\right]$$

Equating the coefficients on both sides:

$$y(x,t) = \frac{3y_0}{4} \cdot \sin\frac{\pi x}{l} \cos\frac{\pi at}{l} - \frac{y_0}{4} \sin\frac{3\pi x}{l} \cos\frac{3\pi at}{l}.$$

$0 = c_2 \sin \lambda l (c_3 \cos \lambda at + c_4 \sin \lambda at)$

Here, $c_3 \cos \lambda at + c_4 \sin \lambda at \neq 0$

Either $c_2 = 0$ (or) $\sin \lambda l = 0$.

Suppose we take $c_2 = 0$ & we have $c_1 = 0$

then we get a trivial soln.

$\therefore$ we consider $c_2 \neq 0$ and $\sin \lambda l = 0$.

$\sin \lambda l = \sin n\pi$.

$$\lambda l = n\pi \Rightarrow \lambda = \frac{n\pi}{l}.$$

Now, substituting 'λ' in ③ we get,

$$y(x,t) = c_2 \sin \frac{n\pi x}{l} \left[c_3 \cos \frac{n\pi at}{l} + c_4 \sin \frac{n\pi at}{l} \right] \text{ ④}$$

Before applying (iii) differentiating ④ partially w.r.t 't' we get

$$\frac{\partial y(x,t)}{\partial t} = c_2 \sin \frac{n\pi x}{l} \left[c_3 \sin \frac{n\pi at}{l} \left(\frac{n\pi a}{l}\right) + c_4 \cos \frac{n\pi at}{l} \left(\frac{n\pi a}{l}\right) \right]$$

Now apply (iii) we get,

$$\frac{\partial y(x,0)}{\partial t} = c_2 \cdot \sin \frac{n\pi x}{l} \, c_4 \frac{n\pi a}{l} = 0.$$

Here $c_2 \neq 0$, $\sin \frac{n\pi x}{l} \neq 0$, $\frac{n\pi a}{l} \neq 0$

$\therefore c_4 = 0$.

Now, Substituting $c_4 = 0$ in ④, we get.

$$y(x,0) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \cos \frac{n\pi a(0)}{l}$$

$$= \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} = k(lx - x^2) \rightarrow (6)$$

To find c_n:

Expand $k(lx - x^2)$ in a half range fourier series in the interval $(0, l)$:

$$f(x) = \sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{n\pi x}{l}\right) \rightarrow (7)$$

$$b_n = \frac{2}{l} \int_0^l f(x) \cdot \sin\left(\frac{n\pi x}{l}\right) dx$$

From (6) & (7) we get, $b_n = c_n$

$$\therefore c_n = \frac{2}{l} \int_0^l k(lx - x^2) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2k}{l} \int_0^l (lx - x^2) \sin \frac{n\pi x}{l} \cdot dx$$

$$= \frac{2k}{l} \left[(lx - x^2) \left(-\cos \right. \right.$$

$$= \frac{2k}{l} \left[(lx - x^2) \cdot \left(\frac{-\cos \frac{n\pi x}{l}}{n\pi/l} \right) - (l - 2x) \left(- \frac{\sin \frac{n\pi x}{l}}{n^2\pi^2/l^2} \right) + (-2) \left(\frac{\cos n\pi x/l}{n^3\pi^3/l^3} \right) \right]_0^l$$

$$= \frac{2k}{l} \left[\left(0 + 0 - \frac{2\cos n\pi}{n^3\pi^3/l^3} \right) - \left(0 + 0 - \frac{2}{n^3\pi^3/l^3} \right) \right]$$

$$= \frac{2k}{l}\left[\frac{-2l\cos n\pi}{n^3\pi^3/l^3} + \frac{2}{n^3\pi^3/l^3}\right]$$

$$= \frac{2k}{l}\left[\frac{l^3}{n^3\pi^3}\left(-2\cos n\pi + 2\right)\right] = \frac{4kl^2}{n^3\pi^3}\left(1-\cos n\pi\right)$$

$$C_n = \frac{4kl^2}{n^3\pi^3}\left(1-\cos n\pi\right)$$

$$C_n = \begin{cases} 0 & \text{when 'n' is even} \\ \dfrac{8kl^2}{n^3\pi^3} & \text{when 'n' is odd.} \end{cases}$$

Substituting C_n in (2)

$$y(x,t) = \sum_{n=1,3,5}^{\infty} \frac{8kl^2}{n^3\pi^3} \sin\frac{n\pi x}{l} \cos\frac{n\pi a t}{l}$$

$$= \frac{8kl^2}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin\frac{n\pi x}{l} \cos\frac{n\pi a t}{l}.$$

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Maths-3

1. Define Fourier Series:.

The Fourier Series for the function $f(x)$ in the interval $C < x < C + 2\pi$ is given by $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ where

$$a_0 = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{c}^{c+2\pi} f(x) \sin nx\,dx$$

2. Dirichlet Conditions:.

Suppose that

* $f(x)$ is defined and Single valued Possible at a finite no. of Points in $(-\pi, \pi)$
* $f(x)$ is Periodic with Period 2π

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Then the fourier Series Converges to

(a) $f(x)$ if x is a point of Continuity

(b) $\frac{f(x+0)+f(x-0)}{2}$ if x is a Point of dis Continuity.

3. Periodic function :-

A function $f(x)$ is Said to have a period T if for all x, $f(x+T) = f(x)$, where value of $T>0$ is Called the Period of $f(x)$

eg $f(x) = \sin x = \sin(x+2\pi) = \sin(x+$

$4\pi) = \ldots\ldots\ldots$

Therefore the function has periods $2\pi, 4\pi, 6\pi$, etc...

however, 2π is the least value and 2π is the Period of $f(x)$. III^ly $\cos x$ is a

... Period 2π and

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4. $f(x) = A - x$ in $0 \leq x \leq \pi$

given: $f(x) = \pi - x \quad 0 \leq x \leq \pi$

$$a_0 = \frac{1}{\pi} \int_0^{A} f(x)\, dx$$

$$= \frac{1}{A} \int_0^{A} (\pi - x)\, dx$$

$$= \frac{1}{A} \left\{ -\frac{(A - x)^2}{2} \right\}_0^{A}$$

$$= \frac{1}{A} \left(-0 + \frac{\pi^2}{2} \right)$$

$$= \frac{1}{A} \left(\frac{\pi^2}{2} \right)$$

$$= \frac{A^2}{2A} = \pi/2$$

$$\boxed{a_0 = \pi/2}$$

5. formula for the fourier Series in $(0, 2l)$

The fourier Series $f(x)$ in $(0, 2l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum b_n$$

(i) f must be absolutely integrable over a period

(ii) f must be of bounded variation in any given bounded interval.

finite number of

periodic function with

tanx has Period π

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$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\,dx$$

6.

(i) bernoullis formulae

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \ldots$$

where u and v are function of x

$u' = \frac{du}{dx}$ $v_1 = \int v\,dx$

$u'' = \frac{d^2u}{dx^2}$ $v_2 = \int v_1\,dx$

$u''' = d^2$ $v_3 = \int v_2\,dx$

7. b. The sine Series of f(x) in (0, l) is

$$f(x) = \sum b_n \sin\frac{n\pi x}{l}$$

To find bn

$$b_n = \frac{2}{l}\int f(x)\sin\frac{n\pi x}{l}\,dx$$

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To find a_0:-

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} (x^2)\,dx$$

$$= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^3}{3} - 0\right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}} \rightarrow (2)$$

To find a_n:

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right)$$

$$+ 2\left(\frac{-\sin nx}{n^3}\right)_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2x\cdot\cos nx}{n^2}\right]_0^{\pi}$$

$$a_n = \frac{4}{n^2}(-1)^n \quad - (3)$$

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from eqn (1), (2), (3) we get

The required fourier series is

$$f(x) = \frac{2\pi^2}{3\times 2} + \sum_{n=1}^{\infty} \frac{4}{n^2}(-1)^n \cos nx$$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$

$$\therefore f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$

To find a_0:

$$a_0 = \frac{1}{l}\int_0^l f(x)\,dx$$

$$= \frac{1}{l}\int_0^l x\,dx$$

$$= \frac{1}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{1}{l}\cdot\frac{l^2}{2}$$

$$a_0 = l/2 \quad \rightarrow (2)$$

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10.

b.

Step 1:

The fourier series given by

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \rightarrow (1)$

Step 2:

To find a_0

$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$

$= \frac{2}{\pi} \int_0^{\pi} x\,dx$

$= \frac{2}{\pi} \left[\frac{x^2}{2}\right]_0^{\pi}$

$= \frac{2}{\pi} \left[\frac{\pi^2}{2}\right]$

$= \pi \rightarrow (2)$

Step 3:

To find a_n

$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx$

$= \frac{2}{\pi} \int_0^{\pi} x \cos nx\,dx$

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$$\bar{y}^2 = \frac{1}{2}\sum b_n^2$$

$$\frac{2}{l}\int [f(x)]^2 dx = \sum \frac{4l^2}{n^2\pi^2}(-1)^{2n+2}$$

~~$\frac{2}{l}\int_0^l x^2 dx = \sum \frac{4l^2}{n^2}$~~

$$\frac{2}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{4l^2}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{2^2}+\cdots\right]$$

$$\frac{2l^2}{3}\times\frac{\pi^2}{4l^2} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$$

$$\frac{\pi^2}{6} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\cdots$$

8.

9.

Soln

The half range Cosine Series function f(x) is given by

$$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx \quad \text{——— (1)}$$

To find a_0:

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

To find b_n U20CS238

$$b_n = \frac{1}{l} \int_0^l f(x) \sin \frac{n\pi}{l} x$$

$$= \frac{1}{\lambda} \int_0^{\lambda} x \left(\frac{-\cos \frac{n\pi}{\lambda} x}{n\pi / x} \right) \cdot \left(\frac{\sin \frac{n\pi}{\pi n}}{(n\pi/\pi)^2} \right)_0^l$$

$$= \frac{1}{\lambda} \left[l \left(\frac{\cos n\pi}{n\pi / l} \right) \right]$$

$$= \frac{1}{x} \frac{l^2}{n\pi} (-1)^n$$

$$\boxed{b_n = \frac{l}{n\pi} (-1)^n} \rightarrow (4)$$

Subtiate (2), (3), (4) in (1)

$$f(x) = \frac{l/2}{2} + \sum_{n=1}^{\infty} \frac{l}{\pi^2 n^2} \left[(-1)^n - 1 \right] \cos \frac{n\pi}{l} x + \sum_{n=1}^{\infty} \frac{l}{n\pi} (-1)^n \sin \frac{n\pi}{x} x.$$

$$\Rightarrow f(x) = \frac{l}{4} + \frac{l}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[(-1)^n - 1 \right] \cos \frac{n\pi}{x} + \frac{l}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^n \sin \frac{\pi n}{\lambda} x$$

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$$= \frac{2}{l} \int_0^l x \sin \frac{n\pi x}{l} \, dx$$

$$= \frac{2}{l} \left[x \left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right) - (1) \left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}} \right) \right]_0^l$$

$$= \frac{2}{l} \left[\frac{-x \cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right]_0^l$$

$$= \frac{-2}{l} \left[\frac{l \cos \frac{n\pi l}{l}}{\frac{n\pi}{l}} - 0 \right]$$

$$= \frac{-2}{l} \, l (-1)^n \frac{l}{n\pi}$$

$$b_n = \frac{2l (-1)^{n+1}}{n\pi}$$

$$\therefore f(x) = \sum \frac{2l (-1)^{n+1}}{n\pi} \sin \frac{n\pi x}{l}$$

$$= \frac{2l}{\pi} \left[\sin \frac{\pi x}{l} - \frac{\sin \frac{2\pi x}{l}}{2} + \cdots \right.$$

Since it is a half range ...

$= \frac{2}{\pi} \left[\frac{\cos \pi n}{n^2} - \frac{\cos(0)}{n^2} \right]$

$= \frac{2}{\pi n} \left[\frac{\cos n\pi}{n^2} - \frac{\cos.(0)}{n^2} \right]$

$= \frac{2}{n^2 \pi} \left[\cos \pi n - \cos(0) \right]$

$a_n = \begin{cases} \frac{-4}{n^2\pi}, & \text{if } n \text{ is odd} \\ 0 & \text{if is even.} \end{cases}$

Step4:-

The required Fourier Series

Substating (2) and (3) in (1) we get

$f(x) = \pi/2 \cdot \sum_{n=odd}^{\infty} \frac{4}{n^2} \cos nx$

$f(x) = \pi/2 - 4/n \left[\frac{\cos x}{2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots \right] \rightarrow (4).$

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To find an:

$$a_n = \frac{1}{l}\int_0^l f(x) \cos \frac{n\pi x}{l}\, dx$$

$$a_n = \frac{1}{l}\int_0^l x \cos \frac{n\pi}{l} x\, dx$$

$$= \frac{1}{l}\left[x \cdot \frac{\sin \frac{n\pi x}{l}}{n\pi/l} - (1)\left(\frac{-\cos \frac{n\pi}{l}}{(n\pi/l)^2} \right) \right]_0^l \quad \begin{matrix} u = x \\ u' = 1 \end{matrix}$$

$$dv = \cos \frac{n\pi}{l} x\, dx$$

$$v = \sin \frac{\frac{n\pi}{l} x}{x}$$

$$v_2 = \frac{-\cos \frac{n\pi}{l}}{\left(\frac{n\pi}{l}\right)^2}$$

$$= \frac{1}{l}\left[\frac{\cos n\pi}{n^2\pi^2} \cdot l^2 - \frac{l^2}{n^2\pi^2} \right]$$

$$= \frac{1}{l} \cdot \frac{l^2}{n^2\pi^2}\left[(-1)^n - 1 \right]$$

$$\Rightarrow a_n = \frac{l}{\;\cdot\;}\left[(-1)^n \ldots \right]$$

Assignment - 1

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Section : C

Transform and Boundary Value Problems.

1) Find the Fourier series of period 2π of the function $f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 2 & \pi \le x \le 2\pi \end{cases}$ and hence find the sum of the series $1/1^2 + 1/3^2 + 1/5^2 + \dots + \infty$

Given

$$f(x) = \begin{cases} 1, & 0 \le x \le \\ 2, & \pi \le x \le 2\pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x \quad \text{---} (1)$$

To find a_0 :

$$a_0 = 1/\pi \int_0^{\pi} f(x).dx + 1/\pi \int_{\pi}^{2\pi} f(x).dx$$

$$= 1/\pi \int_0^{\pi} 1.dx + 1/\pi \int_{\pi}^{2\pi} 2dx$$

$$= 1/\pi (x)_0^{\pi} + 1/\pi (2x)_{\pi}^{2\pi}$$

$$= 1/\pi (\pi - 0) + \frac{2}{\pi}(2\pi - \pi)$$

$$= \pi/\pi + 2\pi/\pi$$

$$= 3.$$

To find a_n :

$$a_n = 1/\pi \int_0^{\pi} f(x).\cos nx dx + 1/\pi \int_{\pi}^{2\pi} f(x).\cos nx dx$$

$$= 1/\pi \int_0^{\pi} 1.\cos nx dx + 1/\pi \int_{\pi}^{2\pi} 2.\cos nx dx$$

$$= 1/\pi \left[\frac{\sin nx}{n}\right]_0^{\pi} + 2/\pi \left[\frac{\sin nx}{n}\right]_{\pi}^{2\pi}$$

$$= 1/\pi \left[\frac{\sin n\pi}{n} - 0\right] + \frac{2}{\pi}\left[\left(\frac{\sin 2n\pi}{n}\right) - \left(\frac{\sin n\pi}{n}\right)\right]$$

$$= 0.$$

To find b_n : $\frac{1}{\pi}\int_0^{\pi} f(x)\sin nx\,dx + \frac{1}{\pi}\int_{\pi}^{2\pi} f(x)\cdot\sin nx\cdot dx$

$$= \frac{1}{\pi}\int_0^{\pi} 1\cdot\sin nx\,dx + \frac{1}{\pi}\int_{\pi}^{2\pi} 2\cdot\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi} + \frac{2}{\pi}\left[-\frac{\cos nx}{n}\right]_{\pi}^{2\pi}$$

$$= \frac{1}{\pi}\left(-\frac{(-1)^n}{n} + \frac{1}{n}\right] + \frac{2}{\pi}\left[\frac{-1}{n} + \frac{(-1)^n}{n}\right)$$

$$= \frac{(-1)^n}{n\pi} + \frac{1}{n\pi} + \frac{(-2)}{n\pi} + \frac{2(-1)^n}{n\pi}$$

$$= \frac{(-1)^n}{n\pi} - \frac{1}{n\pi}$$

$$= \frac{(-1)^n - 1}{n\pi}.$$

Sub, $a_0 = 3$, $a_n = 0$, $b_n = \frac{(-1)^n - 1}{n\pi}$ in (1)

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} 0 + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi}\sin nx.$$

$$f(x) = 3/2 + \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi}\cdot\sin nx \Rightarrow \begin{cases} 3/2 & \text{if } n \text{ is even} \\ 3/2 + \sum_{n=odd} \frac{-2}{n\pi}\sin nx & \text{if } n \text{ is odd.} \end{cases}$$

[illegible]

put $x = \pi/2$

$$1 = 3/2 + \sum_{n=odd}^{\infty} \frac{-2}{n\pi}\sin n(\pi/2)$$

$$-1/2 = \frac{-2}{\pi}\sum_{n=odd}^{\infty} \frac{1}{n}$$

$$\pi/4 = \sum_{n=odd}^{\infty} 1/n.$$

2) Find the fourier series for function

$$f(x) = \begin{cases} kx & 0 < x < l \\ 0 & l \le x \le 2l \end{cases}$$

Given,

$$f(x) = \begin{cases} kx & 0 \le x \le l \\ 0 & l \le x \le 2l \end{cases}$$

The fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n . \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n . \sin\frac{n\pi x}{l} \quad \text{---} (1)$$

To find a_0 :

$$a_0 = 1/l \int_0^l f(x)\,dx + 1/l \int_l^{2l} f(x)\,dx$$

$$= 1/l \int_0^l kx\,dx + 1/l \int_l^{2l} 0.dx$$

$$= \frac{k}{l}\left(\frac{x^2}{2}\right)_0^l + \frac{1}{l}(0)$$

$$= \frac{k}{l}\left(\frac{l^2}{2} - 0\right) \Rightarrow \frac{kl^2}{2l}$$

$$= \frac{kl}{2}$$

To find a_n :

$$a_n = \frac{1}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx + \frac{1}{l}\int_l^{2l} f(x)\cos\frac{n\pi x}{l}.dx$$

$$= 1/l \int_0^l kx.\cos\frac{n\pi x}{l}.dx + \frac{1}{l}\int_l^{2l} 0.\cos\frac{n\pi x}{l}.dx$$

$$= \frac{k}{l}\int_0^l x.\cos\frac{n\pi x}{l}.dx + 0$$

$u = x$ $\quad dv = \cos\frac{n\pi x}{l}\cdot dx$

$u' = 1$

$u'' = 0$ $\quad v = \frac{\sin\frac{n\pi x}{l}}{n\pi/l} \Rightarrow v_1 = -\frac{\cos(n\pi \cdot x/l)}{(n\pi/l)^2}$

$$\int u\,dv = uv - u'v_1 + \cdots$$

$$a_n = \frac{K}{l}\left[x\left(\frac{\sin(n\pi x/l)}{n\pi x/l}\right) - 1\cdot\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{(n\pi x/l)^2}\right)\right]_0^l$$

$$= \frac{K}{l}\left[\frac{\cos(n\pi x/l)}{(n\pi x/l)^2}\right]_0^l$$

$$= \frac{K}{l}\left[\frac{\cos(n\pi l/l)}{(n\pi)^2} - \frac{\cos(n\pi 0/l)}{(n\pi)^2}\right]$$

$$= \frac{K}{l}\left[\frac{l^2}{1}\times\frac{\cos n\pi}{(n\pi)^2} - \frac{\cos 0}{(n\pi)^2}\times\frac{l^2}{1}\right].$$

$$= \frac{K}{l}\left(\frac{l^2((-1)^n - 1)}{n^2\pi^2}\right)$$

To find b_n:

$$b_n = \frac{1}{l}\int_0^l f(x)\cdot\sin\frac{n\pi x}{l}\,dx + \frac{1}{l}\int_l^{2l} f(x)\cdot\sin\frac{n\pi x}{l}\cdot dx$$

$$= \frac{1}{l}\int_0^l kx\cdot\sin\frac{n\pi x}{l}\cdot dx + \frac{1}{l}\int_0^{2l} 0\cdot\sin\frac{n\pi x}{l}\cdot dx$$

$$= \frac{K}{l}\int_0^l x\cdot\sin\frac{n\pi x}{l}\cdot dx + 0$$

$u = x$ $\quad dv = \sin\frac{n\pi x}{l}$

$u' = 1$ $\quad v = \frac{-\cos\frac{n\pi x}{l}}{n\pi/l} \Rightarrow v_1 = \frac{-\sin(n\pi x/l)}{(n\pi/l)^2}.$

$$b_n = \frac{k}{l}\left[x\left(-\frac{\cos(n\pi x/l)}{n\pi/l}\right) - 1\left(-\frac{\sin(n\pi x/l)}{(n\pi/l)^2}\right)\right]_0^l$$

$$= \frac{k}{l}\left[x \cdot \frac{-\cos(n\pi x/l)}{n\pi/l}\right]_0^l$$

$$= \frac{k}{l}\left[l - \frac{\cos(n\pi l/l)}{n\pi/l} - 0\right]$$

$$= \frac{k}{l}\left[l^2 - \frac{(-1)^n}{n\pi}\right]$$

$$b_n = \frac{-kl(-1)^n}{n\pi}.$$

$$a_0 = \frac{kl}{2}$$

$$a_n = \begin{cases} 0 & , \text{if } n \text{ is even} \\ -2kl/n^2\pi^2 & \text{if } n \text{ is odd.} \end{cases}$$

$$b_n = \begin{cases} -kl/2n, & \text{if } n \text{ is even} \\ kl/2\pi & \text{if } n \text{ is odd.} \end{cases}$$

Sub in (1)

$$f(x) = \frac{kl/2}{2} + \sum_{n=1}^{\infty} \frac{-2kl}{n=1} \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} \frac{-k(-1)^n}{n\pi} \cdot \sin\frac{n\pi x}{l}$$

$$f(x) = \frac{kl}{4} + \left(-\frac{2kl}{\pi^2}\right) \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\frac{n\pi x}{l} - \frac{k}{n} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\frac{n\pi x}{l}$$

$$f(x) = \frac{kl}{4} - \frac{2kl}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\frac{n\pi x}{l} - \frac{k}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cdot \sin\frac{n\pi x}{l}.$$

3) A function is defined as follows $f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 \le x \le \pi \end{cases}$.

Deduce that $\sum_{n=1}^{\infty} \frac{1}{(2x-1)^2} = \frac{\pi^2}{8}$.

Given, $f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 \le x \le \pi \end{cases}$

$\pi \ge x > 0$

$0 \ge x \ge -\pi$

$f(x) = f(-x)$. $\therefore$ it is an even function.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot dx$$

$$= 2/\pi \int_0^{\pi} x\, dx \Rightarrow \frac{2}{\pi} \left[\frac{x^2}{2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \cdot \frac{\pi^2}{2}$$

$$\boxed{a_0 = \pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \cos nx\, dx$$

$u = x$ $dv = \cos nx$

$u' = 1$ $v_1 = \frac{\sin nx}{n}$

$u'' = 0$ $v_2 = \frac{-\cos nx}{n^2}$

$$a_n = \frac{2}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - 1 \left(-\frac{\cos nx}{n^2} \right) + 0 \right]_0^{\pi}$$

$$= 2/\pi \left[x \left(\frac{\sin nx}{n} \right) + \frac{\cos nx}{n^2} \right]_0^{\pi}$$

$$= 2/\pi \left[\frac{(-1)^n}{n^2} - \frac{1}{n^2} \right]$$

$$a_n = \frac{2[(-1)^n - 1]}{n^2 \pi}$$

$$a_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{-4}{n^2\pi} & \text{if } n \text{ is odd.} \end{cases}$$

Sub a_0 & a_n in eq (1)

$$f(x) = \frac{\pi}{2} + \sum_{n=1,3,5}^{\infty} \frac{-4}{n^2\pi} \cos nx$$

$$= \pi/2 - 4/\pi \sum_{n=1,3,5} \frac{-1}{n^2} \cos nx$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{1}{1^2}\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \dots\right]$$

if $x = 0$

$$0 = \frac{\pi}{2} - \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\frac{\pi}{2} = \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\pi^2/8 = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad \text{--- (2)}$$

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \quad \text{--- (3)}$$

$$\frac{1}{(2(1)-1)^2} + \frac{1}{(2(2)-1)^2} + \frac{1}{(2(3)-1)^2} + \dots = \frac{\pi^2}{8}$$

$$1/1^2 + 1/3^2 + 1/5^2 + \dots = \frac{\pi^2}{8}$$

from (2) & (3)

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}}$$

(7)

4) Find the fourier series with Period 2 to represent $f(x) = 2x - x^2$ in the range $(0,2)$.

Given $f(x) = 2x - x^2$ $(0,2) \Rightarrow (0, 2l)$.

Here $2l = 2$

$\boxed{l = 1}$

$c + 2l = 2$

$\boxed{c = 0}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} \quad \text{--- (1)}$$

To find a_0:

$$a_0 = 1/l \int_0^2 f(x).dx$$

$$a_0 = \frac{1}{1}\int_0^2 (2x - x^2)\,dx$$

$$\Rightarrow \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = \left[\left(4 - \frac{8}{3}\right) - (0)\right]$$

$$\Rightarrow \frac{12-8}{3} = 4/3$$

$\therefore \boxed{a_0 = 4/3}$

$$a_n = 1/l \int_0^2 f(x) \cos\frac{n\pi x}{l}.dx$$

$$a_n = 1/1 \int_0^2 (2x - x^2) \cos\frac{n\pi x}{1}.dx$$

By Bernaulis formula:

$$\int u\,dv = uv - u'v_1 + u''v_2 - \cdots$$

$u = 2x - x^2$ $\qquad dv = \cos n\pi x$

$u' = 2 - 2x$ $\qquad v = \sin n\pi x$

$u'' = -2$ $\qquad v_1 = \frac{-\cos n\pi x}{(n\pi)^2} \Rightarrow v_2 = \frac{-\sin \pi x}{(n\pi)^3}$

$$a_n = \left[(2x-x^2)\frac{\sin n\pi x}{n\pi} - (2-2x)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right) + (-2)\left(\frac{\sin n\pi x}{(n\pi)^3}\right)\right]_0^2$$

$$a_n = \left[0 - 2\cdot\frac{(-1)^{2n}}{(n\pi)^2} + 0 - \frac{2}{(n\pi)^2}\right]$$

$$a_n = \frac{-4}{(n\pi)^2}.$$

To find b_n:

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}\cdot dx$$

$$= \frac{1}{1}\int_0^2 (2x-x^2)\sin n\pi x$$

$u = 2x - x^2$ $\quad$ $dv = \sin n\pi x$

$u' = 2 - 2x$ $\quad$ $v = -\frac{\cos n\pi x}{n\pi} \Rightarrow v_1 = -\frac{\sin n\pi x}{(n\pi)^2}$

$u'' = -2$

$$b_n = \left[(2x-x^2)\left(-\frac{\cos n\pi x}{n\pi}\right) - (2-2x)\left(-\frac{\sin n\pi x}{(n\pi)^2}\right)\right] + (-2)\cdot\frac{\cos n\pi x}{(n\pi)^3}.$$

$$b_n = \left[0 + 0\cdot\frac{-2}{(n\pi)^3}\cdot\left(\frac{-2}{(n\pi)^3}\right)\right]$$

from ①

$$f(x) = \frac{4/3}{2} + \sum_{n=1}^{\infty}\cdot\frac{-4}{(n\pi)^2}\cdot\cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty}(0)\cdot\sin\frac{n\pi x}{l}$$

$$\boxed{2x - x^2 = \frac{2}{3} - \frac{4}{\pi^2}\cdot\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\cos n\pi x.}$$

5) Find the Fourier series expansion of $f(x) = x+x^2$ in $-1 < x < 1$.

Given,

$f(x) = x + x^2$ in $(-1, 1)$

Here, $l = 1$

the fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right)$$

Here, $l = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos n\pi x + b_n \cdot \sin n\pi x \quad \text{—} \quad (1)$$

To find a_0:

$$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \cdot dx$$

$$= 1/l \int_{-l}^{l} (x+x^2)\, dx \Rightarrow \left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{2} + \frac{1}{3}\right)$$

$$a_0 = 1/3 + 1/3 \Rightarrow 2/3$$

$$\boxed{a_0 = 2/3}$$

To find a_n:

$$a_n = 1/l \int_{-l}^{l} f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$$

$$a_n = 1/l \int_{-1}^{1} (x+x^2) \cos\left(\frac{n\pi x}{l}\right) dx$$

$u = x + x^2$ $\qquad$ $dv = \cos n\pi x \cdot dx$

$u' = 1 + 2x$ $\qquad$ $v = \frac{\sin n\pi x}{n\pi} \Rightarrow v_1 = -\frac{\cos n\pi x}{(n\pi)^2}$

$$\int u\,dv = uv - u'v_1 + u''v_2 - \cdots$$

$$a_n = \left[(x+x^2)\cdot\frac{\sin n\pi x}{n\pi} - (1+2x)\cdot\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + 2\cdot\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_{-1}^{1}$$

$$a_n = \left[(1+2x)\cdot\frac{\cos n\pi x}{n^2\pi^2}\right]_{-1}^{1}$$

$$= \left((1+2)\cdot\frac{\cos n\pi}{n^2\pi^2}\right) - \left((1-2)\cdot\frac{\cos n\pi}{n^2\pi^2}\right)$$

$$= \frac{3(-1)^n}{n^2\pi^2} + \frac{(-1)^n}{n^2\pi^2}$$

$$\boxed{a_n = \frac{4(-1)^n}{n^2\pi^2}}$$

To find b_n:

$$b_n = 1/l\int_{-l}^{l} f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$b_n = \int_{-l}^{l}(x+x^2)\cdot\sin(n\pi x)\,dx$$

$u = x+x^2$ $\quad dv = \sin n\pi x\,dx$

$u' = 1+2x$ $\quad v = -\frac{\cos n\pi x}{n\pi} \Rightarrow v_1 = -\frac{\sin \pi x}{n^2\pi^2} \Rightarrow v_2 = \frac{\cos n\pi x}{n^3\pi^3}$

$$b_n = \left[(x+x^2)\cdot\left(-\frac{\cos n\pi x}{n\pi}\right) - (1+2x)\cdot\left[-\frac{\sin \pi x}{n^2\pi^2}\right] + 2\cdot\left[\frac{\cos n\pi x}{n^3\pi^3}\right]\right]_{-1}^{1}$$

$$= \left[(x+x^2)\left[-\frac{\cos n\pi x}{n\pi}\right] + 2\cdot\left(\frac{\cos n\pi x}{n^3\pi^3}\right)\right]_{-1}^{1}$$

$$= \left[(1+1)\left[-\frac{\cos n\pi}{n\pi}\right] + 2\cdot\frac{\cos n\pi}{n^3\pi^3}\right] - \left[-(-1+1)\cdot\frac{\cos n\pi}{n\pi} + 2\cdot\frac{\cos n\pi}{n^3\pi^3}\right]$$

$$b_n = \frac{-2(-1)^n}{n\pi}.$$

Substitute $a_0 = 2/3$, $a_n = \frac{4(-1)^n}{n^2\pi^2}$, $b_n = \frac{-2(-1)^n}{n\pi}$ in (i)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos n\pi x + b_n \sin n\pi x\right]$$

$$= \frac{2/3}{2} + \sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^2\pi^2}\cos n\pi x - \frac{2(-1)^n}{n\pi}\cdot \sin n\pi x\right]$$

$$f(x) = 1/3 + \sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^2\pi^2}\cdot \cos n\pi x - \frac{2(-1)^n}{n\pi}\sin n\pi x\right].$$

Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed-to-be University under section 3 of UGC Act, 1956)
BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
SCHOOL OF AERONAUTICAL ENGINEERING

NBA Accredited

CONSOLIDATED INTERNAL MARK STATEMENT

Program: Aerospace Engineering
Sub.Code/Name: U20MABT03-TBVP

S.No	Register No	Student Name	CLA-1 (10 Marks)	CLA-2 (15 Marks)	CLA-3 (15 Marks)	CLA-4 (10 Marks)	INTERNAL (50 Marks)
1	U21AS001	AMEEN NOORDEEN A	8	8	8	10	34
2	U21AS002	BOLLINA RAM PRANAV TEJ	7	11	11	10	39
3	U21AS003	CHETAN SINGH	10	15	15	10	50
4	U21AS004	CHINTALAPUDI KUMAR SATYA CHANDRAMOULI	9	13	13	10	45
5	U21AS005	CHIRRAVURI BHANUTEJA	9	15	15	10	49
6	U21AS006	CYNTHIA RAI	10	15	15	10	50
7	U21AS007	DANIEL INFANT RAJ A	10	15	15	10	50
8	U21AS008	DHANUSH S	10	15	15	10	50
9	U21AS010	ETLAM DHEERAJ REDDY	9	13	12	10	44
10	U21AS011	GOKULRAJ SRAVANTHI	9	11	11	10	41
11	U21AS012	HARESHSIVA M	9	12	12	10	43
12	U21AS013	JASHEEMA BEGAM S	9	14	13	10	46
13	U21AS014	KAILASH D	10	15	15	10	50
14	U21AS015	KISHORE C S	9	13	13	10	45
15	U21AS016	MADHAN RAJ W	9	11	12	10	42
16	U21AS017	MANOKARAN N	9	12	13	10	44
17	U21AS018	MERLYN REJI	9	13	13	10	45
18	U21AS019	MOHAMED FAZIL M	9	13	13	10	45
19	U21AS020	PHANI SRI NAGA DURGA ARAVA	10	15	15	10	50
20	U21AS021	POTHEESWARAN E	9	11	12	10	42
21	U21AS022	PUNDI THATHARAO	9	11	12	10	42
22	U21AS023	PUPPALA LAKSHMAN KARTHIK	7	9	10	10	36
23	U21AS024	SAKTHI KUMAR A	9	14	14	10	47
24	U21AS025	SANDHIYA S	9	13	13	10	45
25	U21AS026	SANTHOSH R	6	9	9	10	34
26	U21AS027	SATHEESH S	9	13	13	10	45
27	U21AS028	SENTHILNATHAN A G S	10	15	15	10	50
28	U21AS029	SUBIN SAMUEL A	8	11	11	10	40
29	U21AS030	SUJIRTHA P	9	13	13	10	45
30	U21AS031	THOROTU SAHITH	8	10	10	10	38
31	U21AS032	VENKATA GURUDATTA SARMA P	10	15	15	10	50
32	U21AS033	SANSKAR SINGH	9	15	14	10	48
33	U21AS034	SHAIK MOHAMMAD SADIK	7	9	8	10	34
34	U21AS035	PRIYADHARSHINI M	8	10	10	10	38
35	U21AS036	SARIPALLI ROHITH	8	9	9	10	36
36	U21AS037	ARUN MOZHI VARMA S J	5	8	7	10	30
37	U21AS038	HIMANSHU SAI PRAKASH YADAV	10	15	15	10	50
38	U21AS039	V.SANDEEP KUMAR	5	10	10	10	35
39	U21AS040	BITTU KUMAR	9	12	11	10	42
40	U21AS041	G SIRISHA	0	0	0	0	0
41	U21AS042	PRINCE KUMAR	9	12	11	10	42
42	U20AS030	SOWMIYA K	10	13	13	10	46

SUBJECT IN-CHARGE **HOD-AEROSPACE**

ASSIGNMENT – I

DEPARTMENT OF IT

TRANSFORMS AND BOUNDARY VALUE PROBLEMS

1 Find $Z^{-1}\left[\frac{z^2}{(z+2)(z^2+4)}\right]$

$$F(z) = \frac{z^2}{(z+2)(z^2+4)}$$

$$\frac{F(z)}{z} = \frac{z}{(z+2)(z^2+4)} = \frac{A}{z+2} + \frac{Bz+C}{z^2+4}$$

Put $z = 2$

$-2 = A(8) + 0$

$A = -1/4$

Put $z = 0$

$0 = A(0+4) + C(2)$

$0 = 4A + 2C$

$= 4\left(-\frac{1}{4}\right) + 2C$

$C = \frac{1}{2}$

Eq on both sides z^2

$0 = A + B$

$0 = -\frac{1}{4} + B$

$B = \frac{1}{4}$

$$\frac{F(z)}{z} = \frac{-\frac{1}{4}}{z+2} + \frac{\frac{1}{4}z + \frac{1}{2}}{z^2+4}$$

$$Z[F(z)] = \frac{-1}{4}\left[\frac{z}{z+2}\right] + \frac{1}{4}\left[\frac{z^2}{z^2+4}\right] + \frac{1}{2}\left[\frac{z}{z^2+4}\right]$$

Taking z^{-1} on both sides

$$z^{-1}[F(z)] = \frac{-1}{4} z^{-1}\left[\frac{z}{z+2}\right] + \frac{1}{4} z^{-1}\left[\frac{z^2}{z^2+4}\right] + \frac{1}{4} z^{-1}\left[\frac{z^2}{z^2+4}\right]$$

$$= \left[\frac{-1}{4}(-2)^n + \frac{1}{4}\left(2^n \cos\frac{n\pi}{2}\right) + 2^n \sin\frac{n\pi}{2}\right]$$

2) Find $z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$ using convolution method

$$z^{-1}\left[\frac{z^2}{(z-a)^2}\right] = z^{-1}\left[\frac{z}{z-a} \cdot \frac{z}{z-a}\right]$$

$$= a^n * a^n$$

$$= \sum_{r=0}^{n} (a)^r \times (a)^{n-r}$$

$$= a^n + a \cdot a^{n-1} + \dots a^n$$

$$= a^n + a^{n-1+1} \dots a^n$$

$$= a^n + a^n \dots a^n$$

$$z^{-1}\left[\frac{z^2}{(z-a)^2}\right] = (n+1)a^n$$

3) Using z Transform $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$$

Apply z transform both sides

$$z[y_{n+2}] + z[6y_{n+1}] + z[9y_n] = z[2^n]$$

$$\{z^2 y(z) - z^2 y_0 - z y_1\} - 6[z(y(z)) - z y_0] + 9y(z) = \frac{z}{z-2}$$

$$y(z)[z^2 - 6z + 9] = \frac{z}{z-2}$$

$$y(z) = \frac{z}{(z-2)(z-3)^2}$$

$$= \frac{1}{(z-2)(z-3)^2} = \frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{(z-3)^2}$$

$$= \frac{A(z-3)^2 + B(z-2)(z-3) + C(z-2)}{(z-3)^2(z-2)}$$

$$1 = A(z-3)^2 + B(z-2)(z-3) + C(z-2)$$

put $z=2$ $\quad 1 = A(2-3)^2$

$A=1$

put $z=3$

$1 = 0 + 0 + C(1) \quad = C = 1$

$z = 0 \quad 1 = 1(9) + B(-2)(-3) + 1(-2)$

$1 = 9 - 2 + 6B$

$B = -6$

$$\frac{y(z)}{z} = \frac{1}{z-2} + \frac{+}{z-3} + \frac{1}{(z-3)^2}$$

$$y(z) = \frac{z}{z-2} - \frac{z}{z-3} + \frac{1}{3} \cdot \frac{3z}{(z-3)^2}$$

$$z^{-1}\{y(z)\} = z^{-1}\left[\frac{z}{z-3}\right] - z^{-1}\left[\frac{z}{z-3}\right] + \frac{1}{3} z^{-1}\left[\frac{3z}{(z-3)^2}\right]$$

$$y_n = 2^n - 3^n + \frac{1}{3} \cdot 3^n \cdot n$$

$$= 2^n + 3^n\left[-1 + \frac{n}{3}\right]$$

---

4) using z transform $y_{x+2} - 5y_{k+1} + 6y_n = 36$ $\quad y(0) = y(1) = 0$

$y_{k+2} - 5y_{n+1} + 6y_n = 36$

Taking z on both sides

$$[y_{k+2}] - 5[z\, y_{k+1}] + 6\, z y_n] = 36z$$

$$= z^2 y(z) - 0 - 0 - 5zy(z - 0 + 6y(z) = 36z$$

$$y(z)\,[z^2 - 5z + 6] = 36z$$

$$y(z) = \frac{36z}{(z-3)(z-2)}$$

$$\frac{y(z)}{z} = \frac{36}{(z-3)(z-2)} = \frac{A}{z-3} + \frac{B}{z-2}$$

$$\frac{36}{(z-3)(z-2)} = \frac{A(z-2) + B(z-3)}{(z-3)(z-2)}$$

$$36 = A(z-2) + B(z-3)$$

$$36 = 6 + B(-1)$$

$$B = -36$$

Put $z = 3$

$$36 = A(3-2) + B(3-3)$$

$$36 = A(1)$$

$$A = 36$$

$$y(z) \quad \frac{36}{z-3} + \frac{36}{z-2}$$

$$= 36 \frac{z}{z-3} - 36 \frac{z}{z-2}$$

$$36 \cdot 3^n - 36 \cdot 2^n$$

$$= \boxed{36 [3^n - 2^n]}$$

5) using inverse transform $\frac{z^3 - 20z}{(z-2)^3(z-4)}$

$$F(z) = \frac{z^3 - 20z}{(z-2)^3(z-4)}$$

$$\frac{F(z)}{z} = \frac{z^2 - 20}{(z-2)^2(z-4)} = \frac{A}{z-4} + \frac{B}{(z-2)} + \frac{C}{(z-2)^3}$$

$$A(z-2)^3 + B(z-2)^2(z-4) + C(z-4) = z^2 - 20$$

Put $z = 4$

$$A(4-2)^3 + B(0) + C(0) = 4^2 - 20$$

$$A(2)^3 = 16 - 20$$

$$8A = 4$$

$$A = \frac{4}{8}\,2$$

$$\boxed{A = 1/2}$$

Put $z = 2$

$$A(1) + B(0) + C(2-4) = 2^2 - 20$$

$$C(-2) = 4 - 20$$

$$C(+2) = +16$$

$$\boxed{C = 8}$$

Put $z = 0$

$$A(0-2)^3 + B(0-2)^2(0-4) + C(0-4) = 0 - 20$$

$$-8A + 16B + C(-4) = -20$$

$$16B - 36 = -20$$

$$16B = 16$$

$$\boxed{B = 1}$$

$$\frac{F(z)}{z} = \frac{1/2}{z-4} + \frac{1}{(z-2)} + \frac{8}{(z-2)^3}$$

$$F(z) = z\left[\frac{1}{2(z-4)}\right] + z\frac{1}{(z-2)} + \frac{8z}{(z-2)^3}$$

$$= \frac{z}{2(z-4)} + \frac{z}{z-2} + \frac{4\cdot 2z}{(z-2)^3}$$

Taking z^{-1} on both sides

$$z^{-1}y(z) = z^{-1}\left[\frac{z}{2(z-4)}\right] + z^{-1}\left[\frac{1}{z-2}\right] + z^{-1}\left[\frac{4\cdot 2z}{(z-2)^3}\right]$$

$$= 2\cdot 4^n + 2^n + 4\cdot 2^n$$

$$= 2\cdot 2^{2n} + 2^n + 2^2\cdot 2^n$$

$$= 2^{n+1} + 2^n + 2^{n+1}$$

ASSIGNMENT

II.

Name: P.Umesh

12-12-2022

Reg No: U21[illegible]T013

Subject: U20MABT03 - TBVP

5/5

① Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| \ge 1 \end{cases}$ as a fourier Integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$ and find the value of $\int_0^\infty \frac{\sin\lambda}{\lambda} d\lambda$

Sol:

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$f(x) = 1 \qquad -1 < x < 1$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} 1 \,(\cos sx + i \sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} \cos sx \, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} \sin sx \, dx$$

$\int_{-1}^{1} \cos sx$ is an even function

$\int_{-1}^{1} \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 \cos sx \, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin sx}{s} \right]_0^1$$

$$F(s) = \frac{2}{\sqrt{2\pi}} \left(\frac{\sin s}{s} \right)$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s}{s}\right] (\cos sx - i \sin sx)\, ds$$

$$= \frac{2}{2\pi} \int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx\, ds - \frac{2i}{2\pi} \int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx\, ds$$

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \cos sx$ is an even function

$\int_{-\infty}^{\infty} \frac{\sin s}{s} \sin sx$ is an odd function

$$= \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin s \cos sx}{s}\, ds$$

put $s = \lambda$

$ds = d\lambda$

$$f(x) = \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda}\, d\lambda$$

put $x = 1$ $f(x) = 1$

$$1 = \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin \lambda}{\lambda} \cos \lambda (1)\, d\lambda$$

$$\frac{\pi}{2} = \int_{0}^{\infty} \frac{\sin \lambda}{\lambda}\, d\lambda$$

Hence proved//

(2) Find the fourier transform of $e^{-a^2x^2}$, $a>0$ Hence show that $e^{-x^2/2}$ is a self reciprocal under the fourier transform

Sol: (i) $F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \, e^{isx} \, dx$ $\quad f(x) = e^{-a^2x^2}$

$a > 0$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} e^{isx} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} (\cos sx + i \sin sx) \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} \cos sx \, dx + \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a^2x^2} \sin sx \, dx$$

$\int_{-\infty}^{\infty} e^{-(ax)^2} \cos sx$ is an even function

$\int_{-\infty}^{\infty} e^{-(ax)^2} \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-(ax)^2} \cos sx \, dx$$

$$\therefore F(s) = \frac{2}{\sqrt{2\pi}} \left[\frac{a}{a^2+b^2} \right]^2 \qquad \left(\because \int_0^{\infty} e^{-ax} \cos bx \, dx = \frac{a}{a^2+b^2} \right)$$

(ii) $e^{-x^2/2}$ is a self reciprocal

Fourier transform

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \, e^{isx} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2} e^{isx} e^{s^2/2} e^{-s^2/2} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-x^2/2 + isx + s^2/2} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-\left(\frac{x-is}{\sqrt{2}}\right)^2} dx \quad \left[\because \frac{-x^2+2isx+s^2}{2} = -\frac{(x-is)^2}{2}\right]$$

let $t = \frac{x-is}{\sqrt{2}}$ $\quad dt = \frac{dx}{\sqrt{2}}$ $\quad dt\sqrt{2} = dx$

$x \to -\infty \quad t \to -\infty$

$x \to \infty \quad t \to \infty$

$$= \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2}\, dt$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} \cdot 2 \int_{0}^{\infty} e^{-t^2} dt \quad \left[\because e^{-t^2} \text{ is an even function}\right]$$

$$= \frac{1}{\sqrt{\pi}} e^{-s^2/2} \cdot 2 \left[\frac{\sqrt{\pi}}{2}\right]$$

$$\therefore F(s) = e^{-s^2/2}$$

③ Find the fourier transform of $f(x) = \begin{cases} a-|x| & \text{for } |x|<a \\ 0, & \text{for } |x|>a>0 \end{cases}$

Soln

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$f(x) = a-x \qquad -a<x<a$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a-x)(\cos sx + i \sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} (a-x)\cos sx\, dx + \frac{i}{\sqrt{2\pi}} \int_{-a}^{a} \sin sx (a-x)\, dx$$

$\int_{-a}^{a} (a-x)\cos sx$ is an even function

$\int_{-a}^{a} (a-x) \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_{0}^{a} (a-x)\cos sx\, dx$$

$$= \frac{2}{\sqrt{2\pi}}\left[(a-x)\frac{\sin sx}{s} + (-1)\frac{\cos sx}{s^2}\right]_0^a$$

$$= \frac{2}{\sqrt{2\pi}}\left[-\frac{\cos sa}{s^2} + \frac{1}{s^2}\right]$$

$$F(s) = \frac{2}{\sqrt{2\pi}}\left[\frac{1-\cos sa}{s^2}\right]$$

Inverse Fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{2}{\sqrt{2\pi}}\left[\frac{1-\cos sa}{s^2}\right](\cos sx - i\sin sx)\, ds$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\cos sx\, ds - \frac{i}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\sin sx\, ds$$

WKT

$\Rightarrow \int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\cos sx$ is an even function

$\int_{-\infty}^{\infty}\left(\frac{1-\cos sa}{s^2}\right)\sin sx$ is an odd function

$$= \frac{2}{\pi}\int_0^{\infty}\left[\frac{1-\cos sa}{s^2}\right]\cos sx\, ds \qquad \left[\because 1-\cos\theta = 2\sin^2\theta/2\right]$$

$$a-x = \frac{2}{\pi}\int_0^{\infty}\frac{2\sin^2\frac{sa}{2}}{s^2}\cos sx\, ds$$

put $x = 0$

$$a = \frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin\frac{sa}{2}}{s}\right)^2 (1)\, ds$$

put $\frac{sa}{2} = t$, $s = \frac{2t}{a}$ $\quad ds = \frac{1}{a}\cdot 2\cdot dt$

$$a = \frac{4}{\pi}\int_0^{\infty}\left(\frac{\sin t}{2\cdot\frac{2t}{a\cdot a}}\right)^2\cdot\frac{1}{a}2\, dt$$

$$a = \frac{4a}{\pi} \cdot \frac{1}{2} \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$$

$$\frac{\pi a}{2a} = \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$$

$$\frac{\pi}{2} = \int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt \quad //$$

(4) Find the fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| \geq 1 \end{cases}$

Hence show that $\int_0^\infty \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2} \, ds = \frac{3\pi}{16}$. Also show that $\int_0^\infty \frac{(x\cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}$

Soln: (i) Fourier transform

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2)(\cos sx + i\sin sx) \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \cos sx \, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-x^2) \sin sx \, dx$$

$\int_{-1}^{1} (1-x^2) \cos sx$ is an even function

$\int_{-1}^{1} (1-x^2) \sin sx$ is an odd function

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 (1-x^2) \cos sx \, dx$$

$$= \frac{2}{\sqrt{2\pi}} \left[(1-x^2) \frac{\sin sx}{s} - 2x \frac{\cos sx}{s^2} + 2\frac{\sin sx}{s^3} \right]_0^1$$

$$= \frac{2}{\sqrt{2\pi}} \left[-\frac{2\cos s}{s^2} + \frac{2\sin s}{s^3} \right]$$

$$F(s) = \frac{4}{\sqrt{2\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right]$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{4}{\sqrt{2\pi}}\left[\frac{\sin s - s\cos s}{s^3}\right](\cos sx - i\sin sx)\, ds$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s + s\cos s}{s^3}\right)\cos sx\, ds - \frac{2i}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin s - s\cos s}{s^3}\right)\sin sx\, ds$$

$\int_{-\infty}^{\infty} \frac{\sin s - s\cos s}{s^3} \cos sx$ is an even function

$\int_{-\infty}^{\infty} \frac{\sin s - s\cos s}{s^3} \sin sx$ is an odd function

$$1 - x^2 = \frac{4}{\pi}\int_0^{\infty} \frac{\sin s - s\cos s}{s^3}\cos sx\, ds \quad \text{-②}$$

sub for eq ①

put $x = \frac{1}{2}$ is continuous

$$1 - \left(\frac{1}{2}\right)^2 = \frac{4}{\pi}\int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\frac{3}{4} \times \frac{\pi}{4} = \int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds$$

$$\frac{3\pi}{16} = \int_0^{\infty}\frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\, ds \,//$$

(ii) Using parseval's identity

$$\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |F(s)|^2\, ds$$

$$\int_{-1}^{1} (1-x^2)^2\, dx = \int_{-\infty}^{\infty}\left(\frac{4}{\sqrt{2\pi}}\left(\frac{\sin s - s\cos s}{s^3}\right)\right)^2 ds$$

$$2\int_0^1 (1-x^2)^2\,dx = 2\cdot\frac{16}{2\pi}\int_0^\infty \left(\frac{\sin s - s\cos s}{s^3}\right)^2 ds$$

Put $s = x$

$ds = dx$

$$2\int_0^1 (1^2 + x^4 - 2x^2)\,dx = \frac{16}{\pi}\int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$2\left[x + \frac{x^5}{5} - 2\frac{x^3}{3}\right]_0^1 \times \frac{\pi}{16} = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{16}{15} \times \frac{\pi}{16} = \int_0^\infty \left(\frac{\sin x - x\cos x}{x^3}\right)^2 dx$$

$$\frac{\pi}{15} = \int_0^\infty \frac{(\sin x - x\cos x)^2}{x^6}\,dx$$

$$\frac{\pi}{15} = \int_0^\infty - \frac{(x\cos x - \sin x)^2}{x^6}\,dx \quad //$$

(5) Find the fourier sine transform of $\frac{1}{x}$

Soln

$$F_s(s) = \frac{1}{x}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{1}{x}\sin sx\,dx$$

~~$2\sqrt{\frac{2}{\pi}}\left[\log x\left(-\frac{\cos sx}{s}\right)\right]_0^\infty$~~

Put $sx = y \qquad x = \frac{y}{s} \qquad dx = \frac{dy}{s}$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin y}{\frac{y}{s}}\cdot\frac{dy}{s}$$

$$\boxed{\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}}$$

$$= \sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin y}{y}\cdot dy = \sqrt{\frac{2}{\pi}}\left[\frac{\pi}{2}\right]$$

$$F(s) = \sqrt{\frac{\pi}{2}} \text{ or } \frac{\sqrt{\pi}}{\sqrt{2}} \quad //$$

Sl.NO	Roll. No	Marks (50)	Is Absent
1	U21IT001	31	NO
2	U21IT002	32	NO
3	U21IT003	31	NO
4	U21IT004	35	NO
5	U21IT005	33	NO
6	U21IT006	31	NO
7	U21IT007	42	NO
8	U21IT008	45	NO
9	U21IT009	30	NO
10	U21IT010	31	NO
11	U21IT011	45	NO
12	U21IT012	31	NO
13	U21IT013	0	YES
14	U21IT014	32	NO
15	U21IT015	34	NO
16	U21IT016	32	NO
17	U21IT017	33	NO
18	U21IT018	32	NO
19	U21IT019	32	NO
20	U21IT020	35	NO
21	U21IT021	0	YES
22	U21IT022	32	NO
23	U21IT023	35	NO
24	U21IT024	32	NO
25	U21IT025	32	NO
26	U21IT026	33	NO
27	U21IT027	47	NO
28	U21IT028	30	NO
29	U21IT029	35	NO
30	U21IT030	31	NO
31	U21IT031	35	NO
32	U21IT032	34	NO
33	U21IT033	32	NO
34	U21IT034	35	NO
35	U21IT035	34	NO
36	U21IT036	33	NO
37	U21IT037	34	NO
38	U21IT038	33	NO
39	U21IT039	33	NO

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESE

B.TECH, II YEAR - 2022 - 2023 (SEM - III)

INTERNAL MARKS - CONSOLIDATED MARK STATEME

U20MABT03 - TRANSFORMS & BOUNDARY VALUE PROBLE

Name of Students	CLA - I	CLA - II
AJAI SURIYA V	28	25
AKASHKUMAR V	25	30
ANANDKUMAR M	27	26
ANNIE NISHITA I	36	29
ATHAVAN V	26	28
BALAJI N	28	26
BATTU SHIVA KRISHNA	41	35
BHUVANESHWARI D	40	44
CHARULATHA P	25	25
DENISH I	26	27
DILFAR NISHA A	44	40
DINESH S	25	28
GINJALA MANJITH REDDY	A	A
GOLLA ALLAIAH	26	31
GUDEBOINA HARSHAVARDHAN	30	27
ITHRISH M	28	25
JAI PRAKASH P	27	29
JESSICA RUPAVATHI M	30	25
JOSHVA A	30	26
JUTTU ARAVIND	31	30
KAKANI BHANU SHANKAR	A	A
KAKANI NIKESH CHOWDARY	26	26
KARTHIKEYAN V	35	29
KATHIRVEL S	30	26
KOLLA VIJAYKUMAR	34	25
KUDITHI SAINITHEESH REDDY	35	27
MADHUPRIYA C	49	40
MANDADAPU YASHWANTH	26	25
MANO RANJAN	33	26
MOHAMEDKADHARUSAIN M	30	25
NANDIGAMA SUPRAJA	33	26
NELAKURTHI VENKATA PHANINDRA	32	28
PANGULURI ANIL KUMAR	30	25
PRANAV KUMAR S	31	28
PRAVEEN KANTH G	31	25
PREMKUMAR S	32	26
PRIYADHARSHINI A	30	29
RAVIPATI DHARMA TEJA	36	26
RISHU KUMAR	35	28

ARCH

NT
MS - IT

CLA - III	CLA - I	CLA - II	CLA - III	CLA - IV
26	6	8	8	10
25	5	9	8	10
27	5	8	8	10
30	7	9	9	10
30	5	8	9	10
25	6	8	8	10
43	8	11	13	10
45	8	13	14	10
26	5	8	8	10
25	5	8	8	10
46	9	12	14	10
25	5	8	8	10
A	0	0	0	0
26	5	9	8	10
32	6	8	10	10
28	6	8	8	10
29	5	9	9	10
28	6	8	8	10
27	6	8	8	10
33	6	9	10	10
A	0	0	0	0
30	5	8	9	10
31	7	9	9	10
28	6	8	8	10
25	7	8	8	10
26	7	8	8	10
50	10	12	15	10
25	5	8	8	10
35	7	8	11	10
26	6	8	8	10
35	7	8	11	10
30	6	8	9	10
29	6	8	9	10
33	6	8	10	10
33	6	8	10	10
28	6	8	8	10
32	6	9	10	10
26	7	8	8	10
26	7	8	8	10

AEROSPACE

Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed-to-be University under section 3 of UGC Act, 1956)
(Vide Notification No. F.9-5/2000 - U.3, Ministry of Human Resource Development, Govt. of India, dated 4th July 2002)

NATIONAL BOARD OF ACCREDITATION Accredited

BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
SCHOOL OF AERONAUTICAL ENGINEERING

CONTINUOUS LEARNING ASSESSMENT - II
U20MABT03–TRANSFORMS & BOUNDARY VALUE PROBLEM

Date	:	23/11/2022
Academic Year / Semester	:	2022-2023/ODD
Duration	:	1:30 Hours
Maximum Marks	:	50

Q.No	Questions	Weightage	CO	Bloom's Level
	Part – A (6×2=12 Marks) Answer All Questions			
1	Write all possible solutions of 1-D wave equation.	2	CO 3	1
2	State Fourier law of heat conduction.	2	CO 3	1
3	Define 2-D heat flow equation.	2	CO 3	1
4	State the Fourier Transform pair.	2	CO 4	1
5	Define Convolution theorem and Parseval's identity for Fourier transform.	2	CO 4	1
6	Write the Fourier cosine transform pair of formulae.	2	CO4	1
	Part – B (3×6=18 Marks) Answer either (a) or (b)			
7(a)	In steady state conditions derive the solution of one dimensional heat flow equation.	6	CO 3	2
7(b)	Classify the partial differential equations (i) $y^2U_{xx} + x^2U_{yy} = 0$. (ii) $4U_{xx} + 4U_{xy} + U_{yy} + 2U_x - U_y = 0$.			
8(a)	If F(s) is the Fourier transform of f(x), find the Fourier transform of F (ax)where a>0.	6	CO 4	2
8(b)	State and Prove Modulation Theorem.			
9(a)	Show that $e^{\frac{-x^2}{2}}$ is a self reciprocal with respect to Fourier transform	6	CO 4	3
9(b)	Find the Fourier sine and cosine transform of e^{-ax} ,a>0			
	Part – C (2×10=20 Marks) Answer either (a) or (b)			
10(a)	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x, 0) = y_0 sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from its position, find the displacement y at any time and any distance from the end $x = 0$.	10	CO 3	3
10(b)	A square plate is bounded by the lines $x = 0, y = 0, x = 20, y = 20$.Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x(20 - x)$, $0 < x < 20$ while other two edges are kept at $0C$. Find the steady temperature distribution in the plate.			

11(a)	Find the Fourier transform of f(x) = 1-|x|, |x| <1 0, |x| > 1 and hence find the value of $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$	10	CO 4	3
11(b)	Find the Fourier transform of f(x) = 1- x^2 ,|x| < 1 0 , |x| > 1 and hence prove that $\int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3} \right)^2 ds = \frac{\pi}{15}$			

CO	Weightage
CO3	22
CO4	28
Total	50

Prepared by	Staff Name Dr. Ch. Nagalakshmi	Signature
Verified by	HoD Dr. S.V Manemaran	Signature

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INSTITUTE OF HIGHER EDUCATION AND RESEARCH

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BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY

SCHOOL OF AERONAUTICAL ENGINEERING

NBA NATIONAL BOARD ACCREDITATION Accredited

CONTINUOUS LEARNING ASSESSMENT - II

U20MABT03–TRANSFORMS & BOUNDARY VALUE PROBLEM

Date	:	28/12/2022
Academic Year / Semester	:	2022-2023/ODD
Duration	:	1:30 Hours
Maximum Marks	:	50

Q.No	Questions	Weightage	CO	Bloom's Level
	Part – A (6×2=12 Marks) Answer All Questions			
1	Find the general solution of $\frac{\partial^2 z}{\partial x^2} = 0$.	2	CO 2	2
2	Form a partial differential equation by eliminating arbitrary constants from $Z = ax + by + a^2 + b^2$.	2	CO 2	2
3	Define one sided Z- transform and inverse Z-transform	2	CO 5	1
4	Write the Z transform of '1' and' $(-1)^n$ '	2	CO 5	2
5	State and prove change of scale property	2	CO 5	1
6	Write the Z transform of $\frac{1}{n}$ and $\frac{1}{n+1}$	2	CO5	1
	Part – B (3×6=18 Marks) Answer either (a) or (b)			
7(a)	Form the PDE by eliminating the arbitrary function $\emptyset$ from $\emptyset(x^2 + y^2 + z^2, x + y + z) = 0$	6	CO 2	2
7(b)	Find the Complete integral of the PDE $Z = px + qy + p^2 + q^2$.			
8(a)	Solve p x + q y = z	6	CO 2	2
8(b)	Solve $(mz - ny)p + (nx - lz)q = ly - mx$			
9(a)	Find the Z transform of(i) (n+1)(n+2) (ii) $(n+1)^2$	6	CO 2	3
9(b)	Find the Z transform of (i) n (ii) na^n			
	Part – C (2×10=20 Marks) Answer either (a) or (b)			
10(a)	Solve $(D^2 - DD' - 20D'^2)z = e^{5x+y} + \sin(4x - y)$.	10	CO 5	3
10(b)	Slove $(D^2 + 2DD' + D'^2)z = x^2 y$			
11(a)	Find $Z^{-1}\left\{\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}\right\}$ by partial fraction method	10	CO 5	3
11(b)	Find the Z-transform of $\cos n\theta$ and hence find $r^n \cos n\theta$			

TRANSFORM AND PARTIAL DIFFERENTIAL EQUATION.

NAME : MERLYN REJI

REG NO : U21AS018

DATE : 16/11/22

YEAR : II nd

DEPT : AEROSPACE

1) i) WRITE THE ONE DIMENSIONAL WAVE EQUATION AND WRITE ALL THE POSSIBLE SOLUTION OF WAVE EQUATION

One dimensional wave equation is,

$$\frac{d^2u}{dt^2} = \frac{a^2 d^2u}{dx^2}$$

Here $a^2 = \frac{T}{m} = \frac{\text{Tension}(T)}{\text{mass}(m)}$

The various solution of one dimensional wave equation.

1) $y(x,t) = (C_1 e^{px} + ce^{-px})(C_3 e^{apt} + C_4 e^{-apt})$

2) $y(x,t) = (C_5 \cos px + C_6 \sin px)(C_7 \cos apt + C_8 \sin apt)$

3) $y(x,t) = (C_9 x + C_{10})(C_{11} t + C_{12})$

Correct solution of the one dimensional wave equation out of all the possible solutions we can choose the correct solution as follows.

Since we are dealing with problems on vibration of strings. The solution should be a periodic function. And the solution must involves trigonometric terms like sines and cosines.

Therefore the correct solution of the wave equation is

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$$

ii) FOURIER'S LAW OF THERMAL CONDUCTION (OR) CONDUCTIVITY:

The fourier law of thermal conduction states that the rate of heat transfer through a material is proportional to the negative gradient (-ve) in the temperature and the area of the surface through which the heat flows.

iii) WRITE THE ONE-DIMENSIONAL HEAT FLOW EQUATION AND WRITE ALL THE POSSIBLE SOLUTION OF HEAT EQUATION

One dimensional heat equation is given by,

$$\frac{\partial u}{\partial t} = \frac{c^2 \partial^2 u}{\partial x^2}$$

Where $c^2 = k/\rho c$

Here, k - Thermal Conductivity

ρ - Density

c - Specific heat Capacity

where $c^2 = k/\rho c$

Here, In steady state $\frac{\partial u}{\partial t} = 0$

$\therefore$ The two dimensional heat flow becomes

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial^2 y} = 0$$

which is known as Laplace Equation

The various possible solutions of Laplace Equation in 2D heat flow equation are,

1. $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$
2. $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$
3. $u(x,y) = (Ax+B)(Cy+D)$

If the non-zero boundary conditions are along the x-axis are a line parallel to x-axis.

(i.e)., $u(x,0) = u(x,l) = f(x)$.

The Correct solution is

$u(x,y) = (A\cos px + B\sin px) \cdot (Ce^{py} + De^{-py})$

The various possible solutions of one dimensional Heat equations are

1. $U(x,t) = (Ae^{px} + Be^{-px})e^{apt}$
2. $u(x,t) = (A\cos px + B\sin px) \cdot e^{-a^2p^2t}$
3. $u(x,t) = Ax+B$

Since 'u' decreases as time 't' increases

The only suitable solution of the heat Equation is

$u(x,t) \cdot (A\cos px + B\sin px) e^{-a^2p^2t}$.

iv) WRITE THE TWO-DIMENSIONAL HEAT FLOW EQUATION AND WRITE ALL POSSIBLE SOLUTION OF TWO-DIMENSIONAL HEAT EQUATION.

If the temperature distribution at any point is independent of the z-coordinate then the heat flow is called two dimensional heat flow.

$$\frac{\partial u}{\partial t} = c^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

) WRITE THE FOURIER TRANSFORM PAIR (FOURIER TRANSFORM AND INVERSION OF FOURIER TRANSFORM)

FOURIER TRANSFORM:

Let f(x) be defined in $(-\infty, \infty)$ and pieces-wise continuous and absolutely integrable in $(-\infty, \infty)$ then the fourier transform of f(x) is defined as,

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x) \cdot e^{isx} dx$$

It is denoted by $F(f(x))$ or $F(s)$

$$(i.e)\ F(f x) = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} dx \rightarrow ①$$

Where 's' is the parameter sometimes 'p' (or) 'ω' are also used instead of 's'

INVERSION FORMULA FOR FOURIER TRANSFORM:

Inversion formula for fourier transform if the $F[f(x)]$, $f(s)$ then the inverse fourier transform of $F(s)$ is defined as $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx}$, it is denoted by $F^{-1}[F(s)]$ or $f(x)$

$$(i.e)\ f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds \rightarrow ②$$

Here the Equ ① + ② together are called as 'Fourier Transform pairs'.

2. A STRING IS STRETCHED AND FASTENED TO TWO POINTS $x=0$ AND $x=l$ APART. MOTION IS STARTED BY DISPLACING THE STRING INTO THE FORM $y = k(lx - x^2)$ FROM WHICH IT IS RELEASED AT TIME $t=0$. FIND THE DISPLACEMENT OF ANY POINT ON THE STRING AT A DISTANCE OF x FROM ONE END AT TIME t.

The one dimensional wave equation

$$\frac{d^2 u}{dt^2} = a^2 \frac{d^2 y}{dx^2}$$

The Correct solution of one dimensional wave equation is

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt) \rightarrow ①$$

The Boundary Conditions are

1) $y(0,t) = 0,\ t \geq 0$

2) $y(l,t) = 0,\ t \geq 0$

$$\Rightarrow C_2 \sin\left(\frac{n\pi}{l}\right)x\left(C_4\left(\frac{n\pi a}{l}\right)\right)=0$$

Since $C_2 \neq 0$, then

$$C_4\left(\frac{n\pi a}{l}\right)=0$$

$$\therefore \boxed{C_4 = 0} \qquad (\because n\pi a/l \neq 0)$$

Sub $\boxed{C_4=0}$ in eq ③,

$$y(x,t) = \left[C_2 \sin\left(\frac{n\pi}{l}\right)x\right]\left[C_3 \cos\left(\frac{n\pi a}{l}\right)t\right]$$

$$y(x,t) = C_2 - C_3 \left(\sin\left(\frac{n\pi}{l}\right)x\right)\left(\cos\left(\frac{n\pi a}{l}\right)t\right)$$

$$y(x,t) = C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t \longrightarrow (A)$$

The most general solution of given problem is

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t \longrightarrow ④$$

Applying 4th boundary condition in ④

$$\Rightarrow y(x,0) = f(x) = k(lx - x^2)$$

$$\Rightarrow y(x,0) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos(0) = k(lx - x^2)$$

$$\Rightarrow y(x,0) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x = k(lx - x^2)$$

$$\Rightarrow k(lx - x^2) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x$$

This is in the form of half range sine series in $(0,l)$

Here, $$C_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx.$$

$$C_n = \frac{2}{l}\int_0^l k(lx - x^2)\sin\left(\frac{n\pi}{l}\right)x\,dx$$

$$C_n = \frac{2k}{l}\left[(lx-x^2)\left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{(n\pi/l)}\right) - (l-2x)\left(\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right) + (0-2)\left(\frac{\cos(n\pi/l)x}{(n\pi/l)^3}\right)\right]_0^l$$

$$= \frac{2k}{l}\left[\left(0+0-\frac{2l^3}{n^3\pi^3}(-1)^n\right] - \left[0+0-\frac{2l^3}{n^3\pi^3}(1)\right]$$

3) $\frac{d}{dt} y(x,0) = 0, \quad 0 \le x \le l$

4) $y(x,0) = f(x) = k(lx - x^2)$

Applying boundary Condition ① in equation ①

$\Rightarrow y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$

$\Rightarrow y(0,t) = (C_1(1) + C_2 \sin(0))(C_3 \cos apt + C_4 \sin apt)$

$\Rightarrow y(0,t) = C_1 (C_3 \cos apt + C_4 \sin apt) = 0$

$\boxed{C_1 = 0}$

$\because (C_3 \cos apt + C_4 \sin apt \neq 0)$

Sub $\boxed{C_1 = 0}$ in ①, we get

$y(x,t) = C_2 \sin px (C_3 \cos apt + C_4 \sin apt) \rightarrow ②$ $(\because C_1 = 0)$

Applying 2nd boundary Condition in ②

$\Rightarrow y(l,t) = C_2 \sin pl (C_3 \cos apt + C_4 \sin apt) = 0$

$\Rightarrow C_2 \sin pl = 0$

$(\because C_2 \neq 0$, If $C_2 = 0$, then we get zero solution)

$\Rightarrow \sin pl = 0$

$\Rightarrow \sin pl = \sin n\pi \quad \Rightarrow pl = n\pi$

$\boxed{p = n\pi / l}$

Sub $p = n\pi/l$ in ②, we get

$y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_3 \cos a\left(\frac{n\pi}{l}\right)t + C_4 \sin a\left(\frac{n\pi}{l}\right)t\right) \rightarrow ③$

Partially differentiating equ ③ with respect to t.

$\frac{dy(x,t)}{dt} = C_2 \sin\left(\frac{n\pi}{l}\right)x \left[C_3 \sin\left(\frac{n\pi a}{l}\right)t \left(\frac{n\pi a}{l}\right) + C_4 \cos\left(\frac{n\pi a}{l}\right)t \left(\frac{n\pi a}{l}\right)\right]$

Applying 3rd Condition in above equation

$\frac{d}{dt} y(x,0) = 0$

$\Rightarrow C_2 \sin\left(\frac{n\pi}{l}\right)x \left(0 + C_4 \frac{n\pi a}{l}\right) = 0$

$$= \frac{2k}{l}\left[\frac{-2l^3}{n^3\pi^3}(-1)^n + \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{2k}{l}\left[\frac{2l^3}{n^3\pi^3}\right]\left[1-(-1)^n\right]$$

$$= \frac{4kl^2}{n^3\pi^3}(1-(-1)^n)$$

$$\therefore C_n = \frac{4kl^2}{n^3\pi^3}(1-(-1)^n)$$

$$C_n = \begin{cases} 0, & \text{if } n \text{ is even} \quad (-1)^n \Rightarrow 1-(-1)^n = 1-1=0 \\ \frac{8kl^2}{n^3\pi^3}, & \text{if } n \text{ is odd} \quad (-1)^3 = -1 \Rightarrow 1-(-1)^n = 1-(-1) = 1+1 = 2 \end{cases}$$

Sub C_n values in eqn (4), we get

$$y(x,t) = \sum_{n=\text{odd}}^{\infty} \frac{8kl^2}{n^3\pi^3} \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi}{l}\right)t$$

which is the required general series of given problems

3) A TIGHTLY STRETCHED STRING WITH FIXED END POINT $x=0$ & $x=l$ IS INITIALLY IN A POSITION GIVEN BY $y(x,0) = y_0 \sin^3\left(\frac{\pi}{l}\right)x$ IF IT IS RELEASED FROM REST FROM THIS POSITION, FIND THE DISPLACEMENT y AT ANY DISTANCE x FROM ONE END AT ANY TIME t.

The one dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = \frac{a^2 \partial^2 y}{\partial x^2}$$

The correct solution of one dimensional wave equation is

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt) \longrightarrow (1)$$

The Boundary Conditions are

i) $y(0,t) = 0, \quad t \geq 0$

ii) $y(l,t) = 0, \quad t \geq 0$

iii) $\frac{d}{dt} y(x,0) = 0, \ 0 \le x \le l$

iv) $y(x,0) = f(x) = k(lx - x^2)$

Applying boundary Condition ① in eqn ①

$\Rightarrow y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$

$\Rightarrow y(0,t) = (C_1(1) + C_2(0))(C_3 \cos apt + C_4 \sin apt) = 0$

$\Rightarrow C_1 (C_3 \cos apt + C_4 \sin apt) = 0$

$\therefore \boxed{C_1 = 0} \quad (C_3 \cos apt + C_4 \sin apt \neq 0)$

Sub $\boxed{C_1 = 0}$ in ①, we get

$y(x,t) = C_2 \sin px (C_3 \cos apt + C_4 \sin apt) \rightarrow ②$

Applying 2nd boundary condition in ②,

$y(l,t) = C_2 \sin pl (C_3 \cos apt + C_4 \sin apt) = 0$

$\Rightarrow C_2 \sin pl = 0$

$\Rightarrow \sin pl = \sin n\pi$

$\Rightarrow pl = n\pi$

$\boxed{p = n\pi/l}$

Sub $p = n\pi/l$ in ②, we get

$y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_3 \cos\left(\frac{n\pi a}{l}\right)t + C_4 \sin\left(\frac{n\pi a}{l}\right)t\right) \rightarrow ③$

Partially diff with resp to t

$y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left[C_3 \cos\left(\frac{n\pi a}{l}\right)t + C_4 \sin\left(\frac{n\pi a}{l}\right)t\right]$

$\frac{dy}{dt}(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left[C_3\left(-\sin\left(\frac{n\pi a}{l}\right)t\right)\left(\frac{n\pi a}{l}\right) + C_4\left(\cos\left(\frac{n\pi a}{l}\right)t\right)(n\pi a/l)\right]$

Applying 3rd boundary condition in above equation

$\frac{d}{dt} y(x,0) = 0$

$\Rightarrow C_2 \sin\left(\frac{n\pi}{l}\right)x \left(0 + C_4 \frac{n\pi a}{l}\right) = 0$

$\Rightarrow C_2 \sin\left(n\pi/l\right)x \left(C_4\left(\frac{n\pi a}{l}\right)\right) = 0$

Since $\boxed{C_2 \neq 0}$ then;

$\Rightarrow C_4\left(\frac{n\pi a}{l}\right) = 0$

$\boxed{C_4 = 0}$

Sub $C_4 = 0$ in ③,

$y(x,t) = \left[C_2 \sin\left(\frac{n\pi}{l}\right)x\right]\left[C_3 \cos\left(\frac{n\pi a}{l}\right)t\right]$

$= (C_2 - C_3) \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$

$= C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$

The most general solution is

$y(x,t) = \sum_{n=1} C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t \longrightarrow ④$

Applying 4th boundary Condition in ④,

$\Rightarrow y(x,0) = f(x) = y_0 \sin^3\left(\frac{\pi}{l}\right)x$

$\Rightarrow \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{l}\right)x = y_0 \sin^3\left(\frac{\pi}{l}\right)x$

$\Rightarrow C_1 \sin(\pi/l)x + C_2 \sin\left(\frac{2\pi}{l}\right)x + C_3 \sin(3\pi/l)x + C_4 \sin(4\pi/l)x + \cdots$

$\Rightarrow y_0\left[\frac{1}{4}(3\sin(\pi/l)x - \sin(3\pi/l)x\right]$

By Comparing Corresponding Coefficients of $\sin(\pi/l)x$, $\sin(3\pi/l)$ on both sides

$C_1 = \frac{3y_0}{4}, \quad C_2 = 0, \quad C_3 = -y_0/4, \quad C_4 = 0$

Sub, we get.

In the most general solution is (i-e) eqn ④

$y(x,t) = \sum_{n=1} C_n \sin(n\pi/l)x \cdot \cos(n\pi a/l)t$

$= C_1 \sin(\pi/l)x \cos(\pi a/l)t + C_3 \sin(3\pi/l)x \cdot \cos(\frac{3\pi a}{l})t$

$\therefore y(x,t) = \frac{340}{4} \sin(\pi/l)x \cdot \cos(\pi a/l)t - 40/4 \sin(3\pi/l)x - \cos(\frac{3\pi a}{l})t$

which is the required general solution of the given problem

4) A SQUARE PLATE IS BOUNDED BY THE LINE X=0, Y=0 AND X=Y=20, ITS FACES ARE INSULATED. THE TEMPERATURE ALONG UPPER HORIZONTAL LINE IS GIVEN BY $U(x-20) = x(20, x)$ WHEN $0 < x < 20$, WHILE OTHER 3 EDGES ARE KEPT AT 0°C. FIND STEADY STATE TEMPERATURE IN THE PLATE:

Let $u(x,y)$ be the temperature at any point x,y then $u(x,y)$ satisfies the laplace equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

The correct solution is

$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

The boundary conditions are,

1) $u(0,y) = 0, \; 0 < x < l$
2) $u(l,y) = 0, \; 0 < y < l$
3) $u(x,0) = 0, \; 0 < x < l$
4) $u(x,l) = f(x) = x(l-x)$

$\Rightarrow u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py}) \rightarrow ①$

Applying 1st boundary condition in ①

$u(0,y) = 0$

$\Rightarrow (A\cos 0 + B\sin 0)(Ce^{py} + De^{-py}) = 0$

$\Rightarrow A(Ce^{py} + De^{-py}) = 0$

$A = 0 \qquad (Ce^{py} + De^{-py} \neq 0)$

Substitute $A=0$ in ①

$U(x,y) = B\sin px\,(Ce^{py}+De^{-py}) \longrightarrow ②$

Applying 2nd Boundary Condition in ②,

$u(l,y)=0$

$B\sin pl\cdot(Ce^{py}+De^{-py})=0$ $\quad$ ($\because B\neq 0$, if $B=0$ the we get zero solution)

$\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi$

$\boxed{p = n\pi/l}$

Substitute $p = n\pi/l$ in eq ②

$u(x,y) = B\sin(n\pi/l)x\,\left(Ce^{(n\pi/l)y}+De^{-(n\pi/l)y}\right) \longrightarrow ③$

Applying 3rd boundary Condition in eqn ③

$u(x,0)=0$

$B\sin(n\pi/l)x\,\left(Ce^{(n\pi/l)0}+De^{-(n\pi/l)0}\right)=0$

$B\sin(n\pi/l)x\,(C+D)=0$

$C+D=0 \Rightarrow \boxed{D=-C}$

Substitute $D=-C$ in eq

$U(x,y) = B\sin(n\pi/l)x\,\left(Ce^{(n\pi/l)y} - Ce^{(-n\pi/l)y}\right)$

$= +B\sin(n\pi/l)x\left[Ce^{(n\pi/l)y} - Ce^{-(n\pi/l)y}\right]$

× by and ÷ by 2

$\Rightarrow \quad = 2BC\sin(n\pi/l)x\left[\frac{e^{(n\pi/l)y} - e^{-(n\pi/l)y}}{2}\right]$

$= 2BC\sin(n\pi/l)x\,\sinh(n\pi/l)y$

$u(x,y) = C_n\sin(n\pi/l)x\,\sinh(n\pi/l)y$

The most general equation is,

$U(x,y) = \sum_{n=1}^{\infty} C_n\sin(n\pi/l)x\,\sinh(n\pi/l)y \longrightarrow ④$

Applying 4th boundary Conditions in eqn ④

$u(x,l) = f(x) = x(l-x)$

$\Rightarrow \sum_{n=1}^{\infty} C_n \sin(n\pi/l)x \cdot \sinh(n\pi/l)l = x(l-x)$

$\Rightarrow \sum_{n=1}^{\infty} C_n \sin(n\pi/l)x \cdot \sinh(n\pi) = x(l-x)$

Let $C_n \sinh n\pi = b_n$

$\Rightarrow x(l-x) = \sum_{n=1}^{\infty} b_n \sin(n\pi/l)x$

$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin(n\pi/l)x$ in $(0,l)$

This is in the form of half range sine series.

Now,

$$b_n = \frac{2}{l}\int_0^l f(x)\sin(n\pi/l)x\,dx$$

$$= \frac{2}{l}\int_0^l x(l-x)\sin(n\pi/l)x\,dx$$

Applying the bernoulli's theorem $uv_1 - u'v_2 + u''v_3 + \cdots$

$$= \frac{2}{l}\left[\int_0^l lx\sin(n\pi/l)x\,dx - \int_0^l x^2\sin(n\pi/l)x\,dx\right]$$

$$= \frac{2}{l}l\left[\int_0^l x\sin\left(\frac{n\pi}{l}\right)x\,dx\right] - \frac{2}{l}\left[\int_0^l x^2\sin(n\pi/l)x\,dx\right]$$

$$= 2\left[x\left(\frac{-\cos(n\pi/l)x}{(n\pi/l)}\right) - 1\,\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right]_0^l$$

$$- \frac{2}{l}\left[x^2\left(\frac{-\cos(n\pi/l)x}{n\pi/l}\right) - 2x\left(\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right) + 2\left(\frac{\cos(n\pi/l)x}{(n\pi/l)^3}\right)\right]_0^l$$

$$= 2\left[\left(\frac{-l^2}{n\pi}(-1)^n + 0\right) - (0+0)\right] - \frac{2}{l}\left[\left(\frac{-l^3}{n\pi}(-1)^n + 0 + \frac{2l^3(-1)^n}{n^3\pi^3}\right) - \left(0+0+\frac{2l^3}{n^3\pi^3}\right)\right]$$

$$= \frac{-2l^2(-1)^n}{n\pi} - \frac{2}{l}\left[\frac{-l^3(-1)^n}{n\pi} + \frac{2l^3}{n^3\pi^3}(-1)^n - \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{2l^2(-1)^n}{n\pi} - \frac{2}{l}\left[\frac{-l^3}{n\pi}(-1)^n + \frac{2l^3}{n^3\pi^3}\left[(-1)^n - 1\right]\right]$$

$$= \frac{-2l^2(-1)^n}{n\pi} + \frac{2l^2(-1)^n}{n\pi} - \frac{4l^2}{n^3\pi^3}\left[(-1)^n - 1\right]$$

$$b_n = \frac{-4l^2}{n^3\pi^3}\left[(-1)^n - 1\right]$$

if $\begin{cases} n \text{ is odd} : b_n = \frac{8l^2}{n^3\pi^3} \\ n \text{ is even} : b_n = 0 \end{cases}$

$$\therefore U(x,y) = \sum_{n=1,3,5}^{\infty} \frac{1}{\sinh n\pi}\left(\frac{8l^2}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{4}\right)x \sin\left(\frac{n\pi}{l}\right)y$$

This is from, $b_n = C_n \sinh n\pi$

$$C_n = \frac{b_n}{\sinh n\pi}$$

$\therefore$ In most general solution

$$u(x,y) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \sin h\left(\frac{n\pi}{l}\right)y$$

Substitute the C_n value

$$u(x,y) = \sum_{n=\text{odd}}^{\infty} \frac{1}{\sinh n\pi}\frac{8l^2}{n^3\pi^3}\sin\left(\frac{n\pi}{l}\right)x \sin\left(\frac{n\pi}{l}\right)y$$

$$u(x,y) = \sum_{n=\text{odd}}^{\infty} \frac{1}{\sinh n\pi}\frac{3200}{n^3\pi^3}\sin\left(\frac{n\pi}{20}\right)x \sin\left(\frac{n\pi}{20}\right)y \quad (l = 20)$$

$\therefore$ It is the required Solution.

5. a) Derive change of scale property and modulation theorem of fourier series.

STATE AND PROVE CHANGE OF SCALE PROPERTY:

STATEMENT:

If $F[f(x)] = F(s)$, then $F[f(ax)] = \frac{1}{|a|}F(s/a)$

where $a \neq 0$

PROOF:

we know that

Fourier Transform

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x) - e^{isx}\,dx$$

Now $(x = ax)$

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax) \cdot e^{isx} dx$$

Case (i): If $a > 0$

By using Substitution method.

Let $ax = t$ $\quad t/a = x$

$a\,dx = dt \Rightarrow dx = dt/a$

The limit

i) When $x = -\infty \Rightarrow t = ax = -\infty$

ii) When $x = \infty \Rightarrow t = ax = \infty$

Now,

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{is(t/a)} dt/a$$

$$= \frac{1}{a} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s)t} \cdot dt \right]$$

$$= \frac{1}{a} \cdot F[s/a]$$

$\therefore F[f(ax)] = \frac{1}{a} F(s/a)$ when $a > 0$

Case (ii): $a < 0$

Now, $F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{-\infty} f(t) \cdot e^{is(t/a)} \cdot dt/a$

$$= \frac{1}{a} \cdot \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s/a)t} dt$$

$$F[f(ax)] = -\frac{1}{a} F(s/a) \text{ when } a < 0$$

From Case (i) & Case (ii)

$$F[f(ax)] = \frac{1}{|a|} F(s/a), \quad a \neq 0$$

STATE AND PROVE MODULATION THEOREM:

STATEMENT:

If $F[f(x)] = F(s)$ then

$$F[f(x)\cos ax] = \frac{1}{2}[F(s+a)+F(s-a)]$$

PROOF:

We know that

Fourier Transform,

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}dx$$

x by $\cos ax$

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot \cos ax\cdot e^{isx}dx$$

$\therefore \cos x = \frac{e^{ix}+e^{-ix}}{2}$

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\left(\frac{e^{iax}+e^{-iax}}{2}\right)e^{isx}dx$$

$$F[f(x)\cos ax] = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}\cdot e^{iax}dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}\cdot e^{-iax}dx\right]$$

$$= \frac{1}{2}[F(s+a)+F(s-a)]$$

$$\therefore F[f(x)\cos ax] = \frac{1}{2}[F(s+a)+F(s-a)]$$

b. FIND THE FOURIER TRANSFORM OF THE FUNCTION $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ AND DEDUCE THAT $\int_0^{\infty}\frac{\sin t}{t}dt = \pi/2$.

Given function

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

We know that, Fourier Transform

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1) \cdot e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{e^{isx}}{is} \right)_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{is} \left[(\cos ax + i \sin as) - (\cos ax - i \sin as) \right]$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{is} \left[2i \sin ax \right]$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sin as}{s}$$

By inversion formula,

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} ds$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \frac{\sin as}{s} \cdot e^{isx} ds$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} \cdot e^{isx} ds.$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} \left[\cos x - i \sin sx \right] ds.$$

$$= \frac{1}{\pi} \left[\int_{-\infty}^{\infty} \frac{\sin as}{s} \cos sx \, ds - i \int_{-\infty}^{\infty} \frac{\sin as}{s} \sin sx \, ds \right]$$

$$= \frac{1}{\pi} \left[2 \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds \right]$$

$$= \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds.$$

$$\therefore f(x) = \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, dx.$$

$$\therefore \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Put $x = 0$

$$\frac{2}{\pi}\int_0^\infty \frac{\sin as}{s}\,ds = 1$$

$$\int_0^\infty \frac{\sin as}{s}\,ds = \pi/2$$

Put $(t = as)$

$as = t$

$a\,ds = dt$

$ds = dt/a$

$s = t/a$

Now,

$$\int_0^\infty \frac{\sin as}{s}\,ds = \pi/2$$

$$\int_0^\infty \frac{\sin t}{(t/a)}\,\frac{dt}{a} = \pi/2$$

$$\therefore \int_0^\infty \frac{\sin t}{t}\cdot dt = \pi/2$$

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U21AS001

TPDE - Assignment-1

1. i) write one-dimensional wave equation and write all possible solutions of wave equation

ii) fourier's law of thermal condition

iii) write two-dimensional heat flow equation and write all possible solutions of two-dimensional heat equation

iv) write one-dimensional heat equation and write all possible solutions of heat equation

v) write fourier transform pair (fourier transform and Inversion of fourier transform.

Solution:-

i) The one-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Here, $a^2 = \frac{T}{m} \left[\frac{\text{Tension}}{\text{Mass}} \right]$

The Various (or) All possible solutions of wave equations are:-

1) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{apt} + c_4 e^{-apt})$

2) $y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos pta + c_8 \sin pta)$

3) $y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$

ii) Fourier law of thermal conduction:-

The fourier's law of thermal conduction states that the rate of heat transfer through a material is proportional to the negative gradient in the temperature and the area of the surface through which the heat flows iv, The one dimensional heat equation is given by

$$\frac{du}{dt} = c^2 \frac{d_2 u}{dx^2} , \quad \text{where } c^2 = K/\rho c$$

K = Thermal conductivity

ρ = density

c = Specific heat

The all possible solutions of one-dimensional heat equation are:-

1, $u(x,t) = (Ae^{px} + Be^{-px}) e^{a^2 p^2 t}$

2, $u(x,t) = (A \cos px + B \sin px) e^{-a^2 p^2 t}$

3, $u(x,t) = Ax + B$

iii, Two-dimensional heat flow equation

If the temperature distribution at any point is independent of the z-coordinate, then the heat flow is called two-dimensional heat flow ie, the equation of to two dimensional heat flow equation is

$$\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) , \quad \text{where } c^2 = \frac{K}{\rho c}$$

where,

K - thermal conductivity

ρ - density

c - Specific heat

In steady state $\frac{\partial u}{dt} = 0$

Therefore, the 2D heat flow equations become,

$$0 = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$, which is known as laplace equation (or) 2D heat flow equation under steady state.

The all possible solution of 2D heat flow equation or laplace equation are:

1, $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

2, $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

3, $u(x,y) = (Ax + B)(Cy + D)$

4, Fourier transform

Let $f(x)$ be defined in $(-\infty, \infty)$ and piece-wise continuous and absolutely integrable in $(-\infty, \infty)$, then the fourier transform of $f(x)$ is defined as,

$\frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{isx} dx$. It is defined by $F(f(x))$ or $F(s)$, that is

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot dx \quad -(1)$$

Inversion formula for fourier transform:-

If $f[f(x)] = F(s)$, then the inversion fourier of $f(s)$ is defined as $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} ds$. It is denoted by

$F(x)$ or $F^{-1}[f(x)]$

(i.e) $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{isx} \cdot dx$ — ②

① and ② together are called as fourier transform pair.

2) A string is streched and frustrated to two points $x=0$ and $x=l$ apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which is it released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time t.

Solution :-

The one-dimensional wave equation is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}, \text{ Here } a^2 = \frac{T}{m} \left(\frac{\text{tension}}{\text{mass}}\right)$$

The correct solution of 1D wave equation is,

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos apt + c_4 \sin apt) \quad \text{—①}$$

The boundary conditions are :-

1) $y(0,t) = 0, \ t \geq 0$

2) $y(l,t) = 0, \ t \geq 0$

3) $\frac{\partial (y(x,0))}{\partial t} = 0, \quad 0 \leq x \leq l$

4) $y(x,0) = f(x) = k(lx - x^2)$

Applying boundary condition ① in equation ①

$$y(0,t) = (c_1(1) + c_2(0))(c_3 \cos apt + c_4 \sin apt) = 0$$

$$= (C_1)\underset{a}{} \; (C_3 \cos apt)\underset{b}{} \; (C_4 \sin apt) = 0$$

$C_1 = 0$ [∵ If $a \times b = 0$, then either $a = 0$ (or) $b = 0$]

$(∵ C_3 \cos apt + C_4 \sin apt) \neq 0$

Sub:

$C_1 = 0$ in equ ①, we get

$$y(x,t) = C_2 \sin px \,(C_3 \cos apt + C_4 \sin apt) \quad -②$$

$∵ C_1 = 0$

Applying 2nd boundary condition in equ ②

$$y(l,t) = C_2 \sin pl \,(C_3 \cos apt + C_4 \sin apt) = 0$$

$$\Rightarrow C_2 \sin pl = 0$$

$$= \sin pl = 0$$

$(∵ C_2 \neq 0$ If $C_2 = 0$, then we get trivial zero in solution)

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = n\pi / l$

[$\sin pl = \sin 0$

$pl = 0$

not possible]

substituting $p = n\pi/l$ in equ ②, we get

$$y(x,t) = C_2 \sin px \,(C_3 \cos apt + C_4 \sin apt)$$

$$= C_2 \sin\left(\frac{n\pi}{l}\right)x \cdot \left(C_3 \cos\left(\frac{n\pi a}{l}\right)t + C_4 \sin\left(\frac{n\pi a}{l}\right)t\right) \quad -③$$

$$y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left[C_3\left(-\sin\left(\frac{n\pi a}{l}\right)t\right)\left(\frac{n\pi a}{l}\right) + C_4 \cos\left(\frac{n\pi a}{l}\right)t.\left(\frac{\pi a}{l}\right)\right.$$

Applying 3rd condition in the above equation

$$\frac{\partial}{\partial t} y(x,0) = 0,$$

$$C_2 \sin\left(\frac{n\pi}{l}\right)x \left(0 + C_4 \cdot \frac{n\pi a}{l}\right) = 0$$

$$C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_4 \left(\frac{n\pi a}{l}\right)\right) = 0$$

since $C_2 \neq 0$

then, $C_4 \left(\frac{n\pi a}{l}\right) = 0$

$C_4 = 0$, $\left(\because \frac{n\pi}{l} \neq 0\right)$; Sub $C_4 = 0$ in eqn ③

$$y(x,t) = \left(C_2 \cdot \sin\left(n\pi/l\right)x\right)\left(C_3 \cos\left(\frac{n\pi a}{l}\right)t\right.$$

$$y(x,t) = C_2 \cdot C_3 \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

$$y(x,t) = C_n \cdot \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

∴ The most general solution of the given problem is

$$(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t \quad - ④$$

Applying 4th boundary condition in eqn ④

$$y(x,0) = f(x) = k(lx - x^2)$$

$y(x,t)$

$$k \cdot (lx - x^2) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \qquad \left[\because \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x\right]$$

$$b_n = 2/l \int_0^l \sin\left(\frac{n\pi}{l}\right)x \, dx$$

This is in the form of half range sine series.

here, $C_n = \frac{2}{l} \int_0^l f(x) \cdot \sin\left(\frac{n\pi}{l}\right)x \, dx$

$$C_n = \frac{2}{l} \int_0^l k(lx - x^2) \cdot \sin\left(\frac{n\pi}{l}\right)x \, dx$$

$$= \frac{2k}{l}\left[(lx - x^2)\left(\frac{-\cos\left(\frac{n\pi}{l}\right)l}{\left(\frac{n\pi}{l}\right)}\right) - (l - 2x)\left(\frac{-\sin\left(\frac{n\pi}{l}\right)l}{\left(\frac{n\pi}{l}\right)^2}\right) + (0-2)\left(\frac{\cos\left(\frac{n\pi}{l}\right)l}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^l$$

$$= 2k/l\left[\left(0+0 - 2\times\frac{l^3}{n^3\pi^3}(-1)^n\right) - \left(0+0-\frac{2l^3}{n^3\pi^3}(1)\right)\right]$$

$$= \frac{2k}{l}\left[\frac{-2l^3}{n^3\pi^3}(-1)^n + \frac{2l^3}{n^3\pi^3}\right]$$

$$= 2k/l \times \frac{2l^3}{n^3\pi^3}\left[1 - (-1)^n\right]$$

$$= \frac{4kl^2}{n^3\pi^3}\left[1 - (-1)^n\right]$$

$$\therefore C_n = \frac{4kl^2}{n^3\pi^3}\left[1 - (-1)^n\right]$$

$$\therefore C_n = \begin{cases} 0, & \text{if } n \text{ is even} \quad (-1)^n = 1 \Rightarrow 1 - (-1)^n = 1 - 1 = 0 \\ \frac{8kl^2}{n^3\pi^3}, & \text{if } n \text{ is odd} \quad (-1)^3 = 1 - (-1)^n = 1 + 1 = 2 \end{cases}$$

Sub C_n value in equ (4)

$$y(x,t) = \sum_{n=odd}^{\infty} \frac{8kl^2}{n^3\pi^3} \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t$$

which is the required general solution of the given problem.

3/ A tightly stretched string with fixed end points $x=0$ & $x=l$ is initially in a position given by $y(x,0) = y_0 \sin^3 (\pi/l) x$. If it is released from rest from this position. Find the displacement y at any distance x from one end at any time t.

The 1D wave equation is

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad a = t/m \left(\frac{\text{tension}}{\text{mass}}\right)$$

The current solution of 1D wave equation is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos apt + c_4 \sin apt) \quad -①$$

The boundary conditions are

1) $y(0,t) = 0, \quad t \geq 0$

2, $y(l,t) = 0, \quad t \geq 0$

3, $\frac{\partial}{\partial t} y(x,0) = 0, \quad 0 \leq x \leq l$

4, $y(x,0) = f(x) = y_0 \sin^3 (\pi/l) x$

Applying boundary condition ① in equation ①

$$y(0,t) = (c_1(1) + c_2(0))(c_3 \cos apt + c_4 \sin apt) = 0$$

$$= (c_1)(c_3 \cos apt + c_4 \sin apt) = 0$$

$\boxed{c_1 = 0}$

C: $(c_3 \cos apt + c_4 \sin apt \neq 0)$ [∵ If $a \times b = 0$, then either $a=0$ (or) $b=0$]

Sub: $c_1 = 0$ in equation ①, we get

$$y(x,t) = c_2 \sin px (c_3 \cos apt + c_4 \sin apt) \quad -② \quad (\because c_1 = 0)$$

Applying 2nd boundary Condition in equation (2)

$y(l,t) = C_2 \sin pl \, (C_3 \cos pt + C_4 \sin apt) = 0$

$\Rightarrow C_2 \sin pl = 0$ (∵ $C_2 \neq 0$ if $C_2 = 0$ then we get trivial (zero) solution)

c.b

$\Rightarrow \sin pl = 0$

($\sin pl = \sin 0$, $pl = 0$ not possible)

$\sin pl = \sin n\pi$

$pl = n\pi$

$\boxed{p = n\pi/l}$

Substituting $p = n\pi/l$ in equation (2), we get

$y(x,t) = C_2 \sin px \, (C_3 \cos apt + C_4 \sin apt)$

$= C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_3 \cos\left(\frac{n\pi a}{l}\right)t + C_4 \sin\left(\frac{n\pi a}{l}\right)t\right)$

$y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_3 \cos\left(\frac{n\pi a}{l}\right)t + C_4 \sin\left(\frac{n\pi a}{l}\right)t\right.$

$= \frac{\partial y}{\partial t}(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_3\left(-\sin\left(\frac{n\pi a}{l}\right)t\right)\left(\frac{n\pi a}{l}\right)\right)$

$+ C_4 \cos\left(\frac{n\pi a}{l}\right)t \cdot \frac{n\pi a}{l}\Big)$

Applying 3rd Condition in the above equ

$\frac{\partial y}{\partial t}(x,0) = 0$

$C_2 \sin\left(\frac{n\pi}{l}\right)x \left(0 + C_4 \cdot \frac{n\pi a}{l}\right) = 0$

$C_2 \sin\left(\frac{n\pi}{l}\right)x \left(C_4\left(\frac{n\pi a}{l}\right)\right) = 0$

since, $C_2 \neq 0$

then, $c_4\left(\frac{n\pi a}{l}\right) = 2$

$c_4 = 0$, $\left[\because \frac{n\pi a}{l} \neq 0\right]$

sub $c_4 = 0$ in equ ③

$$y(x,t) = \left(c_2 \sin\left(\frac{n\pi}{l}\right)x\right)\left(c_3 \cos\left(\frac{n\pi a}{l}\right)\right) \quad (\because c_4 = 0)$$

$$y(x,t) = c_2 \cdot c_3 \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

$$y(x,t) = c_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

The 3rd boundary conditions are same as the 1st equation

The most general solution is,

$$y(x,t) = \sum_{n=1}^{\infty} c_n \cdot \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t \quad —④$$

Applying the 4th boundary condition in equation ④

$$y(x,0) = f(x) = y_0 \sin^3\left(\pi/l\right)x$$

$$\Rightarrow \sum_{n=1}^{\infty} c_n \cdot \sin\left(\frac{n\pi}{l}\right)x = y_0 \sin^3\left(\pi/l\right)x$$

$$\left[\because \sin^3 x = \tfrac{1}{4}\left(3\sin x - \sin 3x\right)\right]$$

$$\Rightarrow c_1 \sin\left(\pi/l\right)x + c_2 \sin\left(\frac{2\pi}{l}\right)x + c_3 \sin\left(3\pi/l\right)x + c_4 \sin\left(4\pi/l\right)x + \ldots$$

$$= y_0\left[\frac{1}{4}\left(3\sin\left(\pi/l\right)x - \sin\left(3\pi/l\right)x\right]\right. \qquad \because x = \pi/l \, x$$

By comparing corresponding coefficients of $\sin\left(\pi/l\right)x$, $\sin\left(3\pi/l\right)x$ on both sides.

$c_1 = \frac{3y_0}{4}$, $c_2 = 0$, $c_3 = \frac{-y_0}{4}$, $c_4 = 0$

Substitute $C_1 = \frac{3y_0}{4}$, $C_2 = 0$, $C_3 = -y_0/4$, $C_4 = 0$, $C_5 = 0$

in the most general solution in equ. (4)

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

$$= C_1 \sin(\pi/l)x \cdot \cos(\pi a/l)t + C_3 \sin(3\pi/l)x \cdot \cos\left(\frac{3\pi a}{l}\right)t$$

$$[\because C_2 = C_4 = C_5 + \dots = 0]$$

$$\therefore y(x,t) = \frac{3y_0}{4} \sin(\pi/l)x \cdot \cos\left(\frac{\pi a}{l}\right)t - \frac{y_0}{4} \sin\left(\frac{3\pi}{l}\right)x \cdot \cos\left(\frac{3\pi a}{l}\right)t$$

which is the required general solution of the given problem.

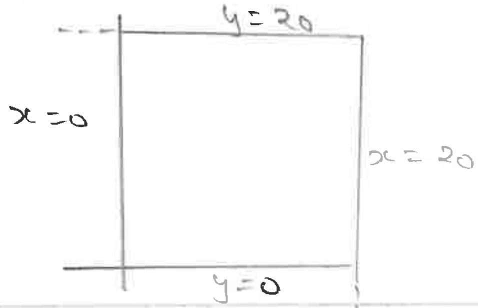
4/ A square plate is bounded by the lines $x=0$, $y=0$ and $x=y=10$, its faces are insulated. The temperature along upper horizontal line is given by $u(x,10) = x(10-x)$, when $0 < x < 10$. while other 3 edges are kept at 0°C find steady state temperature in the plate.

Solution:

Let $u(x,y)$ be the temperature at any point (x,y). Then $u(x,y)$ statisfies the laplace equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

The correct solution is $u(x,y) = (A\cos px + B\sin px)\cdot(Ce^{py} + De^{-py})$ — (1)



let $l = 20$

The boundary conditions are,

1, $u(0,y) = 0, \quad 0 < x < l$

2, $u(l,y) = 0, \quad 0 < y < l$

3, $u(x,0) = 0, \quad 0 < x < l$

4, $u(x,l) = f(x) = x(l-x)$

$u(x,l) = f(x) = x(l-x)$

$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

Applying 1st b.c in equation ①

$u(0,y) = 0$

$\Rightarrow (A\cos 0 + B\sin 0)(Ce^{py} + De^{-py}) = 0$

$\Rightarrow A.(Ce^{py} + De^{-py}) = 0$ [∵ If $a-b=0$ then either $a=0$ or $b=0$]

$\Rightarrow A = 0 \quad (\because Ce^{py} + De^{-py} \neq 0)$

Sub $A=0$ in eqn ①

$u(x,y) = B\sin px\,(Ce^{py} + De^{-py})$ - ②

Applying 2nd B.C in equ②

$u(l,y) = 0$

$\Rightarrow B\sin pl\,(Ce^{py} + De^{-py}) = 0$

$\Rightarrow B\cdot\sin pl = 0$

$\Rightarrow \sin pl = 0$

$\Rightarrow \sin pl = \sin n\pi$

$\Rightarrow pl = n\pi$

$p = n\pi / l$

Sub, $P = n\pi/l$ in equ ②

$$u(x,y) = B\sin(n\pi/l)x\left(Ce^{(n\pi/l)y} + De^{-(n\pi/l)y}\right) \quad -③$$

$u(x,0) = 0$

$$\Rightarrow B\sin\left(\frac{n\pi}{l}\right)x\left(Ce^{(n\pi/l)0} + De^{-(n\pi/l)0}\right)$$

$$\Rightarrow B\cdot\sin\left(\frac{n\pi}{l}\right)x\,(C+D) = 0$$

$$\Rightarrow C + D = 0 \quad \Rightarrow \boxed{D = -C}$$

sub $D = -C$ in equ ③

$$u(x,y) = B\sin(n\pi/l)x\left(Ce^{(n\pi/l)y} + De^{-(n\pi/l)y}\right)$$

$$= B\sin(n\pi/l)x\left[Ce^{(n\pi/l)y} - Ce^{-(n\pi/l)y}\right]$$

$$= 2BC\sin\left(\frac{n\pi}{l}\right)x\left[e^{(n\pi/l)y} - e^{-(n\pi/l)y}\right]$$

$$= 2BC\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y \qquad \left[\frac{e^{x}-e^{-x}}{2} = \sinh x\right]$$

$$\therefore u(x,y) = C_n\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y$$

[$\because 2BC = C_n$ say]

The most general solution is,

$$u(x,y) = \sum_{n=1}^{\infty} C_n\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y \quad -④$$

Apply the 4 B.C in ④ equ

$$u(x,l) = f(x) = x(l-x)$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n\cdot\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh n\pi = x(l-x)$$

[let $C_n\cdot\sinh n\pi = b_n$]

$$f(x) = \sum_{n=1}^{\infty} b_n\cdot\sin\left(\frac{n\pi}{l}\right)x \text{ in } (0,l)$$

This is in the form of half range sine Series

$$\text{Now}, b_n = 2/l \int_0^l f(x) - \sin\left(\frac{n\pi}{l}\right)x \, dx$$

$$= 2/l \int_0^l x(l-x) \cdot \sin(n\pi/l)x \cdot dx$$

$$= 2/l \int_0^l x(l-x) \cdot \sin\left(\frac{n\pi}{l}\right)x \cdot dx$$

By Bernoulli's formula

$$\int uv dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \cdots$$

$u = x(l-x)$

$u' = x(0-1) + (l-x)$

$= l - 2x$

$u'' = -2$

$v_1 = \left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{(n\pi/l)}\right)$

$v_2 = \left(\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right)$

$v_3 = \left(\frac{\cos(n\pi/l)x}{(n\pi/l)^3}\right)$

$$b_n = \frac{2}{l}\left[(xl - x^2)\left(\frac{-\cos(n\pi/l)x}{(n\pi/l)}\right) - (l-2x)\left(\frac{-\sin(n\pi/l)x}{(n\pi/l)^2}\right) + (-2)\left(\frac{\cos(n\pi/l)x}{(n\pi/l)^3}\right)\right]_0^l$$

$$= 2/l\left[\left(0 + 0 - \frac{2l^3}{n^3\pi^3}\cos n\pi\right) - \left(0 + 0 - \frac{2l^3}{n^3\pi^3}\right)\right]$$

$$= 2/l\left(\frac{2l^3}{n^3\pi^3}\right)\left[1 - (-1)^n\right]$$

$$b_n = 4l^2/n^3\pi^3\left[1 - (-1)^n\right]$$

$$b_n = \begin{cases} 0, & \text{if } n \text{ is even } b_n = \frac{4l^2}{n^3\pi^3}(1-(-1)^n) = \frac{4l^2(1-1)}{n^3\pi^3} = 0 \\ \frac{8l^2}{n^3\pi^3}, & \text{if } n \text{ is odd } b_n = \frac{4l^2}{n^3\pi^3}(1-(-1)^n) = \frac{4l^2 \cdot 2}{n^3\pi^3} = \frac{8l^2}{n^3\pi^3} \end{cases}$$

we know that, $b_n = C_n \cdot \sinh n\pi$

$$\Rightarrow C_n = 1/\sinh n\pi \cdot b_n$$

The most generalised solution

$$u(x,y) = \sum_{n=1}^{\infty} C_n \cdot \sin\left(\frac{n\pi}{l}\right)x \cdot \sin\left(\frac{n\pi}{l}\right)y$$

Sub

C_n value in the above equation.

$$\Rightarrow u(x,y) = \sum_{n=odd}^{\infty} \frac{b_n}{\sinh n\pi} \sin\left(\frac{n\pi}{l}\right)x \cdot \sin\left(\frac{n\pi}{l}\right)y$$

$$= \sum_{n=odd}^{\infty} \frac{\frac{8l^2}{n^3\pi^3}}{\sinh n\pi} \cdot \sin\left(n\pi/l\right)x \cdot \sin\left(\frac{n\pi}{l}\right)y$$

$$= \sum_{n=odd}^{\infty} \frac{8l^2}{n^3\pi^3}\left(\frac{1}{\sinh n\pi}\right) \cdot \sin\left(\frac{n\pi}{l}\right)x \cdot \sin\left(\frac{n\pi}{l}\right)y$$

$l = 20$

$$\Rightarrow u(x,y) = \sum_{n=odd}^{\infty} \frac{8(20)^2}{n^3\pi^3}\left(\frac{1}{\sinh n\pi}\right) \cdot \sin\left(\frac{n\pi}{20}\right)x \cdot \sin\left(\frac{n\pi}{20}\right)y$$

$$u(x,y) = \sum_{n=odd}^{\infty} \frac{3200}{n^3\pi^3}\left(\frac{1}{\sinh n\pi}\right) \cdot \sin\left(\frac{n\pi}{20}\right)x \cdot \sin\left(\frac{n\pi}{20}\right)y$$

5/a) Derive change of scale property and modulation theorem of fourier transform.

b) find the fourier transform of $f(x)$, if $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence find the value of $\int_0^{\infty} \frac{\sin t}{t} dt$

Solution :-

a, Scale property

Statement : If $F[f(x)] = F(s)$, then $F(f(ax)) = \frac{1}{|a|} F(s/a)$,

where $a \neq 0$

Proof : we know that

$$F[f(x)] = F(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot dx$$

Now, $$F[f(ax)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot dx$$

Case (i) If $a > 0$

let $ax = t \Rightarrow x = t/a$

$\Rightarrow a\,dx = dt$

$\Rightarrow dx = dt/a$

where $x = -\infty \Rightarrow t = ax = -\infty$

where $x = \infty \Rightarrow t = ax = \infty$

Now $$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{is(t/a)} \, dt/a$$

$$= 1/a \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s/a)t} \cdot dt \right.$$

$$= 1/a \; F(s/a)$$

$\therefore F[f(ax)] = 1/a \; F(s/a)$ when $a > 0$

Case (ii) If $a < 0$,

let $ax = t \Rightarrow x = t/a$

$\Rightarrow a\,dx = dt$

$dx = dt/a$

L.L when $x = -\infty \Rightarrow t = ax = \infty$

U.L when $x = +\infty \Rightarrow t = ax = -\infty$

Now $$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{is(t/a)} \, dt/a$$

$$= \frac{1}{a}\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s/a)t} \cdot dt\right]$$

$$\left[\because \int_{\infty}^{-\infty} f(x)\,dx = -\int_{-\infty}^{\infty} f(x)\,dx\right]$$

$$= -1/a \; F(s/a)$$

$\therefore F(f(ax)] = 1/a \; F(s/a)$, when $a < 0$

$\therefore$ from case (i) and (ii)

$$F(f(ax)] = \frac{1}{|a|} F(s/a) \text{ where } a \neq 0$$

$\Rightarrow$ Modulation Theorem :-

Statement :

If $F[f(x)] = F(s)$, then $F[f(x) \cdot \cos ax)$

$$= 1/2 \, [F(s+a) + F(s-a)]$$

proof :- we know that

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot dx$$

$$F[f(x) \cdot \cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot \cos ax \cdot e^{isx} \cdot dx$$

since $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$

$e^{isx} \cdot e^{iax}$
$= e^{(isx)} + e^{(iax)}$
$= e^{i(s+a)x}$

$$\Rightarrow F[f(x) \cdot \cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot \left(\frac{e^{iax} + e^{-iax}}{2} \right) e^{isx} \cdot dx$$

$$= \frac{1}{2} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot e^{iax} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot e^{-ias} dx \right.$$

$$= 1/2 \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{i(s+a)x} dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{i(s-a)x} \cdot dx \right.$$

$$= \frac{1}{2} [F(s+a) + F(s-a)]$$

$$\therefore F[f(x) \cdot \cos ax] = \frac{1}{2} [F(s+a) + F(s-a)]$$

5 - Solution! -

Given function: $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$

we know that

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1 \cdot e^{isx} \cdot dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{isx}}{is} \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \times \frac{1}{is} \left[a^{isa} - e^{-isa} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{is} 2 \sin as$$

$$= \sqrt{2/\pi} \cdot \frac{\sin as}{s}$$

$$\therefore f[f(x)] = f(s) = \sqrt{2/\pi} \cdot \frac{\sin as}{s}$$

By Inversion Formula:-

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} \cdot ds$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{2/\pi} \cdot \frac{\sin as}{s} \cdot e^{isx} \cdot ds$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} [\cos sx - i \sin sx] \cdot dx$$

$$= \frac{1}{\pi} \left[\int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \cos x \right) ds \cdot i \int_{-\infty}^{\infty} \left(\frac{\sin as}{s} \cdot \sin s \right) ds \right.$$

$f(s)$ $g(s)$

$$= \frac{1}{\pi} \left[2 \int_{0}^{\infty} \frac{\sin as}{s} \cos sx ds \right]$$

$$= 2/\pi \int_{0}^{\infty} \frac{\sin as}{s} \cos sx ds$$

$$\therefore f(x) = 2/\pi \int_{0}^{\infty} \frac{\sin as}{s} \cos sx ds$$

$$= 2/\pi \int_{0}^{\infty} \frac{\sin as}{s} \cdot \cos x dx = \begin{cases} 1, & |x| < a \\ 0, & |x| > 0 \end{cases}$$

put $x = 0$

$$\Rightarrow 2/\pi \int_{0}^{\infty} \frac{\sin as}{s} ds = 1$$

$$= 2/\pi \int_{0}^{\infty} \frac{\sin as}{s} ds = 1$$

$$\Rightarrow \int_{0}^{\infty} \frac{\sin as}{s} ds = \pi/2$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{isa} = \cos as + i \sin as$

$e^{isc} = \cos as - i \sin as$

$f(s) = \frac{\sin as}{s} \cos sx$

$f(-s) = \frac{\sin as}{s} \cos sx$

$f(-s) = f(x)$ (even)

$\int_{-\infty}^{\infty} () = 2 \int_{0}^{\infty} ()$

$g(-s) = \frac{-\sin as}{-s} - \sin sx$

$= -g(s)$

$\int_{-\infty}^{\infty} = 0$

$\Rightarrow -a < x < a$

Since

$-a < 0 < a$

$\therefore \cos 0 = 1$

put $as = t \Rightarrow s = t/a$

$\Rightarrow a\,ds = dt/a$

L-L

$s = 0 \Rightarrow t = as = 0$

U-L

$s = \infty \Rightarrow t = as = \infty$

Now,

$$\int_0^\infty \frac{\sin t}{(t/a)} \cdot dt/a = \pi/2$$

$$\int_0^\infty \frac{\sin t}{t}\, dt = \pi/2$$

NAME: SATHEESH . S

REG NO: U21AS027.

SUBJECT: TRANSFORMS AND BOUNDARY VALUE.

SUB CODE: U20MABT03

DATE: 16/11/22.

ASSIGNMENT-2

1. i) Write the One dimensional wave Equation and write all the possible solutions of wave Equation

ONE DIMENSIONAL WAVE EQUATION:

One dimensional Wave equation is,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}.$$

Here $a^2 = \frac{T}{m} = \frac{\text{Tension}(T)}{\text{mass}(m)}$

The various solution of One dimensional wave equation:

1) $y(x,t) = (C_1 e^{px} + c e^{-px})(C_3 e^{apt} + C_4 e^{-apt})$

2.) $y(x,t) = (C_5 \cos px + C_6 \sin px)(C_7 \cos apt + C_8 \sin apt)$

3.) $y(x,t) = (C_9 x + C_{10})(C_{11} t + C_{12})$.

Correct solution of the One dimensional wave Equation out of all the possible solutions we can choose the correct solution as follows:

Since we are dealing with problems on vibration of Strings.

The solution should be a periodic function. And the solution must involve Trignometric terms like Sines and Cosines.

Therefore the correct solution of the Wave Equation is

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$$

ii) Fourier's law of Thermal Conduction (or) Conductivit

The fouriers law of thermal conduction states that the rate of Heat Transfer through a material is proportional to the Negative gradient (-ve) in the temperature and the area of the Surface through which the heat flows.

iii) Write the One-dimensional heat flow equation and write all the possible solution of heat equation.

ONE-DIMENSIONAL HEAT EQUATION:

The One-dimensional Heat Equation is given by

$$\frac{du}{dt} = c^2 \frac{d^2u}{dx^2}.$$

where $c^2 = \frac{K}{ec}$

Here
K - Thermal Conductivity
e - Density
c - Specific Heat Capacity.

where = $c^2 = \frac{K}{ec}$

Here, K - Thermal Conductivity
e - Density
c - Specific Heat

In steady state $\frac{\partial u}{\partial t} = 0$

∴ the two dimensional Heat flow becons.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0$$

which is known as "Laplace Equation"

The Various possible Solutions of Laplace Equation (Two dimensional Heat flow Equation) are

1.) $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

2.) $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

3) $U(x,y) = (Ax+B)(Cy+D)$

The Correct Solution of Laplace Equation

If the non-zero boundary Conditions are along the x-axis are a line parallel to x-axis

ie), $u(x,0) = u(x,l) = f(x)$, then

The correct solution is

$$u(x,y) = (A\cos px + B\sin px)\cdot(Ce^{py} + De^{-py})$$

The Various (or) all possible solutions of one dimensional Heat Equations are.

1) $u(x,t) = (Ae^{px} + Be^{-px})\cdot e^{apt}$

2) $u(x,t) = (A\cos px + B\sin px)\cdot e^{-a^2p^2t}$

3) $u(x,t) = Ax+B$.

The correct solutions of One dimensional Heat Equation out of all the three possible solution. we have to choose the solution which is consistent with the physical nature of the problem.

Since 'u' decreases as time 't' increases

The Only Suitable Solution of the Heat Equation is

$$u(x,t) = (A\cos px + B\sin px)e^{-a^2p^2t}.$$

(5)

iv) Write the two-dimensional heat flow Equation and write all possible Solutions of two-dimensional heat Equation.

Two Dimensional Heat Flow Equation:

If the temperature distribution at any point is independent of the Z-Coordinate then the heat flow is called two-dimensional Heat flow.

ie) The equation of the two dimensional heat flow is

$$\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right).$$

v) Write the fourier Transform pair (Fourier Transform and Inversion of fourier Transform)

Fourier Transform:

Let $f(x)$ be defined in $(-\infty, \infty)$ and piecewise continuous and absolutely integrable in $(-\infty, \infty)$ then the fourier transform of $f(x)$ is defined as

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \, dx.$$

It is denoted by $F(f(x))$ (or) $F(S)$

ie) $$F(f(x)) = F(S) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x) \cdot e^{isx}\, dx \longrightarrow (1)$$

where 'S' is the parameter Sometimes 'p' (or) 'w' are also used instead of 's'

INVERSION FORMOLA FOR FOURIER TRANSFORM:

Inversion formula for fourier transform if the $F(f(x))$, $f(s)$, then the inverse fourier transform of $F(S)$ is defined as

$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(S) \cdot e^{-isx}\, dx$, it is denoted by '$F^{-1}[F(S)]$ or $f(x)$'

ie) $$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds \longrightarrow (2)$$

Here the Equation (1) and (2) together are called as 'Fourier Transform Pairs'.

2.) A string is streached and fastened to two points $x=0$ and $x=l$ motion is started by displacing the string into the form $y = K(lx - x^2)$ from which it is released and at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at time 't'

The One dimensional wave equation is,

$$\frac{\partial^2 y}{\partial t^2} = \frac{a^2 \partial^2 y}{\partial x^2}.$$

The correct soultion of one dimensional wave equation is,

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt) \quad \text{--- (1)}$$

The boundary condition are,

1) $y(0,t) = 0 \quad t > 0$

2) $y(l,t) = 0, \quad t > 0$

3) $\frac{\partial}{\partial t} y(x,0) = 0, \quad 0 \le x < l.$

4) $y(x,0), f(x) = K(lx - x^2)$.

Applying boundary condition ① in equation ①

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$$

$$y(0,t) = (C_1 \cos p(0) + C_2 \sin p(0))(\cos apt + C_4 \sin apt)$$

$$y(0,t) = (C_1 \cos 0 + C_2 \sin(0))(C_2 \cos apt + C_4 \sin apt).$$

$$y(0,t) = (C_1)\, C_3(\cos apt + C_4 \sin apt) = 0$$

$$\boxed{C_1 = 0}$$

$$\therefore (C_3 \cos apt + C_4 \sin apt \neq 0)$$

Substitude $C_1 = 0$ equation (1) we, get —②

$$y(x,t) = C_2 \sin p(x)\,(C_3 \cos apt + C_4 \sin apt) \quad —③$$

$$\therefore (C_1 = 0)$$

Applying 2nd boundary Condition in Equation —②

$$y(l,t) = C_2 \sin pl\,(C_3 \cos apt + C_4 \sin apt) = 0$$

$$C_2 \sin pl = 0.$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$p = \frac{n\pi}{l}.$$

$\therefore C_2 \neq 0$ if $C_2 = 0$ then we get trivial zero Solution.

(9)

Substituting $p = \frac{n\pi}{l}$ in equation (2) we, get,

$$Y(x,t) = C_2 \sin px \,(C_3 \cos apt + C_4 \sin apt)$$

$$= C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 \cos\left(\frac{n\pi a}{l}\right) t + C_4 \sin\left(\frac{n\pi a}{l}\right) t\right) \quad (3)$$

partially differential Equation (3) with respect to on both sides.

$$Y(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 \cos\left(\frac{n\pi a}{l}\right) t\right) + \left(C_4 \left(\sin\left(\frac{n\pi a}{l}\right) t\right)\right)$$

$$\frac{\partial y}{\partial x}(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right) x \quad C_3 - \sin\left(\frac{n\pi a}{l}\right) t\Big)\left(\frac{n\pi a}{l}\right) + \left(C_4 \cos\left(\frac{n\pi a}{l}\right) t \left(\frac{n\pi a}{l}\right)\right)$$

Applying 3rd Condition in the above equation,

$$\frac{\partial}{\partial t}(x,0) = 0$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 - \sin\left(\frac{n\pi a}{l}\right) 0\right)\left(\frac{n\pi a}{l}\right) + C_4 \left(\cos\left(\frac{n\pi a}{l}\right) 0\right)\left(\frac{n\pi a}{l}\right)\Big)$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 (-\sin 0)\left(\frac{n\pi a}{l}\right)\right) + \left(C_4 (\cos 0)\left(\frac{n\pi a}{l}\right)\right)$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 \cdot 0 \left(\frac{n\pi a}{l}\right) + C_4 (1)\left(\frac{n\pi a}{l}\right)\right) = 0$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \left(0 + C_4 \left(\frac{n\pi a}{l}\right)\right) = 0$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \cdot C_4 \left(\frac{n\pi a}{l}\right) = 0.$$

$\left(\because \sin C_2 \neq 0\right.$, then $C_4\left(\frac{n\pi a}{l}\right)$ must be $\left.0\right)$

$$C_4 = 0 \quad \left[\left(\frac{n\pi a}{l}\right) \neq 0\right]$$

Substitude $C_4 = 0$ in equation (3).

$$y(x,t) = C_2 \cdot C_3 \sin\left(\left(\frac{n\pi}{l}\right)x\right) \cdot \left(\cos\left(\frac{n\pi a}{l}\right)t\right)$$

$$y(x,t) = C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t$$

The most general solution of the given problem is

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)t \quad \text{—— (4)}.$$

Applying 4th boundary condition in equation (4)

$$y(x,0) = F(a)$$

$$y(x,0) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \cos\left(\frac{n\pi a}{l}\right)0 = K(lx - x^2)$$

$$K(lx - x^2) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \quad \because (\cos 0 = 1)$$

This is the form of half rang sine series is

$$C_n = \frac{2}{l}\int_0^l b(x) \sin\left(\frac{n\pi}{l}\right)x \, dx.$$

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$$C_n = \frac{2}{l}\int_0^l k(lx - x^2)\sin\left(\frac{n\pi}{l}\right)x\,dx$$

$$C_n = \frac{2k}{l}\left[(lx - x^2)\left(-\cos\left(\frac{n\pi}{l}\right)x - (-2x)\left(\frac{-\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right) + (0-2)\left(\frac{\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^3}\right)\right)\right]_0^l$$

$$C_n = \frac{2k}{l}\left[\left(0+0\ \frac{-2l^3}{n^3\pi^3}(-1)^n\right) - \left(0+0\ \frac{-2l^3}{n^3\pi^3}(1)\right)\right]$$

$$C_n = \frac{2k}{l}\left[\frac{-2l^3(-1)^n}{n^3\pi^3}(-1)^n + \frac{2l^3}{n^3\pi^3}\right]$$

$$C_n = \frac{2k}{l}\left[\frac{2l^{3\,2}}{n^3\pi^3}\left[-(-1)^n + 1\right]\right]$$

$$C_n = \frac{4k}{n^3\pi^3}\left[1-(-1)^n\right]$$

$$C_n = \begin{cases} 0 & \text{if } n \text{ is even } (-1)^n = n = 1,3,5 \\ \dfrac{8kl^2}{n^3\pi^3} & \text{if } n \text{ is odd } (-1)^n = n = 2,4,6. \end{cases}$$

Substitute C_n value is Equation (4)

$$Y(x,t) = \sum_{n=\text{odd}}^{\infty} \frac{8kl^3}{n^3\pi^3}\sin\left(\frac{n\pi}{l}\right)x\cos\left(\frac{n\pi a}{l}\right)t \ldots$$

∴ Which is the required general solution of the give problem.

(12)

3. A Tightly Streched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi}{l}\right)x$. If it is released from rest from the position find the displacement y at any distance x from one end at any time t.

The One dimensional wave equation is

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

The correct Solution of One dimensional wave equation is.

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt) \quad \text{--- (1)}$$

The boundary Condition are.

1.) $y(0,t) = 0 \quad t \geq 0$

2.) $y(l,t) = 0, \quad t \geq 0.$

3.) $\frac{\partial}{\partial t} y(x,0) = 0, \quad 0 \leq x \leq l.$

4.) $y(x,0) = f(x) = K(t, x^2) = K(lx - x^2).$

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Applying boundary condition (1) in equation — (1)

$$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \cos apt)$$

$$y(0,t) = (C_1 \cos p(0) + C_2 \sin p(0))(C_3 \cos apt + C_4 \sin apt)$$

$$y(0,t) = (C_1 \cos 0 + C_2 \sin 0)(C_3 \cos apt + C_4 \sin apt)$$

$$y(0,t) = (C_1 (1) + C_2(0))(C_3 \cos apt + C_4 \sin apt).$$

$$C_1 (C_3 \cos apt + C_4 \sin apt) = 0$$

$$C_1 = 0$$

(if $a, b = 0$ then either $a = 0$ (or) $b = 0$)

$$(C_3 \cos apt + C_4 \sin apt \neq 0)$$

Substitute $C_1 = 0$ Equation (1) we get.

$$y(x,t) = C_2 \sin px (C_3 \cos apt + C_4 \sin apt) \text{ — (2)}$$

$(\because C_1 = 0)$

Applying 2nd boundary condition in Equation — (2).

$$y(l,t) = C_2 \sin pl (C_3 \cos apt + C_4 \sin apt) = 0$$

$$C_2 \sin pl = 0$$

($C_2 \neq 0$, if $C_2 = 0$, then we get trivial (zero) solution)

$$\sin pl = 0$$

$$pl = n\pi$$

$$p = n\pi/l$$

Substituting $p = \frac{n\pi}{l}$ in equation (5), we get

$$y(x,t) = C_2 \sin px \left(C_3 \cos apt + C_4 \sin apt\right)$$

$$= C_2 \sin\left(\frac{n\pi}{l}\right) x \left(C_3 \cos\left(\frac{n\pi a}{l}\right) t + C_4 \sin\left(\frac{n\pi a}{l}\right) t\right) \quad \text{—— (3)}$$

partially differential equation (3) with respect to 't' on both sides.

$$y(x,t) = C_2 \left(\sin\left(\frac{n\pi}{l}\right) x\right) \left(C_3 \cos\left(\frac{n\pi a}{l}\right) t + C_4 \sin\left(\frac{n\pi a}{l}\right) t\right)$$

$$\frac{dy}{dt}(x,t) = C_2 \sin\left(\frac{n\pi}{l}\right) x \left[C_3 \sin\left(\frac{n\pi a}{l}\right) t \left(\frac{n\pi a}{l}\right) + C_4 \cos\left(\frac{n\pi a}{l}\right) t \left(\frac{n\pi a}{l}\right)\right] \quad \text{—— (4)}$$

Applying 3rd condition in above equation —— (4).

$$\frac{d}{dt}(x,0) = 0.$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x . \left[C_3 \sin\left(\frac{n\pi a}{l}\right) 0 \left(\frac{n\pi a}{l}\right)\right] + \left[C_4 \left(\cos\left(\frac{n\pi a}{l}\right) 0\right) \left(\frac{n\pi a}{l}\right)\right]$$

$$C_2 \sin\left(\frac{n\pi}{l}\right) x \left[C_3 (\sin 0) \left(\frac{n\pi a}{l}\right) + C_4 \cos(0) \left(\frac{n\pi a}{l}\right)\right]$$

$$C_3 \sin\left(\frac{n\pi}{l}\right) x \left[C_3 (0) \left(\frac{n\pi a}{l}\right) + C_4 (1) \left(\frac{n\pi a}{l}\right)\right]$$

$$\therefore C_2 \sin\left(\frac{n\pi}{l}\right) x \cdot C_4 \left(\frac{n\pi a}{l}\right) = 0.$$

(Sin $C_2 \neq 0$ then $C_4 \left(\frac{n\pi a}{l}\right)$ must be zero)

$C_4 = 0 \quad \left(\frac{n\pi a}{l} \neq 0\right)$

Substitute $C_4 = 0$ in equation (3) we get,

$$y(x,t) = C_2 . C_3 \left(\sin\left(\frac{n\pi}{l}\right)x . \cos\left(\frac{n\pi a}{l}\right)t\right).$$

$$y(x,t) = C_n \sin\left(\frac{n\pi}{l}\right)x . \cos\left(\frac{n\pi a}{l}\right)t.$$

The most general solution of the given problem is

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x . \cos\left(\frac{n\pi a}{l}\right)t \quad \text{——— (5)}.$$

Applying the 4th boundary condition in equ (5).

$$y(x,0) = f(x) = y_0 \sin^3\left(\frac{\pi}{l}\right)x \quad \left(\because \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)\right)$$

$$\sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x = y_0 \sin^3\left(\frac{\pi}{l}\right)x \qquad \text{Here } x = \left(\frac{\pi}{l}\right)x$$

$$C_1 \sin\left(\frac{\pi}{l}\right)x + C_2 \sin\left(\frac{2\pi}{l}\right)x + C_3 \sin\left(\frac{3\pi}{l}\right)x + \dots$$

$$= y_0 \left[\frac{1}{4}\left(3\sin\left(\frac{\pi}{l}\right)x - \sin 3\left(\frac{\pi}{l}\right)x\right)\right]$$

By comparing corresponding co-efficient of $\sin\left(\frac{\pi}{l}\right)x$. $\sin\left(\frac{3\pi}{l}\right)x$ on both sides.

$C_1 = \frac{3}{4} Y_0$, $C_2 = 0$, $C_3 = -\frac{Y_0}{4}$, $C_4 = 0$, $C_5 = 0$

Substitute in the most general equation (5)

$$Y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right) x \cdot \cos\left(\frac{n\pi a}{l}\right) t.$$

$$= C_1 \sin\left(\frac{\pi}{l}\right) x \cos\left(\frac{\pi a}{l}\right) t + C_3 \sin\left(\frac{3\pi}{l}\right) x \cos\left(\frac{3\pi a}{l}\right) t.$$

$(C_2, C_4, C_5 = 0)$

$$Y(x,t) = \frac{3Y_0}{4} \sin\left(\frac{\pi}{l}\right) x \cdot \cos\left(\frac{\pi a}{l}\right) - \frac{Y_0}{4} \sin\left(\frac{3\pi}{l}\right) x \cos\left(\frac{3\pi a}{l}\right) t$$

∴ Which is the required Solution of the given problem.

4.) A Square plate is Bounded by the lines $x=0$, $Y=0$ and $x-Y=20$, its faces are insulated, the temperature along upper horizonal line is given by $u(x, 20) = x(20-x)$ when $0 < x < 20$ while other three edges are kept at Zero $0°c$. Find Steady State Temperature in the plate.

Let

$u(x, y)$ be the temperature at any point x, y then,

$u(x,y)$ satisfies the Laplace Equation.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

The correct solution is.

$$u(x,y) = (A \cos px + B \sin px) \cdot (Ce^{py} + De^{-py})$$

Let

$$u(x, 20) = x(20 - x)$$

$$u(x, l) = x(l - x)$$

The boundary conditions are,

1.) $u(0,y) = 0$, $0 < x < l$.

2.) $u(l,y) = 0$, $0 < y < l$.

3.) $u(x,0) = 0$, $0 < x < l$

4.) $u(x,l) = f(x) = x(l-x)$.

$$u(x,y) = (A \cos px + B \sin px)(Ce^{py} + De^{-py}) \longrightarrow (1).$$

Applying 1st boundary Condition in (1).

$$u(0,y) = 0.$$

$$\Rightarrow (A \cos(0) + B \sin(0))(Ce^{py} + De^{-py}) = 0.$$

$A\left(Ce^{py}+De^{-py}\right)=0$ (If $a-b=0$, then either

$A=0$ $\left(Ce^{py}+De^{-py}\neq 0\right)$

Substitute $A=0$ in equation (1).

$$u(x,y) = B\sin px\left(Ce^{py}+De^{-py}\right) \rightarrow (2)$$

Applying 2nd boundary Condition in equation (2)

$u(l,y)=0$

$B\sin pl \cdot \left(Ce^{py}+De^{-py}\right)=0$ ∴ ($B\neq 0$, if $B=0$, then we get trivial (zero) solution)

$\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = \frac{n\pi}{l}$

Substitute $p=\frac{n\pi}{l}$ in equation (2)

$$u(x,y) = B\sin\left(\frac{n\pi}{l}\right)x\left(Ce^{\left(\frac{n\pi}{l}\right)y}+De^{-\left(\frac{n\pi}{l}\right)y}\right) \rightarrow (3)$$

Applying 3rd boundary Condition in Equation (3)

$u(x,0)=0$

$B\sin\left(\frac{n\pi}{l}\right)x\left(Ce^{\left(\frac{n\pi}{l}\right)0}+De^{-\left(\frac{n\pi}{l}\right)0}\right)=0$

$B\sin\left(\frac{n\pi}{l}\right)x\,(C+D)=0$

$C+D=0 \Rightarrow D=-C$

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Substitute $D = -C$ in equation C.

$$u(x,y) = B \sin\left(\frac{n\pi}{l}\right)x\left(Ce^{\left(\frac{n\pi}{l}\right)y} - Ce^{-\left(\frac{n\pi}{l}\right)y}\right)$$

$$= B \sin\left(\frac{n\pi}{l}\right)x\left[Ce^{\left(\frac{n\pi}{l}\right)y} - Ce^{-\left(\frac{n\pi}{l}\right)y}\right]$$

$\times$ by and $\div$ by $2 \Rightarrow$

$$= 2BC \sin\left(\frac{n\pi}{l}\right)x\left[\frac{e^{\left(\frac{n\pi}{l}\right)y} - e^{-\left(\frac{n\pi}{l}\right)y}}{2}\right]$$

$$= 2BC \sin\left(\frac{n\pi}{l}\right)x \sinh\left(\frac{n\pi}{l}\right)y$$

$$u(x,y) = C_n \sin\left(\frac{n\pi}{l}\right)x \sinh\left(\frac{n\pi}{l}\right)y$$

The most general Equation is.

$$u(x,y) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \sinh\left(\frac{n\pi}{l}\right)y \quad \text{---} (4)$$

Applying 4^{th} boundary condition in equation (4)

$$u(x,l) = f(x) = x(l-x).$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \sinh\left(\frac{n\pi}{l}\right)l = x(l-x)$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \sinh(n\pi) = x(l-x)$$

Let $C_n \sinh n\pi = b_n$

$$\Rightarrow x(l-x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right) x$ in $(0, l)$

This is in the form of Half range sine series.

Now,

$$b_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi}{l}\right) x\, dx$$

$$= \frac{2}{l}\int_0^l x(l-x) \sin\left(\frac{n\pi}{l}\right) x\, dx$$

Applying the bernoull's theorem $uv_1 = u'v_2 + u''v_3 \ldots$

$$= \frac{2}{l}\left[\int_0^l lx \sin\left(\frac{n\pi}{l}\right) x\, dx - \int_0^l x^2 \sin\left(\frac{n\pi}{l}\right) x\, dx\right]$$

$$= \frac{2}{l} l \left[\int_0^l x \sin\left(\frac{n\pi}{l}\right) x\, dx\right] - \frac{2}{l}\left[\int_0^l x^2 \sin\left(\frac{n\pi}{l}\right) x\, dx\right]$$

$$= 2\left[x\left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)}\right) - 1 \frac{-\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right]_0^l$$

$$-\frac{2}{l}\left[x^2\left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)}\right) - 2x\left(\frac{-\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right) + 2\left(\frac{\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^l$$

$$= 2\left[\left(\frac{-l^2}{n\pi}(-1)^n + 0\right) - (0+0)\right] - \frac{2}{l}\left[\left(\frac{-l^3}{n\pi}(-1)^n + 0 + \frac{2l^3(-1)^n}{n^3\pi^3}\right.\right.$$

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$$-\left(0+0+\frac{2l^3}{n^3\pi^3}\right)\Bigg]$$

$$= -\frac{2l^2(-1)^n}{n\pi} - \frac{2}{l}\left[\frac{-l^3(-1)^n}{n\pi} + \frac{2l^3}{n^3\pi^3}(-1)^n - \frac{2l^3}{n^3\pi^3}\right]$$

$$= -\frac{2l^2(-1)^n}{n\pi} - \frac{2}{l}\left[\frac{-l^3}{n\pi}(-1)^n + \frac{2l^3}{n^3\pi^3}\left[(-1)^n-1\right]\right]$$

$$= \frac{-2l^2(-1)^n}{n\pi} + \frac{2l^2(-1)^n}{n\pi} - \frac{4l^2}{n^3\pi^3}\left[(-1)^n-1\right]$$

$$b_n = \frac{-4l^2}{n^3\pi^3}\left[(-1)^n-1\right]$$

if $\begin{cases} n \text{ is odd} & : b_n = \dfrac{8l^2}{n^3\pi^3} \\ n \text{ is even} & : b_n = 0 \end{cases}$

$$\therefore u(x,y) = \sum_{n=1,3,5}^{\infty} \frac{1}{\sinh n\pi}\left(\frac{8l^2}{n^3\pi^3}\right)\sin\left(\frac{n\pi}{l}\right)x \sin\left(\frac{n\pi}{l}\right)y$$

This is from, $b_n = C_n \sinh n\pi$

$$C_n = \frac{b_n}{\sinh n\pi}$$

$\therefore$ In most general solution

$$u(x,y) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cdot \sinh\left(\frac{n\pi}{l}\right)y.$$

Substitute the C_n value.

$$u(x,y) = \sum_{n=odd}^{\infty} \frac{1}{\sinh n\pi} \frac{8l^2}{n^3\pi^3} \sin\left(\frac{n\pi}{l}\right)x \sin\left(\frac{n\pi}{l}\right)y$$

$l = 20$

$$u(x,y) = \sum_{n=odd}^{\infty} \frac{1}{\sinh n\pi} \frac{3200}{n^3\pi^3} \sin\left(\frac{n\pi}{20}\right)x \sin\left(\frac{n\pi}{20}\right)y$$

∴ Required Solution.

5.

a) Derive change of Scale property and Modulation theorem of fourier Series.

STATE AND PROVE CHANGE OF SCALE PROPERTY

STATEMENT:

If $F[f(x)] = F(s)$, then $F[f(ax)] = \frac{1}{|a|} F(s/a)$

where $a \neq 0$.

PROOF:

We know that

Fourier Transform

$$F(f(x)) = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} dx$$

Now $(x = ax)$

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax) \cdot e^{isx} dx$$

Case i) if $a > 0$

By using Substitution Method.

Let $ax = t \qquad t/a = x$

$a\,dx = dt \Rightarrow dx = dt/a$

The limit

i) when $x = -\infty \Rightarrow t = ax = -\infty$

ii) When $x = \infty \Rightarrow t = ax = \infty$

Now

$$F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{is(t/a)} \, dt/a$$

$$= \frac{1}{a} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s)t} \cdot dt \right]$$

$$= \frac{1}{a} \cdot F[s/a]$$

$\therefore F[f(ax)] = 1/a \; F(s/a)$ when $a > 0$.

Case ii) $a < 0$

Now, $F[f(ax)] = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{-\infty} f(t) \cdot e^{is(t/a)} \cdot dt/a$.

$$= \frac{1}{a} \cdot \frac{-1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cdot e^{i(s/a)t} \, dt.$$

$$F[f(ax)] = -1/a \; F(s/a) \quad \text{when } a < 0.$$

From Case i) and case ii)

$$F[f(ax)] = \frac{1}{|a|} F(s/a) \quad , \quad a \neq 0.$$

STATE AND PROVE MODULATION THEOREM:

STATEMENT:

If $F[f(x)] = F(s)$, then

$$F[f(x) \cos ax] = 1/2 \left[F(s+a) + F(s-a)\right]$$

PROOF:

we know that

Fourier Transform.

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \, dx$$

x by Cos ax

$$F\left[f(x)\cos ax\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot \cos ax \cdot e^{isx}\,dx$$

$$\therefore \cos x = \frac{e^{ix}+e^{-ix}}{2}$$

Using this Cosine and Exponential Relation.

$$F\left[f(x)\cos ax\right] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\left(\frac{e^{iax}+e^{-iax}}{2}\right)e^{isx}\,dx$$

$$F\left[f(x)\cos ax\right] = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}\cdot e^{iax}\,dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}\, e^{-iax}\,dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, e^{i(s+a)x}\,dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{i(s-a)x}\,dx\right]$$

$$= \frac{1}{2}\left[F(s+a) + F(s-a)\right]$$

$$\therefore F\left[f(x)\cos ax\right] = \frac{1}{2}\left[F(s+a)+F(s-a)\right]$$

b.) Find the Fourier Transform of the Function

$$f(x) = \begin{cases} 1 & , |x| < a \\ 0 & , |x| > a \end{cases}$$ and deduce that $\int_0^\infty \frac{\sin t}{t}\, dt = \pi/2$.

Given function :

$$f(x) = \begin{cases} 1 & , |x| < a \\ 0 & , |x| > a . \end{cases}$$

We know that,

Fourier Transform

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1) \cdot e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{e^{isx}}{is} \right)_{-a}^{a} .$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{is} \left[(\cos ax + i \sin as) - (\cos ax - i \sin as) \right]$$

$e^{i\theta} = \cos\theta + i\sin\theta$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{is} \left[2i \sin ax \right]$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sin as}{s} .$$

By inversion Formula.

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} \, ds.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \frac{\sin as}{s} \cdot e^{isx} dx$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} \cdot e^{-isx} ds$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} \left[\cos sx - i \sin sx \right] . ds$$

$$= \frac{1}{\pi} \left[\int_{-\infty}^{\infty} \frac{\sin as}{s} \cos sx \, ds - i \int_{-\infty}^{\infty} \frac{\sin as}{s} \sin sx \, ds \right]$$

$$= \frac{1}{\pi} \left[2 \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds \right]$$

$$= \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds$$

$$\therefore f(x) = \frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, dx$$

$$\therefore \frac{1}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \cos sx \, ds = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Put $x = 0$

$$\frac{2}{\pi} \int_{0}^{\infty} \frac{\sin as}{s} \, ds = 1$$

$$\int_0^\infty \frac{\sin as}{s}\, ds = \pi/2.$$

put $(t = as)$

$as = t$.

$a\,ds = dt$.

$ds = dt/a$

$S = t/a$.

limit

U.L

$S = \infty \Rightarrow t \Rightarrow a(\infty) = \infty$

L.L

$S = 0 \Rightarrow t \quad a(0) = 0$

$\therefore$ U.L $= \infty$ L.L $= 0$

Now

$$\int_0^\infty \frac{\sin as}{s}\, ds = \pi/2.$$

$$\int_0^\infty \frac{\sin t}{(t/a)} \frac{dt}{a} = \pi/2.$$

$$\therefore \int_0^\infty \frac{\sin t}{t} \cdot dt = \pi/2.$$

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH
B.TECH, II YEAR - 2022 - 2023
INTERNAL MARKS - CONSOLIDATED MARK STATEMENT
SUBJECT NAME : TRANSFORMS AND BOUNDARY VALUE PROBLEMS SUBJECT CODE: U20MABT03

SL NO.	Reg No.	Marks (50)	Is Absent	Student Name
1	U21EC001	43	NO	ABBANNAGARI JAYACHANDRA
2	U21EC003	47	NO	ADDEPALLI AKHILESH
3	U21EC004	42	NO	AGASTINRAAJ A
4	U21EC005	40	NO	AILURI PRADEEP REDDY
5	U21EC006	41	NO	S AJITH KUMAR
6	U21EC007	42	NO	AKULA DILEEP KUMAR
7	U21EC008	38	NO	ALAVALA NAGI REDDY
8	U21EC009	44	NO	ALURI VENKATA GOPI
9	U21EC010	48	NO	A ESWARA NAGENDRA NATHA RAO
10	U21EC011	31	NO	ANCHULA RAVI TEJA
11	U21EC012	43	NO	ANDE SRUTHI
12	U21EC013	41	NO	ANGATI VIJAYTEJA
13	U21EC014	38	NO	ANU R
14	U21EC015	36	NO	ARI VISHNU C
15	U21EC016	40	NO	ARUM NAVEEN
16	**U21EC017**	0	YES	ATMAKURI VAMSI KRISHNA
17	U21EC018	40	NO	AVVAGARI VENU
18	U21EC020	42	NO	BADDAM VARSHITH REDDY
19	U21EC021	47	NO	BANDAM NAVYA SRI
20	U21EC022	42	NO	BANDARU MADHAVA RAO
21	U21EC023	40	NO	BANDARU VINEETH
22	U21EC024	41	NO	BATHULA DILEEP SIVA
23	U21EC025	43	NO	BATTARUSETTY SREE CHANDANA
24	U21EC026	34	NO	BELLAMKONDA NIKHIL
25	U21EC027	42	NO	BENDU RAJESH
26	U21EC028	37	NO	BHAVANAM SIVA REDDY
27	U21EC029	39	NO	BIMANABOINA SUKUMAR
28	U21EC030	37	NO	BODAGAM MAHESH
29	U21EC031	43	NO	BOGALA NIKESH REDDY
30	U21EC032	42	NO	BOINA SIDDARDHA
31	U21EC033	39	NO	BOJJA VENKATA SIVANNARAYANA
32	U21EC034	38	NO	BOORA SIDDHARDHA
33	U21EC035	35	NO	BOREDDY RAJASIMHA REDDY
34	U21EC036	49	NO	BORRA NARENDRA
35	U21EC037	37	NO	BOYA KATLA HEMASANKAR
36	U21EC038	40	NO	BUJALA RAVI SANKAR REDDY
37	U21EC039	47	NO	BYRAPUNENI VENKATRAO
38	U21EC040	38	NO	BYREDDY RAMA KRISHNA REDDY
39	U21EC041	38	NO	BYREDDY SOWMITH REDDY
40	U21EC042	41	NO	C.VENKATA SAI DURGA RAMA AJITH
41	U21EC044	40	NO	CHANDAN KUMAR MALLIK
42	U21EC045	39	NO	CHEREDDY SURENDRANATH REDDY
43	U21EC046	40	NO	CHERUKU KOTESHWAR
44	U21EC047	39	NO	C.NARASIMHA REDDY
45	U21EC048	41	NO	CHINTHA PAVAN KUMAR REDDY
46	U21EC049	37	NO	CHIRUVOLU JASWANTH
47	U21EC050	37	NO	CHITRACHEDU SATHEESH KUMAR
48	U21EC051	32	NO	CHITTETI SURYA
49	U21EC052	33	NO	DASARI DILEEP KUMAR
50	U21EC053	38	NO	DASARI HAREESH
51	U21EC054	46	NO	DASARI MOHAN KUMAR
52	U21EC055	41	NO	DASARI PRUDVI SAI
53	U21EC056	37	NO	DHARAVATH RAKESH
54	U21EC057	40	NO	DIVYA K
55	U21EC058	38	NO	DODDA PRAVEEN
56	U21EC059	40	NO	DOMMETI DHARMA TEJA
57	U21EC060	37	NO	DOOLAM ROHITH GOUD

58	U21EC061	41	NO	EDIGA SHYAM SUNDAR GOUD
59	U21EC062	40	NO	EEDARA THIRAPATHI REDDY
60	U21EC063	44	NO	EESAMBETI SIVA SAI
61	U21EC064	39	NO	ENUGU ARTHIK REDDY
62	U21EC065	46	NO	ERLA THIRUMALESH
63	U21EC066	39	NO	G.L.V.GIRISH CHANDRA
64	U21EC067	41	NO	G.VENKATA GIRISH CHANDRA
65	U21EC068	42	NO	GALAM GURU HEMANTH
66	U21EC069	42	NO	GALLANKI HEMANTH
67	U21EC070	39	NO	GANDRA GOPI
68	U21EC071	31	NO	GANDURI HANUMANTHA RAO
69	U21EC072	39	NO	G.OMPRADEEP REDDY
70	U21EC073	48	NO	GANNAVARAPU SRINIVASA RAO
71	U21EC074	47	NO	GARAPATI POOJITHA
72	U21EC075	40	NO	GEDELA VASU
73	**U21EC076**	0	YES	GINJUPALLI VISHNU CHARAN
74	U21EC077	40	NO	GOBBILLA BALAJI
75	U21EC078	41	NO	GOBBILLA VAMSI
76	U21EC079	43	NO	GODUGU YELISHA
77	U21EC080	39	NO	GOLLA AJAY
78	U21EC081	42	NO	GORRE GNANA DEEPIKA
79	U21EC082	39	NO	GOSU VAMSI SAI
80	U21EC083	39	NO	GUJJULA BHARATH KUMAR
81	U21EC084	49	NO	GUMMA GURU PRASANNA
82	U21EC085	41	NO	GUNASREE V
83	U21EC086	36	NO	GUNDA BHARATH KUMAR
84	U21EC087	49	NO	GUNDLA UDAY KRISHNA
85	U21EC088	42	NO	G.SRI DURGA VENKATESWARA RAO
86	U21EC089	39	NO	IDAVALAPATI VIJAY KUMAR
87	U21EC090	39	NO	IJJAGIRI VENKATESHWARLU
88	U21EC091	40	NO	ILASAGARAPU YASWANTH
89	U21EC092	39	NO	JAKKU SRUTHI REDDY
90	U21EC093	42	NO	JALADANKI BHAGAVAN
91	U21EC094	46	NO	JEEVAROSHAN BARNS R
92	U21EC095	36	NO	JOTHIHARIPRASAD P
93	U21EC096	36	NO	JUTURU RAMESH BABU
94	U21EC097	39	NO	KAKARAPALLI SURYA SIVA KUMAR
95	U21EC098	40	NO	KALLEPALLI SHIVA SAI
96	U21EC099	42	NO	KALVA BHANUPRASAD
97	U21EC100	41	NO	KALVACHARLA ABHILASH
98	U21EC101	36	NO	KAMANI GOPI
99	U21EC102	35	NO	KAMANI VENKATA MANIKANTA
100	U21EC103	42	NO	KAMBHAMPATI SAMBASIVARAO
101	U21EC104	41	NO	K.SAKETH REDDY
102	U21EC105	40	NO	KANALA DEEKSHITH REDDY
103	U21EC106	42	NO	KANAPARTHI ABHINAY
104	U21EC107	40	NO	KANAPARTHI SIVANANDA REDDY
105	U21EC108	44	NO	KANCHARLAPALLI PRAKASH RAJ
106	U21EC109	40	NO	KANCHERLA VENKATESWARLU
107	U21EC110	36	NO	KANDALA SURENDRA
108	U21EC111	36	NO	KANDULA DIVYA
109	U21EC112	45	NO	KANDULA KIRAN KUMAR
110	U21EC113	41	NO	KANHAIYA KUMAR
111	U21EC114	35	NO	KANKANALA MANIVISHNU
112	U21EC115	41	NO	KARIKI BHARATH KUMAR
113	U21EC116	40	NO	KATRAPALLI BRAHMA GANESH
114	U21EC117	41	NO	KATTA GOWTHAM REDDY
115	U21EC118	42	NO	KATURI VEERENDRA GOPI
116	U21EC119	36	NO	KAVERIPAKAM VINOD KUMAR
117	U21EC120	43	NO	R KEERTHI REDDY
118	U21EC121	41	NO	KINTHADA SAI SAMBHAV
119	U21EC122	39	NO	KIRSAKAR ROHITH
120	U21EC123	40	NO	KIRUBA E
121	U21EC124	43	NO	KISHTIPATI KARTHIK
122	U21EC125	40	NO	KODI YASHWANTH KUMAR
123	U21EC126	46	NO	KODUMUR SAMEER

124	U21EC127	41	NO	KODURU BALAKRISHNA
125	U21EC128	40	NO	KOKKERA GOPI
126	U21EC129	40	NO	KOLASANAKOTA VINOSH BABU
127	U21EC130	40	NO	KOLLA ASRITH CHOWDARY
128	U21EC131	38	NO	KOLLI DASTHA GIRI SAI
129	U21EC133	38	NO	K.VISHNU VARDHAN REDDY
130	U21EC134	35	NO	KOPPULA SHARATH
131	U21EC135	41	NO	KOSURI E K N V S LAKSHMI SATISH
132	U21EC136	40	NO	KOTHA HARSHAVARDHAN PATEL
133	U21EC137	40	NO	KOTHAPALLI HEMANTH
134	U21EC138	40	NO	K.NAGA SAI PRAKASH REDDY
135	U21EC139	44	NO	KUKATLA VISHNU VARDHAN
136	U21EC140	41	NO	KURUVA KUMARA SWAMY
137	U21EC141	40	NO	KURUVA MANOHAR
138	U21EC142	39	NO	LAKSHMIPATHI BALAJI M
139	U21EC143	37	NO	L.SIVA SAI MANI CHAKRAM
140	U21EC144	45	NO	LOCHARLA BHAVYA SRI
141	U21EC145	39	NO	LOKA SHASHANK
142	U21EC146	40	NO	MADDALA KRISHNA KOUSHIK
143	U21EC147	41	NO	MADDINA JAGADEESH KUMAR
144	U21EC148	42	NO	MADDULURI SIVA TEJA
145	U21EC149	38	NO	MADDUR VINAYAKA
146	U21EC150	47	NO	MADDURI DEDEEPYA
147	U21EC152	38	NO	MALEWAR DEVENDER RAVI
148	U21EC153	40	NO	MALLAMPATI SAI RAM
149	U21EC154	37	NO	MALLELA AMAR VENKATA SIDHU
150	U21EC155	40	NO	MALLEMPATI SRIRAM
151	U21EC156	43	NO	MAMIDISETTI HARI KIRAN
152	U21EC157	42	NO	MANAMASI CHENCHUBALU
153	U21EC158	35	NO	MANCHALA SAI SRIKAR
154	U21EC159	41	NO	MANDAVA CHARAN
155	U21EC160	40	NO	M.SURYA HARINADH YESWANTH
156	U21EC161	41	NO	MANGALA RAKSHITHA
157	U21EC163	45	NO	M.SATYA SURYA VARA PRASAD
158	U21EC164	44	NO	MARISETTI SRI SAI KIRAN GOUD
159	U21EC165	41	NO	MARRAPU DINESH
160	U21EC166	39	NO	M.SIVA SATHISH KUMAR SWAMY
161	U21EC167	40	NO	MARUTHI NAVEEN
162	U21EC168	39	NO	MATTA SAKETH
163	U21EC169	34	NO	MOORABOYANA POTHURAJU
164	U21EC170	41	NO	MOPIDEVI KRISHNA MOHAN
165	U21EC171	40	NO	MORRABOINA PEDDA RAJU
166	U21EC172	45	NO	MUDDADA AMMANNAIDU
167	U21EC173	40	NO	MUDDINENI SAI KIRAN
168	U21EC174	39	NO	M.VENKATA TEJESWARA REDDY
169	U21EC175	40	NO	MULAKKAYALA VIJAY SIMHA REDDY
170	U21EC176	40	NO	MUNUBARTHI JITHENDRA
171	U21EC177	32	NO	M. REDDY ASRITH REDDY
172	U21EC178	40	NO	MUTHYALA MANOJ
173	U21EC179	41	NO	MUTTHINENI SAI TEJA
174	U21EC180	38	NO	MUVVA AASRITHA
175	U21EC181	39	NO	NADIKOTA RAJ KUMAR
176	U21EC182	48	NO	NAGANAMONI DEEPTHI CHAITHANYA
177	U21EC183	40	NO	NAGELLI MANIDEEP
178	U21EC184	41	NO	NALAGESIGARI UDAY KIRAN REDDY
179	U21EC185	42	NO	NALAPAREDDY GARI SATHVIKA
180	U21EC186	40	NO	NALLALA LAKSHMI MOKSHITH
181	U21EC187	44	NO	N.SAI HARSHAVARDHAN KUMAR
182	U21EC188	41	NO	NANCHARLA VIKRAMA CHARY
183	U21EC189	40	NO	NASANA RAVINAGA MANIKANTA
184	U21EC190	41	NO	NEMALIDINNE VENKATA RAMANA
185	U21EC192	40	NO	NITHISH KUMAR V
186	U21EC193	40	NO	NULI VENKATA PRASAD
187	U21EC194	0	YES	NUTHALAPATI DEEPAK
188	U21EC195	40	NO	NUTHALAPATI LALITHA SRAVANTH
189	U21EC196	47	NO	OLLEM ANITHA

190	U21EC197	47	NO	PADAMATA NAGA VENKATA SAI
191	U21EC198	45	NO	PAGADALA MANI DEEPAK
192	U21EC199	47	NO	PAINNI SAI RAMA KRISHNA
193	U21EC200	48	NO	P.NAGA VENKATA SAI KUMAR
194	U21EC201	44	NO	PALAM MANIKANTA
195	U21EC202	45	NO	PALANKI UDAYA PHANI
196	U21EC203	47	NO	P.BHARATH KUMAR REDDY
197	U21EC204	44	NO	PALLA VENKATA SHIVA
198	U21EC205	45	NO	PALLAPOTHULA KONDA REDDY
199	U21EC206	48	NO	PALURU POOJITHA
200	U21EC207	47	NO	PAMARTHI NESWANTH GOUD
201	U21EC208	45	NO	PANDITA SURYA
202	U21EC209	46	NO	P.VENKATA KRISHNA REDDY
203	U21EC210	45	NO	PARIMI NAVEEN
204	U21EC211	44	NO	PASUPULETI RAKESH
205	U21EC212	48	NO	PATAN ASLAM KHAN
206	U21EC213	36	NO	PATAN MASTAN VALI
207	U21EC214	44	NO	PATHURI SAI NIKETH
208	U21EC215	43	NO	P.SRI SATYA KUMAR PAVAN MOHAN
209	U21EC216	45	NO	P.NAGASIVA HEMANTHKUMAR
210	U21EC217	34	NO	PERABOINA JASWANTH
211	U21EC218	49	NO	PERLA VENKATA SRI NIKHILA
212	U21EC219	45	NO	PIKKILI VISHNU VARDHAN
213	U21EC220	33	NO	PINNINTI SRI KIREETI
214	U21EC221	45	NO	POCHAMPALLY SWAPNA
215	U21EC222	46	NO	PODAPATI PREM CHAND
216	U21EC223	46	NO	POLEMREDDY BHASKAR REDDY
217	U21EC224	49	NO	PONNAMANDA MOUNIKA
218	U21EC225	48	NO	PONNAPUDI RAVI KUMAR
219	U21EC226	46	NO	POOJA R J
220	U21EC227	49	NO	POTHULA RAGHAVENDRA REDDY
221	U21EC228	48	NO	PRADEEP NAMOJU
222	U21EC229	44	NO	P.VISHNUCHAITHANYA
223	U21EC230	31	NO	PUNYALA VAMSI KRISHNA REDDY
224	U21EC231	46	NO	PUTHUMBAKA PRANAY TEJA
225	U21EC232	49	NO	RACHARLA VINEETH
226	U21EC233	46	NO	RAMUNI KRISHNA REDDY
227	U21EC234	48	NO	RANJITH KUMAR S
228	U21EC235	49	NO	RAUSHAN KUMAR SAHNI
229	U21EC236	47	NO	RAVOORI UDAY KIRAN
230	U21EC237	39	NO	ROHIT KUMAR
231	U21EC238	47	NO	SAGINALA VENKATESWARA RAO
232	U21EC239	48	NO	SANDURI ASHOK KUMAR
233	U21EC240	48	NO	SANGA AKSHAY
234	U21EC241	48	NO	SAPPA BHANU TEJA
235	U21EC242	46	NO	SARIBALA SANDEEP KUMAR REDDY
236	U21EC243	47	NO	SARIDE YESU DEEPAK
237	U21EC244	47	NO	SARUPURU LOKESH
238	U21EC245	47	NO	SATHISH S
239	U21EC246	49	NO	SHAIK KHASIM SHAREEF
240	U21EC247	47	NO	SHAIK MUJAHID PARVEZ
241	U21EC248	49	NO	SHAIK MUNVAR KHAJAVALI
242	U21EC249	32	NO	SHAIK NAFISHA
243	U21EC250	46	NO	SHAIK NAGUR SHARIF
244	U21EC251	49	NO	SHAIK RIZWAN BASHA
245	U21EC252	47	NO	SIDDA SATYA SAI VENKATA TEJA
246	U21EC253	49	NO	SIDDHARAPU PRAVEEN
247	U21EC254	47	NO	SINDHIYA S
248	U21EC255	49	NO	SINGAM GANGA VASUNDHAR REDDY
249	U21EC256	47	NO	SINGAMSETTI LAKSHMI PRANITHA
250	U21EC257	46	NO	SK ASIF
251	U21EC258	46	NO	SODISETTI SIVA CHAITANYA
252	U21EC259	44	NO	SUNKE ADITHYA NETHA
253	U21EC260	48	NO	SUNNAMPALLI KEERTHI REDDY
254	U21EC261	49	NO	SURAM YESHWANTH REDDY
255	U21EC262	46	NO	SUWALA SUNAND KUMAR

256	U21EC263	48	NO	SWARNA SHASHANK
257	U21EC264	48	NO	SWARNA SRUTHIKA
258	U21EC265	46	NO	TAMMANA LAKSHMI GANAPATHI
259	U21EC266	49	NO	TANNERU SURYA
260	U21EC267	47	NO	TATI KARTHIK
261	U21EC268	49	NO	TEKULAPALLY ANIL
262	U21EC269	45	NO	THOTA AJAY
263	U21EC270	49	NO	THOTA REDDAIAH
264	U21EC271	48	NO	THOTLI USHA
265	U21EC272	49	NO	UPPALA ANIL
266	U21EC273	49	NO	UPPALAPATI SATISH
267	U21EC274	44	NO	URUTURU HARSHAVARDHAN
268	U21EC275	47	NO	VADAPALLI PHANINDRA KUMAR
269	U21EC276	48	NO	VAIKANTI VENKATA RAMANA
270	U21EC277	46	NO	VALLAMKONDA SURESH
271	U21EC278	45	NO	VALLAPUDASU VINAY KUMAR
272	U21EC279	46	NO	VALLEPU ARAVIND
273	U21EC280	47	NO	VANGALA SULOCHANA
274	U21EC281	45	NO	VANNAM BHANU PRAKASH
275	U21EC282	45	NO	VATTAM GANESH KUMAR REDDY
276	U21EC283	46	NO	VATTIKONDA CHANDU
277	U21EC284	45	NO	VAVILALA BADRINADH SAI
278	U21EC285	46	NO	VEERAMALLA VAMSI
279	U21EC286	49	NO	VEERAMREDDY SRIDHAR REDDY
280	U21EC287	45	NO	VEERANKI SWARUP KUMAR
281	U21EC288	48	NO	VEMULA NAVEEN
282	U21EC289	42	NO	VENDRA DHINESH
283	U21EC290	50	NO	VENGALANENI RAVINDRA
284	U21EC291	48	NO	V.UMA MAHESWAR REDDY
285	U21EC292	45	NO	VIGNESH K
286	U21EC293	45	NO	VIMAL SURYA S
287	U21EC294	45	NO	Y.HEMANTH KUMAR REDDY
288	U21EC296	46	NO	YARASANI VENKATA REDDY
289	U21EC297	49	NO	YASWANTH R
290	U21EC299	48	NO	YERRAGUNTA ANITHA
291	U21EC300	45	NO	YERVA VENKATA RAMI REDDY
292	U21EC301	46	NO	KEJAL RAI
293	U21EC302	49	NO	ARVAPALLY ARAVIND
294	U21EC303	49	NO	AGATHAMUDI MANIRATNAM
295	U21EC304	49	NO	AMIT KUMAR
296	U21EC305	46	NO	ANKALA NAGARAJU
297	U21EC306	47	NO	ANDAMANI BHOOMIKA
298	U21EC307	46	NO	ANNAPUREDDY NARENDRA REDDY
299	U21EC308	49	NO	ANNEM LIKHITHA
300	U21EC309	46	NO	ARAVA MOHITH KUMAR REDDY
301	U21EC310	45	NO	ARUMALLA HEMANTH REDDY
302	U21EC311	49	NO	ASUNDI MADHURI
303	U21EC312	47	NO	AYILURI SIRISHA
304	U21EC313	45	NO	AZMEERU JASWANTH
305	U21EC314	47	NO	BADURU CHAITANYA
306	U21EC315	47	NO	BANDIREVU NAGESH KUMAR REDDY
307	U21EC316	47	NO	BANGARI KRISHNAPRASAD
308	U21EC317	47	NO	BATHALA DINESH
309	U21EC318	45	NO	BINIDI LALITHA
310	U21EC319	47	NO	BODIPUDI NITHIN CHOWDARY
311	U21EC320	45	NO	BOODU RAM TEJA
312	U21EC321	45	NO	BUDAMGUNTLA MAHESH BABU
313	U21EC322	47	NO	BUDAMGUNTLA SANDEEP
314	U21EC323	47	NO	BUKKARAJU PRIYANKA
315	U21EC324	46	NO	BURRA SREENIVASULU
316	U21EC325	45	NO	CHARALA SRI DATTU
317	U21EC326	46	NO	CHETTI CHARAN TEJA
318	U21EC327	46	NO	CHINTHAM CHAKRADHAR REDDY
319	U21EC328	46	NO	DANDU GANESH
320	U21EC329	45	NO	D. CHENNAKESAVA REDDY
321	U21EC330	48	NO	GAMPA JAGADEESH KUMAR

322	U21EC331	47	NO	G. RAMA GOVINDA ROHITH
323	U21EC332	46	NO	GAYAM HANUMA REDDY
324	U21EC333	45	NO	G. MAHESWARA 17HISHEK
325	U21EC334	46	NO	GOSU SIVA KARTHIK
326	U21EC335	47	NO	GOTTIMUKKULA KEERTHANA
327	U21EC337	43	NO	JAMPALA KOTESWARA RAO
328	U21EC338	45	NO	JHADE SAI JAYANTH KUMAR
329	U21EC340	45	NO	KAMATHAM KARTHIK
330	U21EC341	47	NO	KAPU ANILKUMAR REDDY
331	U21EC342	41	NO	KARAN KUMAR
332	U21EC344	46	NO	KEVIN D
333	U21EC345	45	NO	KILLANA NARESH
334	U21EC346	46	NO	KIRUBA SHANGARI
335	U21EC347	47	NO	KOLA KALPANA
336	U21EC348	46	NO	KOLLA CHANDRASEKHAR REDDY
337	U21EC349	46	NO	KOPPULA JOEL SURESH NEEL
338	U21EC350	45	NO	KOTHAPALLI SRINIVAS CHOWDARY
339	U21EC351	46	NO	KUNSOTH CHANDULAL
340	U21EC352	47	NO	KUNUTHURU DEVENDAR REDDY
341	U21EC353	46	NO	KURRAPOTHULA KARTHIK
342	U21EC354	46	NO	LAVANYA S
343	U21EC355	45	NO	LODANGI NAVEEN
344	U21EC356	47	NO	L.SATYA BHARGAV REDDY
345	U21EC357	45	NO	M.MALLIKARJUNA NAIDU
346	U21EC358	31	NO	M.JAGAN MOHAN KRISHNA
347	U21EC359	47	NO	MANGISHETTI DINESH KUMAR
348	U21EC360	39	NO	MANISH KUMAR YADAV
349	U21EC361	48	NO	MARISE VEERA DURGA
350	U21EC362	47	NO	MULLAH TH17EESH AHAMMAD
351	U21EC363	45	NO	MUNDLAPATI SATWIK
352	U21EC364	46	NO	MUTHUKUMAR L
353	U21EC365	47	NO	NADELLA GOWTHAM
354	U21EC366	48	NO	NAGANDLA NAVYA
355	U21EC367	45	NO	N.VENKATA KESAVA REDDY
356	U21EC368	46	NO	NALLAMALA VENKATA AMULYA
357	U21EC369	48	NO	NALLI MAHENDRA
358	U21EC370	47	NO	NARAYANASETTI PRABASH
359	U21EC371	46	NO	NESHANTH M
360	U21EC372	46	NO	NIDADAVOLU MANIKANTA
361	U21EC373	0	YES	NIKKI KUMAR
362	U21EC374	47	NO	NUSUM ARAVIND REDDY
363	U21EC375	47	NO	PAGADALA MAHENDRA DHONI
364	U21EC376	46	NO	PAGILLA JEEVAN KUMAR
365	U21EC377	46	NO	PAPAIAHGARI NAVEEN
366	U21EC378	48	NO	PASALA SAI SUDHEER
367	U21EC379	46	NO	PENDURTHI HASWANTH
368	U21EC380	46	NO	POLAKAL SOMASEKHAR REDDY
369	U21EC381	49	NO	RAJOLI VENKATA RAMANA
370	U21EC382	47	NO	RAJUPALLI REDDY JYOTHI REDDY
371	U21EC383	45	NO	RAMADUGU DILEEP CHARI
372	U21EC384	45	NO	RAVICHANDRAN S
373	U21EC385	49	NO	RAVULA ANAND
374	U21EC386	46	NO	A ROHITH KUMAR
375	U21EC387	48	NO	SALUPALA MADHAN
376	U21EC388	45	NO	SANAPALA GEETANAND
377	U21EC389	46	NO	SANTHA YASWANTH
378	U21EC390	48	NO	SANTHOSH S
379	U21EC391	46	NO	SARIHADDU SIVA KUMAR
380	U21EC392	46	NO	SARLANA VINOD KUMAR
381	U21EC393	46	NO	SHAIK ANWAR BASHA
382	U21EC395	44	NO	SHAIK HABEEBULLA
383	U21EC396	45	NO	SHAIK HALEEM
384	U21EC397	46	NO	SHYAMALA MAHENDAR REDDY
385	U21EC398	32	NO	SOHRAB ALI
386	U21EC399	47	NO	SONTAM RAMESH REDDY
387	U21EC400	47	NO	SUNKARI SURYADEEP

388	U21EC401	47	NO	SURYAPRAKASH K
389	U21EC402	43	NO	SYED MOHAMMED MUJEEB
390	U21EC403	47	NO	TADIBOYINA KRANTHI SANDEEP
391	U21EC404	44	NO	TAMIZHARASAN R
392	U21EC405	42	NO	TANALA GOWRI PAVAN
393	U21EC406	47	NO	THOTA REVANTH
394	U21EC407	47	NO	VANGALA NARASIMHA REDDY
395	U21EC408	43	NO	VANTAKULA M S D S PAVAN KUMAR
396	U21EC409	48	NO	VEERAMACHU SAI KRISHNA
397	U21EC410	49	NO	V.VEERA DHANA NAGARJUNA
398	U21EC411	48	NO	YALAMA REDDY GARI MOUNIKA
399	U21EC412	45	NO	YUVABALAJI S
400	U21EC413	47	NO	ABDUL AHAD
401	U21EC415	33	NO	BANDHARAPU VIKRANTH GOUD
402	U21EC416	47	NO	ABBURI SHANMUKA SRINIVAS
403	U21EC417	33	NO	ARASTU RAJ
404	U21EC418	46	NO	BAINA JAGADEESH
405	U21EC419	49	NO	BATTULA YEDUKONDALU
406	U21EC420	46	NO	BIRUDUKOTA DHAMARESWARA SAI
407	U21EC421	45	NO	B.VEERA VENKATA VAMSI
408	U21EC422	47	NO	BOLLU JAGADEESH
409	U21EC423	48	NO	BRINDHA K
410	U21EC424	47	NO	BUSI LAKSHMAN
411	U21EC425	49	NO	CHALLA KRISHNA SAMPATH
412	U21EC426	48	NO	CHINTALA PARASURAM
413	U21EC427	48	NO	DASARI RAJESH
414	U21EC428	48	NO	GANGAVARAPU JASWANTH
415	U21EC429	45	NO	GORAVA MALLIKARJUNA
416	U21EC430	48	NO	JAKKAM MOHAN KRISHNA REDDY
417	U21EC431	47	NO	KAILA SIVA RAMA KRISHNA
418	U21EC432	46	NO	KAMBALA JAGADEESH KUMAR
419	U21EC433	47	NO	KODIMOJU RANJITH KUMAR
420	U21EC434	48	NO	KOLLEBOYINA SRINU
421	U21EC435	46	NO	KUNA VENKATESH
422	U21EC436	45	NO	MANJULA PAVAN KUMAR
423	U21EC437	45	NO	MEDA MODITH REDDY
424	U21EC438	47	NO	MEKALA MANI
425	U21EC439	46	NO	MUNNANGI RAJASEKHAR REDDY
426	U21EC440	45	NO	NULU VENKAT RAO
427	U21EC441	0	YES	PARITALA SEETHARAMAIAH
428	U21EC442	47	NO	PITTA LAXMAN
429	U21EC443	48	NO	PODILA AKHIL
430	U21EC444	45	NO	POLISETTY PRASANTH
431	U21EC445	45	NO	POTHA RENUKA SAI
432	U21EC446	49	NO	PRAVEEN NAIK V
433	U21EC447	33	NO	PRIYANSHU KUMAR GIRI
434	U21EC448	44	NO	PRIYANSHU RAJ
435	U21EC449	47	NO	RAMESH KUMAR KOTHWAL
436	U21EC450	46	NO	RUPIREDDY JOY SAM REDDY
437	U21EC451	45	NO	SAMA ARUN
438	U21EC452	49	NO	SAMIREDDY SASIDHAR REDDY
439	U21EC453	33	NO	SHUBHAM KUMAR SINGH
440	U21EC454	48	NO	DINESH REDDY V
441	U21EC455	46	NO	VALLAPUNENI VAMSI KRISHNA
442	U21EC456	35	NO	VALLEPU THIRUMALESH
443	U21EC457	38	NO	VANGALA SAI CHANDU
444	U21EC458	38	NO	VAVITIKALVA VIJAYA KUMAR
445	U21EC459	39	NO	VIJAY KUMAR RAM
446	U21EC461	36	NO	DATTI OMKAR
447	U21EC462	37	NO	T. RAVI KUMAR REDDY
448	U21EC463	38	NO	T.SUDARSHAN REDDY
449	U21EC464	43	NO	DURGAM RAKESH KUMAR REDDY
450	U21EC465	39	NO	KANKARA MANI KESHAVA REDDY
451	U21EC466	0	YES	KARPURAPU VISHNU
452	U21EC467	41	NO	KORADA ESWAR RAO
453	U21EC468	41	NO	MANCHALA SAI CHARAN REDDY

454	U21EC469	43	NO	MANGAMURI LAKSHMI PRASANNA
455	U21EC470	39	NO	NADUPURI SAI
456	U21EC471	46	NO	NARAKATLA REDDEMMA
457	U21EC472	39	NO	NIKHIL GOUD V
458	U21EC473	40	NO	PUTCHAKAYALA SOBHAN REDDY
459	U21EC475	42	NO	REDDYMALLI KAVYA
460	U21EC476	41	NO	SAYYAD JANI
461	U21EC477	37	NO	SIVANGULA CHARAN SAI
462	U21EC479	41	NO	VISHAL KUMAR
463	U21EC480	39	NO	KATIKE SAI KUMAR
464	U21EC481	39	NO	P.CHANDRA RAM MOHAN REDDY
465	U21EC482	36	NO	JADA SIVA KUMAR REDDY
466	U21EC483	40	NO	VELPULA RAGHU VAMSHI
467	U21EC484	39	NO	VIKRAM SENA B
468	U21EC485	38	NO	BOYA BHASKAR NAYUDU
469	U21EC487	37	NO	MAHESHUNI SHESHU
470	U21EC701	39	NO	A.SRICHACAN
471	U21EC702	39	NO	ABDUL RIYAZ S
472	U21EC703	37	NO	BAIRI ARUNKUMAR
473	U21EC704	39	NO	N BHARATH KUMAR REDDY
474	U21EC705	37	NO	BOKKA JAYA SAI
475	U21EC706	43	NO	BOMMANI BHARATH
476	U21EC707	38	NO	DINESHKUMAR. S
477	U21EC708	41	NO	ENJETI KARTHEEK
478	U21EC709	40	NO	N. JAGADEESWAR REDDY
479	U21EC710	33	NO	JEFRIN SHENO R
480	U21EC711	37	NO	K.VENKATA MURLAI KRISHNA
481	U21EC712	43	NO	KALANGI PAVAN KUMAR
482	U21EC713	41	NO	K.PRANAY KUMAR
483	U21EC714	40	NO	KSHATRI HARI RAM SINGH
484	U21EC715	36	NO	S.LOKESH
485	U21EC716	39	NO	M.PHANEENDRA MANI KUMAR
486	U21EC717	37	NO	MALLAH SAGAR KUMAR
487	U21EC718	36	NO	PAKALAPATI JOSHI
488	U21EC719	38	NO	PURUSHOTHAMAN A K
489	U21EC720	36	NO	RISHI KARTHIKEYAN N
490	U21EC721	37	NO	S.SANTHOSH
491	U21EC722	48	NO	SAREDDY KIRAN KUMAR REDDY
492	U21EC723	38	NO	SONTI SAIKUMAR
493	U21EC724	40	NO	SURISETTY SIVA SAI KUMAR
494	U21EC725	40	NO	V.VINOTHKUMAR
495	U21EC726	44	NO	YATHAM AKHILA
496	U21EC729	40	NO	VAMSI
497	U21EC727	39	NO	G. SWAMI
498	U21EC728	38	NO	T. SAIPRAKASH REDDY
499	U21EC730	37	NO	VINUTH BHARATH
500	U21EC731	38	NO	ANUMULA MANOJ KUMAR

Aerospace
CLA-I Answer key

1) Bernoulli's formula.

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

2) Euler's constants of a fourier series in $(0, 2l)$

$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$

$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi}{l}\right)x\,dx$

3) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

4) $b_n = 0$

5) Half range cosine series is $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$.

6) parseval's identity is $\frac{1}{\pi}\int_0^{\pi} (f(x))^2\,dx = \frac{1}{2}\sum b_n^2$

7(a) $f(x) = x^2 \; (-\pi, \pi)$

$a_0 = \frac{2\pi^2}{3}$

$a_n = \frac{4}{n^2}(-1)^n$

$b_n = 0$

8(a) $b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx\,dx = \frac{-2(-1)^n}{n}$.

9) $f(x) = x$ $\quad b_n = \frac{2}{\pi}\int_0^{\pi} x \sin nx\,dx$.

$b_n = \frac{-2(-1)^n}{n}$.

10 b)

$f(x) = x\cos x$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = 0$

$a_n = \frac{1}{\pi}\int_0^{2\pi} x\cos x \cdot \cos nx\,dx = \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n+1)^2}\right]$

9)

$f(x) = x$

$a_0 = \frac{2}{l}\int_0^l f(x)\cdot\cos\left(\frac{n\pi}{l}\right)x\,dx = l$

$a_n = \frac{2}{l}\int_0^l x\cos\left(\frac{n\pi}{l}\right)x\,dx = \frac{2l}{n^2\pi^2}\left((-1)^n - 1\right)$

$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} = \frac{\pi^4}{96}$

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH
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SCHOOL OF AERONAUTICAL ENGINEERING

Name: Himanshu Yadav 13/10/2022

Reg. No.: U21AS038 Sem / Year: III / II

Examination: CLA-1 Subject: TPDE

Invigilator Sign:

47/50

PART- B

7a. Given function $f(x) = x^2$ in $(-\pi, \pi)$

Fourier series in $(-\pi,\pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=2}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot \sin nx\, dx$$

To find a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\, dx$$

$\because$ x^2 is even function

$$= \frac{1}{\pi}\times 2\int_{0}^{\pi} x^2\, dx \qquad \left[\int_{-a}^{a} f(x)\,dx = 2\cdot\int_{0}^{a} f(x)\, dx \text{ if } f(x) \text{ is even}\right]$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x^2\, dx$$

$$a_0 = \frac{2}{\pi} \times \left(\frac{x^3}{3}\right)_0^\pi$$

$$a_0 = \frac{2}{\pi} \times \frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

To find a_n:-

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cdot \cos nx \, dx$$

$x^2 =$ even, $\cos nx =$ even

Even function × Even function = Even function

$$\therefore a_n = \frac{1}{\pi} \times 2 \int_0^\pi x^2 \cdot \cos nx \, dx$$

u v

$$= \frac{2}{\pi} \quad \int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$$

$$= \frac{2}{\pi}\left[x^2 \cdot \frac{\sin nx}{n} - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\cdot\left(-\frac{\sin nx}{n^3}\right)\right]_0^\pi$$

$$= \frac{2}{\pi}\left[0 + \frac{2\pi}{n^2}\cos n\pi - 2(0) - (0 - 0 + 0)\right]$$

$$= \frac{2}{\pi} \times \frac{2\pi}{n^2}(-1)^n$$

$$= \frac{4}{n^2}(-1)^n$$

To find b_n:

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \sin nx \, dx$$

$x^2 =$ even, $\sin nx =$ odd

odd × even = odd $\quad \therefore \int_{-a}^{a} f(x)\,dx = 0$ if $f(x)$ is odd

$\therefore b_n = 0$

Fourier series $= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$= \frac{2\pi^2}{3 \times 2} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cdot \cos nx + 0$

$= \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cdot \cos nx}{n^2}$

8.a. Given $f(x) = x$

Half range sine series in $(0, \pi) = \frac{1}{2} \sum_{n=1}^{\infty} b_n \cdot \sin nx$

where $b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \sin nx \, dx$

$b_n = \frac{2}{\pi} \int_0^{\pi} x \cdot \sin nx \, dx$

Bernoulli's Formula $= \frac{2}{\pi} \int uv \, dx = uv_1 - u'v_2 + u''v_3 - \ldots$

6

$b_n = \frac{2}{\pi} \left[x \cdot \left(-\frac{\cos nx}{n} \right) - 1 \left(-\frac{\sin nx}{n^2} \right) \right]_0^{\pi}$

$b_n = \frac{2}{\pi} \left[-\frac{\pi}{n} (-1)^n + 0 - (0 - 0) \right]$

$b_n = \frac{2}{\pi} \left[-\frac{\pi (-1)^n}{n} \right]$

$b_n = \frac{-2(-1)^n}{n}$ $\therefore$ Sine Series $= \sum_{n=1}^{\infty} \frac{(-2)(-1)^n \cdot \sin nx}{n}$

$= -2 \sum_{n=1}^{\infty} \frac{(-1)^n \cdot \sin nx}{n}$

$\therefore$ Required half range sine series is

$= \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{(-2)(-1)^n}{n} \right)^2 = \frac{1}{2} \sum_{n=1}^{\infty} \frac{4(-1)^{2n}}{n^2}$

$= \frac{-2}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\because (-1)^{2n} = 1$

9.a. $f(x) = x$ — Given

& ~~Fourier series~~

Half range sine series in $(0, \pi) = \frac{1}{2}\sum_{n=1}^{\infty} b_n^2$

where $b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cdot \sin nx \, dx$

$b_n = \frac{2}{\pi}\int_0^{\pi} x \cdot \sin nx \, dx$

Bernoulli's Formula = $\int uv\,dx = u^0 v_1 - u' v_2 + u'' v_3 - \cdots$

$b_n = \frac{2}{\pi}\left[x \cdot \left(-\frac{\cos nx}{n}\right) - 1\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$

$b_n = \frac{2}{\pi}\left[-\frac{\pi}{n}(-1)^n + 0 - (0-0)\right]$

$b_n = \frac{-2(-1)^n}{n}$

$(b_n)^2 = \frac{4(-1)^{2n}}{n^2} = \frac{4}{n^2}$ $[\because (-1)^{2n} = 1]$

By Parseval's Identity

$\frac{1}{\pi}\int_0^{\pi} (f(x))^2 \, dx = \frac{1}{2}\sum_{n=1}^{\infty} b_n^2$

$\frac{1}{\pi}\int_0^{\pi} x^2 \, dx = \frac{1}{2}\sum_{n=1}^{\infty}\frac{4}{n^2}$

$\frac{1}{\pi} \times \left(\frac{x^3}{3}\right)_3^{\pi} = \frac{1 \times 4}{2}\sum_{n=1}^{\infty}\frac{1}{n^2}$

$= \frac{1}{\pi} \times \frac{\pi^3}{3} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

Hence ~~pro~~ deduced

PART – C

10·b. $f(x) = x\cos x$ – given

Fourier series in the interval $(0, 2\pi)$

$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

where $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$

$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cdot \cos nx\, dx$

$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cdot \sin nx\, dx$

To find a_0:

$a_0 = \frac{1}{\pi} \int_0^{2\pi} x \cos x\, dx$

$= \frac{1}{\pi} \{$

Bernoulli's Formula – $\int uv\, dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$= \frac{1}{\pi} \left[x \cdot \frac{\sin x}{} - 1\left(-\frac{\cos x}{}\right) \right]_0^{2\pi}$

$$= \frac{1}{\pi}\left[x\frac{\sin nx}{n} + \frac{\cos nx}{n^2} \right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - \left(0 + \frac{1}{n^2} \right) \right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2} \right]$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x \cdot \cos nx$$

$$a_0 = \frac{1}{\pi}\left[x \cdot \sin x - 1(-\cos x \right]_0^{2\pi}$$

$$a_0 = \frac{1}{\pi}\left[0 + 1 - (0+1) \right]$$

$$a_0 = \frac{1}{\pi}[1-1]$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x \cdot \cos x \cdot \cos nx \, dx$$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Let $A = nx$, $B = x$

~~$\cos(nx - x$~~ $\cos(A-B) + \cos(A+B) = 2\cos A \cos B$

$$\frac{\cos(nx-x) + \cos(nx+x)}{2} = \cos nx \cdot \cos x$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x \cdot \left[\cos(nx-x) + \cos(nx+x) \right] dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} [x \cdot \cos(nx - x) + x\cos(nx + x)]\,dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x \cdot \cos(nx - x)\,dx + \int_0^{2\pi} x \cdot \cos(nx + x)\,dx\right.$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x \cdot \cos(n-1)x\,dx + \int_0^{2\pi} x \cdot \cos(n+1)x\,dx\right]$$

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3$$

$$= \frac{1}{2\pi}\left[\left(x \cdot \frac{\sin(n-1)x}{n-1} - 1\left(\frac{-\cos(n-1)x}{(n-1)^2}\right)\right)_0^{2\pi}\right.$$

$$- \left.\left(x \cdot \frac{\sin(n+1)x}{n+1} - 1\left(\frac{-\cos(n+1)x}{(n+1)^2}\right)\right)_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\left(+\frac{1}{(n-1)^2} - \left(\frac{1}{(n-1)^2}\right)\right) - \left(0 + \frac{1}{(n+1)^2} - \left(\frac{1}{(n+1)^2}\right)\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n-1)^2} - \frac{1}{(n+1)^2} + \frac{1}{(n+1)^2}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n-1)^2}\right]$$

if $n \neq 1$ & $n > 1$

$\therefore$ $a_n = 0$

otherwise

$a_n = \frac{1}{2\pi}$ for $n = 1$ $\quad a_n = \frac{1}{2\pi}\left[\frac{1}{(n-1)^2} - \frac{1}{(n-1)^2}\right]$

where is a_1?

$a_1 = \pi$

Given function $f(x) = x$

Half Range cosine series in $(0, l)$ $= \frac{a_0^2}{4} + \frac{1}{2} \sum a_n^2$

$$a_0 = \frac{2}{l} \int_0^l f(x)\,dx$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cdot \cos \frac{n\pi}{l} dx$$

To find a_0:

$$a_0 = \frac{2}{l} \int_0^l x\,dx$$

$$= \frac{2}{l} \times \left(\frac{x^2}{2} \right)_0^l$$

$$a_0 = \frac{2}{l} \times \frac{l^2}{2} = l$$

$$a_n = \frac{2}{\pi} \int_0^\pi x \cdot \cos nx\,dx$$

$$= \frac{2}{\pi} \quad \int u\,v\,dx = uv_1 - u'v_2 + u''v_3$$

$$= \frac{2}{\pi} \left[x \cdot \frac{\sin nx}{n} - 1 \left(\frac{-\cos nx}{n^2} \right) \right]_0^\pi$$

$$= \frac{2}{\pi} \left[0 + \frac{1}{n^2} - \left(0 + \frac{1}{n^2} \right) \right]$$

$$= \frac{2}{\pi}$$

$$a_n = \frac{2}{l}\int_0^l x \cdot \cos\left(\frac{n\pi}{l}\right)x\,dx$$

$$a_n = \frac{2}{l}\left[x \cdot \frac{\sin\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)} - 1\left(\frac{-\cos\left(\frac{n\pi}{l}\right)x}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^{\pi l}$$

$$a_n = \frac{2}{l}\left[0 + \frac{l^2}{n^2\pi^2}(-1)^n - \left(0 + \frac{l^2}{n^2\pi^2}\right)\right]$$

$$a_n = \frac{2}{l}\left[\frac{l^2(-1)^n}{n^2\pi^2} - \frac{l^2}{n^2\pi^2}\right]$$

$$a_n = \frac{2l^2}{l \times n^2\pi^2}\left[(-1)^n - 1\right]$$

$$a_n = \frac{2l}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$\frac{4l}{n^2\pi^2}$

n is even $a_n = 0$

n is odd

Putting values in Parseval's Identity

$$\frac{1}{l}\int_0^{l}(f(x))^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\Sigma a_n^2$$

$$\frac{1}{l\pi}\int_0^{\pi l} x^2\,dx = \frac{l^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{2l}{n^2\pi^2}((-1)^n - 1)\right]^2$$

$$= \frac{1}{l}\int_0^l x^2\,dx = \frac{l^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{4l^2}{n^4\pi^4}\left((-1)^{2n} + 1 - 2(-1)^n\right)$$

$$= \frac{1}{l} \times \left(\frac{x^3}{3}\right)_0^l = \frac{l^2}{4} + \frac{4l^2}{2\pi^4}\sum_{n=1}^{\infty}\frac{(-1)^{2n} + 1 - 2(-1)^n}{n^4}$$

$$= \frac{l^3}{3l} - \frac{l^2}{4} = \frac{2l^2}{\pi^4}\sum_{n=1}^{\infty}\frac{2 - 2(-1)^n}{n^4}$$

$$\frac{l^2}{3} - \frac{l^2}{4} = \frac{4l^2}{\pi^4} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^4}$$

$$\frac{l^2}{12} = \frac{4l^2}{\pi^4} \left[\frac{1-(-1)}{1^4} + \frac{1-1}{2^4} + \frac{1-(-1)}{3^4} + \frac{1-1}{4^4} \right.$$

All the time n is even will become zero only odd values of n will be taken.

$$\frac{l^2 \times \pi^4}{12 \times 4l^2} = \frac{2}{1^4} + \frac{2}{3^4} + \frac{2}{5^4} + \cdots$$

$$\frac{\pi^4}{48} = 2\left[\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots \right]$$

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

PART-A

1. $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4$

u = differentiable function, v = integrable function

~~$u_1 = \int u\,dx$, $u_2 = \int u_1\,dx$, $u_3 =$~~

$u_1 = \frac{du}{dx}$, $u_2 = \frac{du_1}{dx}$, $u_3 = \frac{du_2}{dx}$ ...

$v_1 = \int v\,dx$, $v_2 = \int v_1\,dx$, $v_3 = \int v_2\,dx$ ...

2. Euler's constants of a fourier series in $(0, 2l)$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x)\, dx$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \cos\left(\frac{n\pi}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \sin\left(\frac{n\pi}{l}\right) dx$$

2

3. If $f(x)$ is a periodic function and satisfies Dirichlet's conditions then it can be represented in the form of an infinite series called Fourier series.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=2}^{\infty} b_n \sin nx$$

The above eqn is trigonometric form of Fourier series.

a_0 in $(-\pi, \pi)$ $= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$

a_n in $(-\pi, \pi)$ $= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos nx\, dx$

b_n in $(-\pi, \pi) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin nx\, dx$

2

4. $f(x) = x^2$ in $(-\pi, \pi)$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \, dx \cdot \sin nx \, dx$$

2 ∵ b_n - even f, $\sin nx$ - odd

~~$b_n = \frac{2}{\pi} \int_0^{\pi} x^2 \, dx \sin nx \, dx$~~

even × odd = odd

∴ $\int_{-a}^{a} f(x) \, dx = 0$ if $f(x)$ is odd

∴ $b_n = 0$

5. Ho cosine series in $(0, l)$ = ~~$\frac{a_0}{2} + \frac{1}{2} \sum a_n^2$~~

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}\right) x$$

2

Sine series in $(0, l)$ = $\sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{n\pi}{l}\right) x$

1

6. Parseval's identity for half range sine series in $(0, \pi)$

$$\frac{1}{\pi} \int_0^{\pi} [f(x)]^2 \, dx = \frac{1}{2} \sum b_n^2$$

2

BHARATH INSTITUTE OF HIGHER EDUCATION AND RESEARCH

(Declared as Deemed to be University under Section 3 of UGC Act 1956)

Selaiyur, Chennai - 600 073, INDIA

SCHOOL OF AERONAUTICAL ENGINEERING

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Examination: CLA - I Subject: TBVP (U20MABT03)

Invigilator Sign: B. [signature]

27/50

* Section - C :-

2.

Given:-

$f(x) = x(2\pi - x)$ in $(0, 2\pi)$.

The fourier Series for function in $(0, 2\pi)$ is:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

were, the fourier Coefficients are;

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx.$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx.$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx.$$

→ To find a_0 :-

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx.$$

$$= \frac{1}{\pi}\int_0^{2\pi} x(2\pi - x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} (2\pi x - x^2)\,dx.$$

$$= \frac{1}{\pi}\left[2\pi\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{2\pi}$$

11 (b) Given:-

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2

where; the fourier series for first two harmonic is;

$$f(x) = \frac{a_0}{2} + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \ldots$$

where: the coefficients are;

$$a_0 = \frac{2}{n} \Sigma y$$

$$a_1 = \frac{2}{n} \Sigma y \cos x$$

$$b_1 = \frac{2}{n} \Sigma y \sin x$$

$$a_2 = \frac{2}{n} \Sigma y \cos 2x$$

$$b_2 = \frac{2}{n} \Sigma y \sin 2x$$

x	y	$\cos x$	$\sin x$	$\cos 2x$	$\sin 2x$	$y \cos x$	$y \sin x$	$y \cos 2x$	$y \sin 2x$
0	1	1	0	1	0	1	0	1	0
$\frac{\pi}{3} = 60°$	1.4	0.5	0.866	-0.5	0.866	0.7	1.2124	-0.7	1.2124
$\frac{2\pi}{3} = 120°$	1.9	-0.5	0.866	-0.5	-0.866	-0.95	1.6454	-0.95	-1.6454
$\pi = 180°$	1.7	-1	0	1	0	-1.7	0	1.7	0
$\frac{4\pi}{3} = 240°$	1.5	-0.5	-0.866	-0.5	0.866	-0.75	-1.299	-0.75	1.299
$\frac{5\pi}{3} = 300°$	1.2	0.5	-0.866	-0.5	-0.866	0.6	-1.0392	-0.6	-1.0392
	$\Sigma y = 8.7$					$\Sigma y\cos x = -1.1$	$\Sigma y \sin x = 0.5196$	$\Sigma y \cos 2x = -0.3$	$\Sigma y \sin 2x = -0.1732$

To find Coefficients:-

$\Rightarrow a_0 = \frac{2}{6}$ [struck out] $\Sigma y \Rightarrow \frac{2}{6} \cdot (8.7)$

$a_0 = 2.9$

$\Rightarrow a_1 = \frac{2}{6} \Sigma y \cos x$

$= \frac{2}{6} (-1.1)$

$a_1 = -0.366$

$\Rightarrow a_2 = \frac{2}{6} \Sigma y \cos 2x$

$= \frac{2}{6} (-0.3)$

$a_2 = -0.1$

$\Rightarrow b_1 = \frac{2}{6} \Sigma y \sin x$

$b_1 = \frac{2}{6} (0.5196)$

$b_1 = 0.1732$

$\Rightarrow b_2 = \frac{2}{6} \Sigma y \sin 2x$

$= \frac{2}{6} (-0.1732)$

$b_2 = -0.057$

Section - A:-

(5) **half range Cosine Series:**

~~$f(x) = \frac{1}{l}\int_0^l f(x)\cos\left(\frac{n\pi}{l}\right)x\,dx$~~

~~half range~~

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}\right)x$$

half range Sine Series in $(0, l)$ is,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

(6.)

$$\frac{1}{2\pi}\int_0^{\pi} \{f(x)\}^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum b_n^2$$

Section - B:-

(7) (b) Given:-

$$f(x) = x \quad in \ (-l, l)$$

The fourier Series is;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}\right)x + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

Since the $f(x) = x$,

were;

$$f(-x) = -x = -f(x) \text{ is an odd function.}$$

the fourier Coefficients are;

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$= 0 \qquad [\because f(x) \text{ is an odd}].$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x) \cos nx\,dx$$

$$= 0 \qquad [\because f(x) \text{ is an odd } \& \cos nx \text{ is an even}; \ e \cdot 0 = 0].$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x) \sin nx\,dx$$

$$= \frac{2}{l}\int_{0}^{l} f(x) \sin nx\,dx \qquad [\because f(x) \text{ is odd } \& \sin nx \text{ is odd}; \ 0 \cdot 0 = e].$$

→ To find b_n:-

$$b_n = \frac{2}{l}\int_{0}^{l} x \cdot \sin nx\,dx$$

$$= \frac{2}{l}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^l$$

$= \frac{2}{\pi}\left[\left(\frac{-x}{n}\cdot(-1)^n + 0\right) - (0+0)\right]$

$= \frac{2}{}\left[\frac{-1}{n}(-1)^n\right]$

$= -\frac{2}{n}\left[(-1)^n\right]$

$\therefore$ The required fourier series is

$f(x) = -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$

(8) Given:-

$f(x) = x$ in $(0,\pi)$

the half range sine series in $(0,\pi)$.

$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

were, fourier cofficients are;

$b_n = \frac{1}{2\pi}\int_0^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{2\pi}\int_0^{\pi} x.\sin nx$

$= \frac{1}{2\pi}\left[x.\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$

$= \frac{1}{2\pi}\left[\left(\pi\cdot\frac{(-1)^n}{n} + 0\right) - (0+0)\right]$

$= \frac{1}{2\pi}\left[\frac{\pi}{n}\cdot(-1)^n\right.$

$b_n = \frac{1}{2n}(-1)^n$

$\therefore$ The required half range sine series is

$f(x) = \sum_{n=1}^{\infty}\frac{1}{2n}(-1)^n$

were;

$$f(x) = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n}.$$

(9) Given:-

$$f(x) = x \quad \text{in } (0, \pi)$$

were; the sine series in $(0,\pi)$ is.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \, dx.$$

& the fourier coefficients are;

$$b_n = \frac{2}{2\pi} \int_0^{\pi} f(x) \sin nx \, dx.$$

$$= \frac{1}{2\pi} \int_0^{\pi} x \cdot \sin nx$$

$$= \frac{1}{2\pi} \left[x \cdot \left(-\frac{\cos nx}{n} \right) - (1) \left(-\frac{\sin nx}{n^2} \right) \right]_0^{\pi}$$

$$= \frac{1}{2\pi} \left[\left(\pi \cdot \left(\frac{(-1)^n}{n} \right) + 0 \right) - (0 + 0) \right]$$

$$= \frac{1}{2\pi} \left[\frac{\pi}{n} (-1)^n \right]$$

$$b_n = \frac{1}{2n} (-1)^n$$

∴ The required sine series is;

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{2n} (-1)^n$$

$$= \frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Now: the Parseval's identity for Sine Series (0, π) is;

$$\frac{1}{\pi}\int_0^{\pi} (f(x))^2 \, dx = \frac{1}{2} \sum b_n^2$$

$$\frac{1}{\pi}\int_0^{\pi} [x]^2 \, dx = \frac{1}{2}.$$

$$\frac{1}{3\pi}(x^3)_0^{\pi} = \frac{1}{2}.$$

$$\frac{1}{3\pi}(\pi^3) = \frac{1}{2}.$$

$$\frac{1}{2} \cdot \frac{\pi^2}{3} =$$

$$\frac{\pi^2}{6} =$$

* Section - A :-

① **Bernoulli's formulae :-**

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 + u'''v_4 + \dots$$

were; u, u', u'' ... are differentiation of function and; $v_1, v_2, v_3, \dots$ are integral of the functions.

② Euler's Constants in function $(0, 2l)$ are;

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi}{l}\right)x\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi}{l}\right)x\,dx.$$

③ The fourier Series in $(-\pi, \pi)$ is.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

were; fourier Coefficients are:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

(4) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx$

Since: $f(x) = x^2$

$f(-x) = -x^2 = f(x)$ is an even function

and $\sin nx$ is an odd function.

Even × odd = Odd.

$b_n = \frac{2}{\pi}\int_{0}^{\pi} x^2 \sin x\,dx$

$= \frac{2}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$

$= \frac{2}{\pi}\left[\left(\pi^2 \cdot \frac{(-1)^n}{n} + 0 + \frac{2}{n^3}(-1)^n\right) - \left(0 + 0 + \frac{2}{n^3}\right)\right]$

~~$= \frac{2}{\pi}\left[\frac{\pi^2}{n}(-1)^n + \frac{2}{n^3}(-1)^n - \frac{2}{n^3}\right]$~~

$= \frac{2}{\pi}\left[\frac{\pi^2}{n} + \frac{2}{n^3}\left((-1)^n - 1\right)\right]$

$= \frac{4\pi}{n^3}\left[(-1)^n - 1\right].$

(5) half

→1) $y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{apt} + c_4 e^{-apt})$

$y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos apt + c_8 \sin apt)$

$y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$

→2) Fourier's law of thermal conduction states that the rate of heat transfer through a material is proportional to the -ve gradient in the temperature and the area of the surface through which the heat flows.

→3) $\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ $c^2 = \frac{k}{\rho c}$ k = thermal conductivity, ρ = density, c = specific heat capacity.

→4) $F(f(x)) = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x).e^{isx}\,dx$, $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s).e^{-isx}\,ds.$

→5) $\int_{-\infty}^{\infty} |f(x)|^2\,dx = \int_{-\infty}^{\infty} |F(s)|^2\,ds.$

→6) $F_c(s) = F_c(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x).\cos sx\,dx$, $F_c^{-1}(F_c(s)) = f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_c(s) \cos sx\,ds.$

---

7) a) $\frac{\partial u}{\partial t} = 0$ ⇒ $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ ⇒ $\frac{\partial^2 u}{\partial x^2} = 0.$

⇒ $\frac{d^2 u}{dx^2} = 0$ ⇒ $\int \frac{d}{dx}\frac{dy}{dx} = \int 0$ ⇒ $\frac{du}{dx} = c_1$ ⇒ $du = c_1\,dx$

⇒ $\boxed{u = c_1 x + c_2}$

---

7 b) (i) $B^2 - 4AC = 0 - 4y^2x^2 = < 0$ elliptic

(ii) $B^2 - 4AC = 16 - 4(4)(1) = 0$ parabola

8a) $F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} f(x).e^{isx}\,dx$, $F(f(ax)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(ax).e^{isx}\,dx$

Case (i) for $a > 0$ $F(f(ax)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t).e^{is(t/a)} \frac{dt}{a} = \frac{1}{a} F(s/a)$

Case (ii) for $a < 0$ $F(f(ax)) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{-\infty} f(t)\, e^{is(t/a)} \frac{dt}{a} = -\frac{1}{a} F(s/a)$

8b) $F(f(x).\cos ax) = \frac{1}{2}[F(s+a) + F(s-a)]$

$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x).e^{isx}\,dx$

Now $F(f(x).\cos ax) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} f(x).\cos ax\,dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} f(x).\left(\frac{e^{iax} + e^{-iax}}{2}\right) dx = \frac{1}{2}\left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} f(x).e^{i(s+a)x}\,dx + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} f(x).e^{i(s-a)x}\right]$

$= \frac{1}{2}[F(s+a) + F(s-a)].$

9a) $F(e^{-x^2/2}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2/2}.e^{isx}\,dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2 - 2isx)}\,dx$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}((x-is)^2 + s^2)}\,dx = \frac{1}{\sqrt{2\pi}} e^{-s^2/2} \int_{-\infty}^{\infty} e^{-\left(\frac{x-is}{\sqrt{2}}\right)^2} dx$ $\int_0^{\infty} e^{-x^2} dx = \sqrt{\pi}/2.$

$\frac{1}{\sqrt{2\pi}} \cdot \sqrt{2} = \frac{1}{\sqrt{\pi}}$ even fn $2/\sqrt{\pi} \cdot \sqrt{\pi}/2$

9b) $F_s(f(x)) = F_s(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-ax}\sin sx\,dx = \sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{a^2+s^2}(-a\sin sx - s\cos sx)\right]_0^\infty$

$F_s(e^{-ax}) = \sqrt{\frac{2}{\pi}}\ \frac{s}{s^2+a^2}$

$F_c(f(x)) = \sqrt{2/\pi}\int_0^\infty e^{-ax}\cos sx\,dx = \sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{a^2+s^2}(-a\cos sx + s\sin sx)\right]_0^\infty$

$F_c(e^{-ax}) = \sqrt{2/\pi}\cdot \frac{a}{s^2+a^2}.$

10a) $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$ $\quad y(x,t) = (c_1\cos px + c_2\sin px)(c_3\cos apt + c_4\sin apt)$

$y(0,t)=0,\ y(l,t)=0,\ \frac{\partial y}{\partial t}(x,0)=0,\ y(x,0)=f(x)=y_0\sin^3\left(\frac{\pi}{l}\right)x.$

$y(0,t)=0 \Rightarrow (c_1=0)$ $\quad$ $\left(\frac{\partial y}{\partial t}=0\ \ c_4=0\right)$ $\quad y(x,0)=f(x)=y_0\sin^3\left(\frac{\pi}{l}\right)x$

$y(l,t)=0 \Rightarrow \left(p=\frac{n\pi}{l}\right)$ $\quad y(x,t)=\sum C_n\sin\left(\frac{n\pi}{l}\right)x\cdot\cos\left(\frac{n\pi a}{l}\right)t$ $\quad \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$ $\quad \frac{3\sin x - \sin 3x}{4}$

$c_1 = 3y_0/4,\ c_3 = -y_0/4,\ c_2, c_4, c_5\cdots = 0$

$y(x,t) = \frac{3y_0}{4}\sin\left(\frac{\pi}{l}\right)x\cos\left(\frac{\pi a}{l}\right)t - \frac{y_0}{4}\sin\left(\frac{3\pi}{l}\right)x\cdot\cos\left(\frac{3\pi a}{l}\right)t$

10b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $\quad u(0,y)=0,\ u(l,y)=0,\ u(x,0)=0,\ u(x,l)=f(x)=x(l-x)$

$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

$u(0,y) = A(Ce^{py}+De^{-py}) = 0 \Rightarrow (A=0)$ $\quad u(l,y)=0 \Rightarrow \left(p=\frac{n\pi}{l}\right).$

$u(x,0)=0 \Rightarrow (D=-C)$ $\quad u(x,y) = 2BC\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y.$

$u(x,y) = \sum b_n\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y \Rightarrow u(x,y) = b_n\sin\left(\frac{n\pi}{l}\right)x\cdot\sinh\left(\frac{n\pi}{l}\right)y$

$u(x,l) = x(l-x) \Rightarrow \sum_{n=1}^\infty C_n\sin\left(\frac{n\pi}{l}\right)x = x(l-x)$ $\quad \left(b_n = \frac{1}{\sinh n\pi}\cdot C_n\right)$

$C_n = 2/l\int_0^l (lx - x^2)\sin\left(\frac{n\pi}{l}\right)x\,dx = \frac{4l^2}{n^3\pi^3}(1-(-1)^n)$

$b_n = \frac{1}{\sinh n\pi}\frac{8l^2}{n^3\pi^3}$ $\quad u(x,y) = \sum\frac{3200}{n^3\pi^3}\frac{1}{\sinh n\pi}\sin\left(\frac{n\pi}{20}\right)x\cdot\sinh\left(\frac{n\pi}{20}\right)y.$

11a) $F(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty f(x)\cdot e^{isx}dx = \frac{1}{\sqrt{2\pi}}\,2\cdot\int_0^1(1-x)\cdot\cos sx\,dx = \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s}{s^2}\right)$

$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty F(s)\cdot e^{-isx}ds = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty\sqrt{2/\pi}\cdot\left(\frac{1-\cos s}{s^2}\right)(\cos sx - i\sin sx)\,ds$

$= \frac{2}{\pi}\int_0^\infty\left(\frac{1-\cos s}{s^2}\right)\cdot\cos sx\,ds \Rightarrow \int_0^\infty\left(\frac{1-\cos s}{s^2}\right)\cdot\cos sx\,ds = \frac{\pi}{2}\cdot f(x)$

Put $x=0$, $\int_0^\infty\frac{1-\cos s}{s^2}ds = \frac{\pi}{2} \Rightarrow \int_0^\infty\frac{2\sin^2 s/2}{s^2}ds = \frac{\pi}{4}$ $\quad (s/2 = t)$

$\int_0^\infty\frac{\sin^2 t}{t^2}dt = \frac{\pi}{2}$ //

Bharath
INSTITUTE OF HIGHER EDUCATION AND RESEARCH
(Declared as Deemed-to-be University under section 3 of UGC Act, 1956)
(Vide Notification No. F.9-5/2000 - U.3, Ministry of Human Resource Development, Govt. of India, dated 4th July 2002)
BHARATH INSTITUTE OF SCIENCE AND TECHNOLOGY
SCHOOL OF AERONAUTICAL ENGINEERING
NBA Accredited

Internal Assessment Test I, ODD semester 2022
U20MABT03/ Transforms and Boundary value problems
Aeronautical/Aerospace

Year/Sem : II/ III | **Date: 13/10/2022**
Duration : 1 ½ Hour | **Max. Marks: 50**

	Part – A (6×2=12 Marks) Answer All Questions	CO	BL
1	Write down the Bernoulli's formula	CO 1	R
2	Write the formulae for finding Euler's constants of a Fourier series in $(0,2l)$	CO 1	R
3	Define Fourier series and Fourier coefficients in $(-\pi,\pi)$	CO 1	R
4	Find the values of b_n, if $f(x) = x^2$, in $-\pi < x < \pi$	CO 1	U
5	Write the half range cosine series and sine series formulae in $(0,l)$	CO 1	R
6	State Parseval's identity for the half range sine series in $(0,\pi)$	CO 1	R
	Part – B (3×6=18 Marks) Answer either (a) or (b)		
7a	Find the Fourier series for the function $f(x) = x^2$ in $-\pi < x < \pi$.	CO 1	U
7b	Find the Fourier series for the function $f(x) = x$ in $(-l,l)$.	CO 1	U
8a	Find the half range sine series for the function $f(x) = x$ in $(0,\pi)$.	CO 1	U
	Find the Fourier cosine series for the function $f(x) = x(l-x)$ in $(0,l)$	CO 1	U
9a	Find the sine series for $f(x) = x$ in $(0,\pi)$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ by using Parseval's identity.	CO 1	A
9b	Find the half range sine series for the function $f(x) = a$ in $(0,l)$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}$... by using Parseval's identity.	CO 1	A
	Part – C (2×10=20 Marks) Answer either (a) or (b)		
10a	Find the Fourier series for the function $f(x) = x(2\pi-x)$ in $(0,2\pi)$ and hence deduce the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...$	CO 1	A
10b	Find the Fourier constants a_0 and a_n for the function $f(x) = x\cos x$ in $(0,2\pi)$	CO1	A
11a	Find the half range cosine series for the function $f(x) = x$ in $(0,l)$ and hence deduce that $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+...=\frac{\pi^4}{96}$, by using Parseval's identity.	CO1	A
11b	Find the first two harmonic of the Fourier series of $f(x)$ given by the following table (see below)	CO1	A

Table for 11b:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1

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SCHOOL OF ... ENGINEERING

Name: P. Thathosao ... Date: 13.1

Reg. No: U21AS022 ... Sem / Year: 2nd

Examination: ... Subject: ...

Invigilator Sign: ...

10/5

PART-A

2 Beonoulli's foomula

$\int uv\,dx = uv_1 + u'v_2 - u''v_3 + u'''v_4 - \cdots$

2

3 Sol

fouoies Seoies

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

coefficient $(-\pi, \pi)$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\,dx$

2

PART-B

7a

Sol

given

$f(x) = x^2$ in $-\pi < x < \pi$

fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

find a_0

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2\, dx$$

$$= \frac{1}{\pi} \left[\frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[\frac{\pi^3}{3} \right] - \frac{1}{\pi} \left[\frac{-\pi^3}{3} \right]$$

$$= \frac{\pi^4}{3} + \frac{\pi^4}{3}$$

$$= \frac{\pi^8}{6}$$

find ~~bn~~ a_n

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx \, dx$$

u v

~~$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{x^3}{3} \cos$~~

Bernoulli's formula

$$\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\left[x^2 \cdot \frac{\cos n\pi}{n\pi} - 2x\left(\frac{-\sin n\pi}{(n\pi)^2}\right) + 2\left(\frac{-\cos n\pi}{(n\pi)^3}\right)\right.$$

3

$$= \frac{1}{\pi}\left[\pi^2 \cdot \frac{\cos n\pi}{n\pi} - 2\pi\left(\frac{-\sin n\pi}{(n\pi)^2}\right) + 2\left(\frac{-\cos n\pi}{n\pi^3}\right)\right.$$

$= \pi^3$ ~~$(1) \cdot = 2\pi$~~

~~π^2~~ $= \pi^3 \cdot (1) + 2\pi(\;1) + 2(-1)$

$= \pi^3 + 2\pi$

$a_n = 2\pi^4$

to find b_n

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx \, dx$$

u v

$$\int uv = uv_1 + u'v_2 - u''v_3 + u'''v_4 - \cdots$$

$$= x^2 \frac{\cos nx}{nx} + 2x \frac{-\sin nx}{(nx)^2} - x \frac{-\cos nx}{(nx)^3}$$

$$= \pi^2\left(\frac{\cos n\pi}{n\pi}\right) + 2\pi\left(\frac{-\sin n\pi}{(n\pi)^2}\right) - \pi\left(\frac{-\cos n\pi}{(n\pi)^3}\right)$$

$$= n\pi^3 (1) + 2n\pi^2 (0) - n\pi^4 (1)$$

$$= n\pi^3 - n\pi^4 \qquad n\pi^7 - n\pi^7$$

$$b_n = n\pi^7$$

fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = \frac{\pi^8}{6} + \sum_{n=1}^{\infty} 2n^4 \cos n\pi + \sum_{n=1}^{\infty} n\pi^7 \sin n\pi$$

②

Part - c

b)

given function

$f(x) = x(2\pi - x)$

fourier Series

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

find a_0

$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$

$= \frac{1}{\pi} \int_0^{2\pi} x(2\pi - x)\, dx$

$= \frac{1}{\pi} \int_0^{2\pi} (2\pi x - x^2) \cdot dx$

$= \frac{1}{\pi} \left(2\pi x^2 - x^3 \right)_0^{2\pi}$

$= \frac{1}{\pi} \left[2\pi(2\pi^2) - (2\pi)^3 \right] - 0$

$= \frac{1}{\pi} \left[4\pi^2 - 2\pi^3 \right]$

$= \frac{1}{\pi} \{ 2\pi \}$

$= 2\pi^2$

find a_n :

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x \cdot \cos x \cdot \cos nx \, dx$$

u v

$$\int uv = uv_1 + u'v_2 - u''v_3 + u'''v_4 - - -$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \frac{\cos}{\sin x} \left(\frac{\sin n\pi x}{(n\pi)}\right) + (-\sin x)\left(\frac{-\cos n\pi x}{(n\pi)^2}\right)$$

$$- \cos x \left(\frac{-\sin n\pi x}{(n\pi)^3}\right)$$

$$= \frac{1}{\pi} 2\pi \cos 2\pi \left(\frac{\sin n\pi 2\pi}{n(2\pi)}\right) + (-\sin 2\pi)\left(\frac{-\cos n\pi 2\pi}{n(2\pi)^2}\right)$$

$$- \cos 2\pi \left(\frac{-\sin n\pi 2\pi}{n(2\pi)^3}\right) - 0$$

$$= \frac{1}{\pi} \cdot 4\pi \cos \left(\frac{\sin 2\pi^2}{2\pi n}\right) + \sin 2\pi \left(\frac{\cos n 2\pi^2}{(2\pi)^2 n}\right)$$

$$+ \cos 2\pi \left(\frac{\sin n 2\pi^2}{(2\pi)^3 n}\right)$$

$$= 4\pi^2 (1)^2 + \sin(1)(0) + 2\pi^2 - 0$$

$$a_n = 6\pi^4$$

10a) given function $f(x) = x(2\pi - x)$

fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

to find a_0

3

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x(2\pi - x)\, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} 2\pi x - x^2\, dx$$

$$= \frac{1}{\pi} \left[2\pi \frac{x^2}{2} - \frac{x^3}{3} \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[2\pi \frac{2\pi^2}{2} - \frac{2\pi^3}{3} - 0 \right]$$

$$= \frac{1}{\pi} \left[\frac{4\pi^2}{2} - \frac{2\pi^3}{3} \right]$$

$$= \frac{1}{\pi} \left(\frac{2\pi}{6} \right)$$

$$= \frac{2\pi^2}{6}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \underset{u}{2\pi x - x^2} \; \underset{v}{\cos nx} \, dx$$

$$\int uv\,dx = uv_1 + u'v_2 - u''v_3 + u'''v_4 - \cdots$$

$$= \frac{1}{\pi}\int_0^{2\pi} 2\pi x - x^2 \frac{-\sin nx}{nx} + x - 2x\left(\frac{-\cos nx}{(nx)^2}\right) - x\left(\frac{\sin nx}{(nx)3}\right)$$

$$= \frac{1}{\pi}\; 2\pi \cdot 2\pi - (2\pi)2 - \frac{\sin 2\pi n}{2\pi n} + 2\pi - 2\pi\left(\frac{-\cos n 2\pi}{n(2\pi)2}\right) - 2\pi\left(\frac{\sin n 2\pi}{n(2\pi)3}\right)$$

$$= \frac{1}{\pi}\; 4\pi - 4\pi^2 (1) - 4\pi (0) - 2\pi (1)$$

$$= 2\pi^2 - 2\pi$$

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SCHOOL OF AERONAUTICAL ENGINEERING

Name: Gokulraj . S Date: 23/11/2022
Reg. No. U21AS011 Sem / Year: III / II
Examination: CLA-2 Subject: Transform & Boundary
Invigilator Sign: psnl

14/50

Part-C

10) a) Sol

$$\frac{d^2u}{dt} = c^2 \frac{d^2x}{dt}$$

$c = t/m.$

Given

$y(x,0) = y_0 \sin^3(\pi x/l)$

$x = 0$ & $x = l.$

the boundary conditions are

$y(0,t) = 0$

$y(l,t) = 0$

$\frac{dy}{dt}(l,t) = 0$

$y(x,0) = y_0 \sin^3(\pi x/l)$

The one-dimensional equation of wave.

$$u(x,y) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt) \rightarrow (1)$$

apply 1st boundary condition

$$y(0,t) = (C_1 \cos p(0) + C_2 \sin p(0))(C_3 \cos apt + C_4 \sin apt).$$

$$= C_1 \cos (C_3 \cos apt + C_4 \sin apt). \rightarrow (2)$$

$$C_1 = a \rightarrow (3)$$

Apply equ (3) in (1) we get

~~$u(x,t) = C_2 \sin$~~

$$u(x,y) = C_2 \sin px (C_3 \cos apt + C_4 \sin apt) \rightarrow (4)$$

$$y(l,t) = C_2^a \sin p^b l (C_3 \cos apt + C_4 \sin apt)$$

$C_2 \neq 0$

$$\sin px = \sin 0 \rightarrow (4)$$

$$C_2 \sin px = 0 \qquad C \;;\; \sin n\pi = 0$$

$$\sin px = \sin n\pi$$

$$px = n\pi$$

$$\boxed{p = n\pi / x.} \rightarrow (5)$$

$y(x,l) = c_2 \sin$

sub equ (B) in (A) we get

$u(x,y) = c_2 \sin \frac{n\pi}{l} x \left(c_3 \cos a\frac{n\pi}{l} t + c_4 \sin a \frac{n\pi}{l} t\right)$

$c_2 \neq 0$

$\frac{\partial x}{\partial t}(x,l) = c_2 \sin \frac{n\pi}{l} x \left[c_3 \cos a\frac{n\pi}{l} x \left(\frac{n\pi}{l}\right) + c_4 \sin a\left(\frac{n\pi}{l}\right) x \left(\frac{n\pi}{l}\right)\right]$

$= c_2 \sin \frac{n\pi}{l} x \left(c_4 \frac{n\pi}{l}\right)$

$c_4 = 0$ we get

$u(x,y) = c_2 \sin \frac{n\pi}{l} x \left[c_3 \cos a\frac{n\pi}{l} t\right]$

$= c_2 c_3 \sin \frac{n\pi}{l} x \cos a \frac{n\pi}{l} t$

$= b_n \sin \frac{n\pi}{l} x \cos \frac{n\pi}{l} t$

$= \sum_{n=1}^{\infty} c_n \sin \frac{n\pi}{l} x \cos \frac{n\pi}{l} t \rightarrow$ (6)

Apply the 4 boundary condition we get

$f(x) = y_0 \sin^3 \left(\frac{n\pi}{l}\right)$

$y_0 \sin^3 = y_0 \left[\frac{1}{4}(3 \sin x - \sin 3x)\right]$

$$= y_0\left[\frac{3\sin\left[\frac{n\pi}{l}\right] - \sin 3\left[\frac{n\pi}{l}\right]}{4}\right].$$

Then

Sub the.

$$y_0\left[\frac{1}{4}\; 3\sin\left[\frac{n\pi}{l}\right] - \sin 3\left[\frac{n\pi}{l}\right]\right] = C_1\frac{n\pi}{l} + C_2\frac{2n\pi}{\lambda} + C_3\frac{3n\pi}{\lambda} + C_4\frac{4n\pi}{\lambda}.$$

$$C_1 = \frac{3y_0}{4} \quad \& \quad C_3 = \frac{y_0}{4}.$$

Apply the above C value in eqn (6) we get

$$= \sum_{n=1}^{\infty} C_n \sin\frac{n\pi}{l}x \cos\frac{a\pi}{l}t$$

The most general equation

$$= \left[C_1 \frac{y_0}{} \sin\frac{3n\pi}{l}x \cos\frac{3a\pi}{\lambda} + C_3\, y_0 \sin\frac{n\pi}{l}x \cos\frac{n\pi}{l}t\right]$$

$$y(x,0) = \left[C_1\frac{3}{4}y_0 \sin\frac{\pi}{l}x \cos\frac{a\pi}{l}t + C_3\frac{y_0}{4}\sin\frac{\pi}{l}x \cos\frac{a\pi}{l}t\right]$$

11 (a)

Given

$f(x) = 1 - x^2, |x| < 1$

$0, |x| > 1$

To ~~show~~ $\int_0^\infty \left[\frac{\sin s - s\cos s}{s^3}\right]^2 ds = \pi/15$

Sol. $\int_0^\infty \frac{\sin^2 t}{t^2} dt$.

$F(s)[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$

3/2

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1-x^2) e^{isx} dx$

~~$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1-x^2) (-a\cos\sin x - b\cos x) dx$~~

~~$= 2\frac{1}{\sqrt{2\pi}} \int_0^\infty (1-x^2) (-a\sin x - b\cos x) dx$~~

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty} (1-x^2) (\cos sx - i\sin sx) dx$

$= 2 \times \frac{1}{\sqrt{2\pi}} \left[\int_0 (1-x^2) \cos sx\, dx - \int_0^1 (1-x^2) i\sin x\, dx \right]$

$= 2 \times \frac{1}{\sqrt{2\pi}} [$

$= 2 \times \frac{1}{\sqrt{2\pi}} \left[(-2x) \frac{\sin sx}{s} \right]_0^1$

$f(s) = 2 \times \frac{1}{\sqrt{2\pi}} \left[\frac{1 - \cos s}{s^2} \right]$

Parseval's Identity

~~$[f(x)]$~~ $\left[f(s)\right]^2 = \left[f(x)\right]^2$

$= \frac{8}{2\pi}$ $= [1 - x^2]^2$

Inversion

$F^{-1}(f(s)) = f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \left(\frac{1 - \cos s}{s^2} \right) e^{-isx} ds$

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \left[\frac{2 \sin^2 s/2}{s^2} \right] e^{-isx} ds$

$f(x) = \begin{cases} 1-x & , |x| < 1 \\ 0 & , |x| > 1 \end{cases}$

P.S.M.L

$$= \frac{1}{\pi} \int_0^\infty \left[\frac{2\sin^2 s/2}{s^2}\right] e^{-isx} \, ds$$

$$= \frac{1}{\pi} \int_0^\infty \frac{2\sin^2 s/2}{s^2} \pi \, ds$$

$$= \int_0^\infty \frac{2\sin^2 s/2}{s^2} \, ds$$

put $t = s/2$ $\quad dt = ds/2$

$$= \int_0^\infty \frac{2\sin^2 t}{t^2} \, dt.$$

(b) Given

$f(x) = e^{-ax}$ $> a$

Sol.

$$f(s) = \sqrt{2/\pi} \int_{-\infty}^{\infty} f(x) \sin sx \, dx$$

$$= \sqrt{2/\pi} \int_{-\infty}^{\infty} e^{-ax} \sin sx \, dx$$

$$= \sqrt{2/\pi} \int_{-0}^{\infty} e^{-ax} \sin sx \, dx.$$

$= 2\sqrt{2/\pi}\left[e^{-ax} \sin x\right]_0^\infty dx$

$= 2\sqrt{2/\pi}\left[e^{-ax} - \cos x\right]_0^\infty dx$

$= 2\sqrt{2/\pi}\left[\frac{s^2}{2}\right]$

the cosine form

$$= \sqrt{2/\pi}\int_0^\infty e^{-ax} \cos sx \, dx.$$

$$= \sqrt{2/\pi}\left[e^{-ax} - \sin sx\right]_\infty^0 dx$$

$$= \sqrt{2/\pi}\left[\frac{a^2}{2}\right]$$

P3 Mul

1) ~~1-D wave equation.~~

2) fourier law of heat conduction:

In the fourier law of heat conduction the heat will ~~flow through the~~ transform to material at any area of surface in a negrative gradiant

3) Convolution theorem

$f * g = 0.$

parseval's idmtity.

$\int [f[s]]^2 ds = \int [f[x]]^2 dx$

1) 1-D wave equation

$$\frac{d^2 u}{dx^2} = a^2 \frac{d^2 u}{dx}.$$

$u(x,y) = ~~C_1 \cos~~ (C_1 apx + C_2 px)(C_3 apt + C_4 apt)$

$u(x,y) = (C_5 ~~\cos apx~~ (C_5 \cos px + C_6 \sin px)(C_7 \cos apt + C_8 ~~\cos~~ \sin apt).$

$\int i\sqrt{2/\pi}\, f(x) \cos x\, dx$

a) ii) The Given

$f(x) = e^{-ax}, a > 0.$

Sol

$= \sqrt{2/\pi} \int_{-\infty}^{\infty} f(x) \cdot \sin x\, dx.$

$= \sqrt{2/\pi} \int_{-\infty}^{\infty} e^{-ax} \cdot \sin x\, dx.$

$= 2\sqrt{2/\pi} \int_{0}^{\infty} e^{-ax} \sin x\, dx.$

$= 2 \times \sqrt{2/\pi} \left[e^{-ax} - \cos x \right]_0^{\infty} \quad e^{-\infty} = \frac{1}{e^{\infty}} = 0.$

$= 2 \times \sqrt{2/\pi} \left[e^{a} - c \right.$

7(a) Steady state condition + Z-coordinate.

In the fourier transform, the heat will does doesn't change at any point by time and it is knowna as steady state condition

P.S.M.

$u(x,y) = (C_9 x^2 + C_{10})(C_{11} t + C_{12})$

3) 2-D heat flow equation

$$\frac{\partial u}{\partial t} = c^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] dt$$

$c = K/\rho s$

$u(x,y) = (C_1 e^{Px} + C_2 e^{-Px})(C_3 e^{Py} + C_4 e^{-Py})$

$= t$

8)
b)

$F\{f(x)\cdot \cos ax\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$

$\cos x = \frac{e^{ix} + e^{-ix}}{2}$ $\left[\frac{e^{isx} \quad e^{iax}}{e^{isx} + e^{iax}} \right]$

~~$\Rightarrow F[f(x)\cos ax]$~~ $= e$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\frac{e^{iax} + e^{-iax}}{2} \right) e^{isx}\, dx$

$= \frac{1}{2} \left[\frac{1}{\sqrt{2\pi}} \right]_{-\infty}^{\infty} f(x) \cdot e^{isx} \cdot e^{iax} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx} \cdot e^{-ias}\, dx$

$= \frac{1}{2} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(e^{i(s+a)x} - e^{i(s-a)x} \right) dx \right]$

$= \frac{1}{2} \left[F(s+a) = F(s-a) \right]$

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5/5

PART-A

1. ALL POSSIBLE SOLUTIONS OF ONE DIMENSIONAL WAVE EQUATION:

The one dimensional wave equation,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Here $a = \frac{\text{Tension } (T)}{\text{mass } (m)}$

The all possible solutions of One Dimensional wave equations are,

$$y(x,t) = (c_1 e^{px} + c_2 e^{-px})(c_3 e^{apt} + c_4 e^{-apt})$$

$$y(x,t) = (c_5 \cos px + c_6 \sin px)(c_7 \cos apt + c_8 \sin apt)$$

$$y(x,t) = (c_9 x + c_{10})(c_{11} t + c_{12})$$

These are the possible solutions of 1-D wave equation.

From the above solutions the correct solution is

$$y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos apt + c_4 \sin apt)$$

2. Fourier law of Heat conduction (or) Thermal Conductivity :

* The fourier law of thermal conduction states that the rate of change of Heat Transfer through a material is proportional to the Negative (-ve) gradient in the Temperature and the area surface through which the Heat flow.

* This is known as Fourier law of Heat Conduction.

3. TWO DIMENSIONAL HEAT FLOW EQUATION:

The two dimensional Heat Flow Equation is

$$\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Here $c = \frac{k}{\rho c}$

where $k \rightarrow$ Thermal Conductivity

$\rho \rightarrow$ Density

$c \rightarrow$ Specific Heat capacity.

The Condition at which the Temperature doesn't changes with the Time is known as Steady State Condition.

$$\frac{\partial u}{\partial t} = 0.$$

Then, the 2-D Heat flow equation will be

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

This is also known as Laplace Equation.

Various possible solutions of Laplace Equation are

$$u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$$

$$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$$

$$u(x,y) = (Ax + B)(Cy + D).$$

Then the correct Solution is

along x direction line parallel (or) Non zero boundary condition.

$$u(x,0) = u(x,l) = f(x).$$

$$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$$

along the y direction the line parallel (or) Non zero boundary condition.

$$u(0,y) = u(l,y) = f(x)$$

$$u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py).$$

4. Fourier Transform pairs :

The function $f(x)$ is defined in $(-\infty, \infty)$, and Continuously differentiable and absolutely integrable in $(-\infty, \infty)$. Then the Fourier Transform is

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx}\, dx \rightarrow (1)$$

The inversion of the fourier Series can be written as $F^{-1}[F(s)]$ or $f(x)$.

$$\therefore f(x) = F^{-1}[F(s)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{-isx} \, ds \rightarrow ②$$

The equation ① and ② are together are known as Fourier Transform pairs. (Fourier Transform and Inverse fourier Transform).

5. CONVOLUTION THEOREM :-

The convolution Theorem of two functions $f(x)$ and $g(x)$ are defined as,

$$f * g = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \, g(x-t) \, dt.$$

Fourier Convolution theorem.

The fourier convolution theorem of two function $f(x)$ and $g(x)$ is product of the fourier transform of each Function.

$$f(x) * g(x) = F(s) \cdot G(s).$$

Parseval's Identity for fourier Transform.

The parseval's Identity for fourier Transform is defined as

$$\int_{-\infty}^{\infty} |f(x)|^2 \, dx = \int_{-\infty}^{\infty} |F(s)|^2 \, ds$$

which is the parseval's Identity for fourier Transform.

Part - B.

7 b) Partial differential Equations.

i) Given

$y^2 u_{xx} + x^2 u_{yy} = 0$ → ①

The general expression for the partial differential equation is

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + f\left(x,y,u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}\right) = 0.$$

Comparing the given equation ① with the general equation, we get,

$A = y^2$, $B = 0$, $C = x^2$

Substitute in =>

$B^2 - 4AC \Rightarrow (0)^2 - 4(y^2)(x^2) = 0$

$-4y^2x^2 < 0$ $\quad B^2 - 4AC < 0$

Therefore the given equation is of the elliptic type

ii) Given equation,

$$4U_{xx} + 4U_{xy} + U_{yy} + 2U_x - U_y = 0.$$

$$4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0.$$

The General Equation / solution for Partial differential equation is

$$A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + C(x,y)\frac{\partial^2 u}{\partial y^2} + f\left(x,y,u,\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}\right) = 0.$$

Comparing the given equation with the General Solution. we get.

$A = 4$, $B = 4$, $C = 1$

Substitute in $B^2 - 4AC = 0$.

$B^2 - 4AC \Rightarrow (4)^2 - 4(4)(1) = 0$.

$16 - 16 = 0$. $0 = 0$

$B^2 - 4AC = 0$.

∴ The given equation is of the parabolic type.

Good

8 b) STATE AND PROVE MODULATION THEOREM:

STATEMENT:

If $F(f(x))$ is equal to $F(s)$ then the $F[f(x)\cos ax]$ is equal to $\frac{1}{2}[F(s+a)+F(s-a)]$

Modulation theorem:

$$F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$$

PROOF:

We know that,

The fourier Transform of the function $f(x)$ is,

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx}\, dx$$

× by $\cos ax$ in both the sides,

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cos ax \cdot e^{isx}\, dx$$

$$\cos ax = \frac{e^{iax}+e^{-iax}}{2}$$

$$F[f(x)\cos ax] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \frac{e^{iax}+e^{-iax}}{2} \cdot e^{isx}\, dx$$

$$F[f(x)\cos ax] = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) (e^{iax}+e^{-iax}) \cdot e^{isx}\, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{iax} \cdot e^{isx}\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{iax}\, e^{isx}\, dx\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{iax+isx}\, dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-ias+isx}\, dx\right]$$

$$F[f(x)\cos ax] = \frac{1}{2}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{i(s+a)x}\, dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{i(s-a)x}\, dx\right]$$

which is of the form of fourier Transform

$$\therefore F[f(x)\cos ax] = \frac{1}{2}[F(s+a) + F(s-a)]$$

∴ Hence the Modulation theorem is proved.

---

9. b) Given function,

$$f(x) = e^{-ax}$$

∴ The fourier Sine Transform of the function f(x) is

$$F_s[f(x)] = F_s(s) = \frac{\sqrt{2}}{\sqrt{\pi}}\int_0^{\infty} f(x)\sin sx\, dx$$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx\, dx$$

Here $f(x) = e^{-ax}$

$$F_s[e^{-ax}] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-ax}\sin sx\, dx$$

we know that,

$$\int_0^{\infty} e^{ax}.\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

[Here $a = -a$, $b = s$]

$$F_s\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{a^2+(s)^2}(-a\sin sx - s\cos sx)\right]_0^\infty$$

$$= \sqrt{\frac{2}{\pi}}\left[\left(0 - \frac{1}{a^2+s^2}(0-s)\right)\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{s}{a^2+s^2}\right].$$

$$F_s\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\,\frac{s}{a^2+s^2} \quad , a>0, \; a\neq 0, \; s\neq 0$$

which is the required Fourier Sine Transform of the given function $f(x) = e^{-ax}$.

We know that,

The fourier cosine Transform of the function $f(x)$ is given by

$$F_c[f(x)] = F_c(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx.$$

Here function $f(x) = e^{-ax}$

$$F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-ax}\cos sx\,dx$$

we know that,

$$\int_0^\infty e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx).$$

Here $a = -a$ and $b = s$.

$$F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{(-a)^2+(s)^2}\left(-a\cos sx + s\sin sx\right)\right]_0^{\infty}$$

$$F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\left[0 - \left(\frac{1}{a^2+s^2}(-a+0)\right)\right]$$

$$F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}}\left[\frac{a}{a^2+s^2}\right]$$

$$F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2+s^2}, \quad a>0 \qquad a \neq 0,\ s \neq 0$$

which is the required fourier Cosine Transform of the function $f(x) = e^{-ax}$

PART-C

10. c)

Given that,

$x=0, \quad x=l$

$y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$

The One dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = a^2\left(\frac{\partial^2 u}{\partial x^2}\right)$$

The correct solution of One dimensional wave equation is,

$$y(x,t) = (A\cos px + B\sin px)(C\cos apt + D\sin apt) \rightarrow (1)$$

The boundary Conditions are,

i) $y(0,t) = 0, \quad t \geq 0$

ii) $y(l,t) = 0, \quad t \geq 0$

iii) $\frac{\partial y}{\partial t}(x,0) = 0, \quad 0 < x < l$

iv) $y(x,0) = f(x)$

Applying boundary Condition (i) in equation ① =>

$y(x,t) = (A\cos px + B\sin px)(C\cos apt + D\sin apt)$

$y(0,t) \Rightarrow (A\cos(0) + B\sin(0))(C\cos apt + D\sin apt) = 0$

$y(0,t) \Rightarrow (A + 0)(C\cos apt + D\sin apt) = 0 \quad \cos(0) = 1, \ \sin(0) = 0$

$y(0,t) \Rightarrow A\,(C\cos apt + D\sin apt) = 0$

Here $A = 0$, $(C\cos apt + D\sin apt \neq 0)$ $a.b = 0$ either $a = 0$ (or) $b = 0$.

Substitute $A = 0$ in equation ①, we get

$y(x,t) = (0 + B\sin px)(C\cos apt + D\sin apt)$

$y(x,t) = B\sin px\,(C\cos apt + D\sin apt) \rightarrow ②$

Applying Second boundary Condition in ②,

$y(x,t) = B\sin px\,(C\cos apt + D\sin apt)$

$y(l,t) = B\sin pl\,(\cos apt + D\sin apt) = 0.$

$(C\cos apt + D\sin apt) \neq 0$

a.b = 0
either a = 0
(or)
b = 0

$B \sin pl = 0$

$B \neq 0$ then $\sin pl = 0$

$\sin pl = \sin n\pi$

$pl = n\pi$

$p = \left(\frac{n\pi}{l}\right)$

Substitute $p = \left(\frac{n\pi}{l}\right)$ in (2) we, get.

$y(x,t) = B\sin\left(\frac{n\pi}{l}\right)x\left(C\cos a\left(\frac{n\pi}{l}\right)t + D\sin a\left(\frac{n\pi}{l}\right)t\right)$

$y(x,t) = B\sin\left(\frac{n\pi}{l}\right)x\left(C\cos\left(\frac{n\pi a}{l}\right)t + D\sin\left(\frac{n\pi a}{l}\right)t\right)$ → (3)

partially differentiate the equation with respect to t, we got,

$\frac{\partial y}{dt}(x,t) = B\left(\sin\left(\frac{n\pi}{l}\right)x\right)\left[\left[C - \sin\left(\frac{n\pi a}{l}\right)t\left(\frac{n\pi a}{l}\right)\right] + D\cos\left(\frac{n\pi a}{l}\right)t\left(\frac{n\pi a}{l}\right)\right]$ → (4)

Applying boundary condition (4) in (4), we get.

$\frac{dy}{dt}(x,0) = B\sin\left(\frac{n\pi}{l}\right)x\left[C\left(-\sin\left(\frac{n\pi a}{l}\right)(0)\left(\frac{n\pi a}{l}\right)\right)\right.$
$\left. + \left[D\cos\left(\frac{n\pi a}{l}\right)(0)\left(\frac{n\pi a}{l}\right)\right]\right]$

$\sin 0 = 0$
$\cos 0 = 1$

$= B \sin\left(\frac{n\pi}{l}\right)x \quad D(1)\left(\frac{n\pi a}{l}\right)$

$\frac{\partial y}{\partial t}(x, 0) = B \sin\left(\frac{n\pi}{l}\right)x \quad D\left(\frac{n\pi a}{l}\right) = 0.$

Here $B \sin\left(\frac{n\pi}{l}\right)x \neq 0.$

$D = 0 \qquad \left(\frac{n\pi a}{l}\right) \neq 0.$

Substitute $D = 0$ in ③, we get.

$y(x,t) = B \sin\left(\frac{n\pi}{l}\right)x \left(C \cdot \cos\left(\frac{n\pi a}{l}\right)t + 0\right)$

$y(x,t) = B \sin\left(\frac{n\pi}{l}\right)x \; C \cos\left(\frac{n\pi a}{l}\right)t$

$y(x,t) = BC \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t$

$y(x,t) = C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t \qquad [BC = C_n]$

The most general equation is

$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t$

$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t$

Applying 4th boundary condition,

$y(x,0) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos(0)$

$y(x,0) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \quad (1) = f(x)$

$f(x) = y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right).$

$$\sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x = y_0 \sin^3\left(\frac{\pi x}{l}\right)$$

$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

$$C_1 \sin\left(\frac{\pi}{l}\right)x + C_2 \sin\left(\frac{2\pi}{l}\right)x + C_3 \sin\left(\frac{3\pi}{l}\right)x \dots$$

$$= y_0\left[\frac{1}{4}\left(3\sin\left(\frac{\pi}{l}\right)x - \sin\left(\frac{3\pi}{l}\right)x\right)\right]$$

10

Comparing the $C_1, C_2, C_3 \dots$ terms we get

$C_1 = \frac{3y_0}{4}$, $C_2 = 0$, $C_3 = \frac{-y_0}{4}$, $C_4 = 0 \dots$

$\therefore$ In most general Equation,

$$y(x,t) = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi}{l}\right)x \cos\left(\frac{n\pi a}{l}\right)t$$

$$= C_1 \sin\left(\frac{\pi}{l}\right)x \cos\left(\frac{\pi a}{l}\right)t + C_2 \sin\left(\frac{2\pi}{l}\right)x \cos\left(\frac{2\pi a}{l}\right)t + C_3 \sin\left(\frac{3\pi}{l}\right)x \cos\left(\frac{3\pi a}{l}\right)t.$$

$$y(x,t) = \frac{3y_0}{4}\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi a}{l}\right)t - \frac{y_0}{4}\sin\left(\frac{3\pi}{l}\right)x \cos\left(\frac{3\pi a}{l}\right)t.$$

which is the required general solution.

11. a)

Given that,

$$f(x) = \begin{cases} 1-|x| , & |x|<1 \\ 0 , & |x|>1 \end{cases}$$

We know that,

The fourier Transform of the function f(x) is

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$-1 < x < 1$

$1-|x| = f(x)$

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} 1-|x|\, e^{isx}\, dx$$

$e^{isx} = \cos sx + i\sin sx$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|)(\cos sx + i\sin sx)\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|) \cos sx\, dx + \frac{i}{\sqrt{2\pi}} \int_{-1}^{1} (1-|x|) \sin sx\, dx$$

e. e — even; e. o — odd

u. v

$$= \frac{1}{\sqrt{2\pi}}\, 2\int_{0}^{1} (1-|x|) \cos sx\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\, 2\left[(1-|x|)\frac{\sin sx}{s} - (0-1)\left(-\frac{\cos sx}{s^2}\right)\right]_0^1$$

$$= \frac{1}{\sqrt{2\pi}} \cdot 2\left[\left(0 - \frac{\cos s}{s^2}\right) - \left(1(0) - \frac{1}{s^2}\right)\right]$$

$$F[f(x)] = \sqrt{\frac{2}{\pi}} \left(\frac{1-\cos s}{s^2}\right)$$

$$F(s) = \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s}{s^2}\right)$$

which is required fourier Transform.

By Applying Inversion for fourier Transform, we get

$$f(x) = F^{-1}[F(s)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s}{s^2}\right) e^{-isx}\, ds$$

$e^{-isx} = \cos sx - i\sin sx$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s}{s^2}\right)(\cos sx - i\sin sx)\, ds$$

$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos s}{s^2}\right)\cos sx\, dx - i\,\frac{1}{\pi}\int_{-\infty}^{\infty}\left(\frac{1-\cos s}{s^2}\right)\sin sx\, ds$$

even odd

$$= \frac{2}{\pi}\int_{0}^{\infty}\left(\frac{1-\cos s}{s^2}\right)\cos sx\, ds$$

for $f(x) = 1-|x|$, $-1 < x < 1$

~~put~~ $x =$

$$\frac{2}{\pi}\int_{0}^{\infty}\left(\frac{1-\cos s}{s^2}\right)\cos s(x)\, ds = 1-|x|$$

put $x = 0$

$\cos(0) = 1$

$$\frac{2}{\pi}\int_{0}^{\infty}\left(\frac{1-\cos s}{s^2}\right)\cos(0)\, ds = 1-|0|$$

$$\frac{2}{\pi}\int_0^\infty \left(\frac{1-\cos s}{s^2}\right) \text{~~...~~} (1)\, ds = 1$$

put $1-\cos s = 2\sin(s/2)$

$$\int_0^\infty \frac{2\sin(s/2)}{s^2} \text{~~...~~} (1)\, ds = \frac{\pi}{2}.$$

Substitution method

put $s/2 = t$ $\quad s = 2t$.

$ds = 2dt$, $\quad$ L.L $= 0$

U.L $= \infty$

U.L $\frac{s}{2} = \infty$

L.L $= s/2 = 0$.

$$\int_0^\infty \frac{2\sin t}{4t^2}\, 2dt \text{~~...~~} = \pi/2$$

$$\int_0^\infty \frac{\sin t}{t^2}\, dt = \pi/2.$$

$$\therefore \int_0^\infty \frac{\sin t}{t^2}\, dt = \pi/2.$$

b) Fourier cosine transform pairs

Fourier cosine Transform

$F_c(f(x)) = F_c(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx$ → (1)

Inversion,

$F^{-1}[F_c(s)] = f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos sx \, dx$ → (2)

(1) and (2) are Fourier cosine Transform pairs

CLA-3 Aerospace

Answer key

1) $z = x f(y) + g(y)$

2) $z = px + qy + p^2 + q^2$, $p = a, q = b$

3) $z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n} = F(z)$

4) Inverse of Z-Transform

$z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n} = F(z) \Rightarrow z^{-1}\{F(z)\} = f(n)$

5) $z\{a^n f(n)\} = \sum_{n=0}^{\infty} f(n) \left(\frac{z}{a}\right)^{-n}$

6) $z\left(\frac{1}{n}\right) = \log\left(\frac{z-a}{z}\right)^{-1}$

7) $(x-y) + (x+1)q + (1-y)p = 0$

$\phi\left(x^2+y^2+z^2, x+y+z\right) = 0$

8) $(mz-ny)p + (nx-lz)q = ly - mx$.

$u = x^2+y^2+z^2$

9) $(n+1)(n+2) = f(n)$

$z\{(n+1)(n+2)\} = z\{n^2+3n+2\}$

$= 2\left(\frac{z}{z-1}\right)^3$

10) $C.F = f(y-x) + x f(y-x)$

$P.I = \frac{1}{\phi(D, D')}$ $\quad f(x,y) = \frac{yx^4}{12} - \frac{x^5}{80}$.

(11) $z^{-1}\left\{ \dfrac{3z^2 - 18z + 26}{(z-[illegible])(z-[illegible])(z-4)} \right\}$

$A = 1, \; B = 1, \; C = 1$

$z^{-1}\left\{ \dfrac{3z^2 - 18z + 26}{(z-[illegible])(z-[illegible])(z-4)} \right\} = \left(\frac{1}{[illegible]}\right) 2^n + \left(\frac{1}{[illegible]}\right) 3^n + \left(\frac{1}{[illegible]}\right) 4^n$

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Selaiyur, Chennai

SCHOOL OF AERONAUTICAL ENGINEERING

Name: SUJIRTHA . D Date: 23/11/22
Reg. No. U21AS030 Sem / Year: II
Examination: CLA - II Subject: TBVP
Invigilator Sign: [signature] 23/11/2022

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(1)

PART - A:-

(1). 1D Wave Equation. (Possible Solutions).

$$y(x,t) = (C_1 e^{px} + C_2 e^{-px})(C_3 e^{apt} + C_4 e^{-apt})$$

$$y(x,t) = (C_5 \sin px + C_6 \cos px)(C_7 \sin apt + C_8 \cos apt)$$

$$y(x,t) = (C_9 x + C_{10})(C_{11} + C_{12} t)$$

(6):-

Fourier Cosine Transform [Pair]:-

$$F\{f(x)\} = F(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx.$$

Inversive of Cosine:-

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F(s)(\cos sx) \, ds.$$

(5) Convolution Theorem States that the product of $F(x) * g(x)$ of $(t) \cdot g(t-x) \, dt$.

Parseval's Identity:-

$$\int_{-\infty}^{\infty} |F(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$= \int_{-\infty}^{\infty} F(s) e^{isx} ds$$:-

④ Fourier Transform ~~Pair~~: Equation.

$$F(f(x)) = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx. \rightarrow (1)$$

Fourier Inverse Transform Equation.

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds. \rightarrow (2)$$

The Combination of Equation (1) & (2) is Called Fourier Transform Pair.

---

③ 2D-heat flow Equation :-

$$\frac{\partial^2 u}{\partial t^2} = (c^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Where c^2 is $\frac{K}{\rho c}$.

where K is

ρ is density

c is Specific heat

Fourier law of heat Conduction :- (2) (Part A) :-

$$\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2}\right). \qquad c = \frac{k}{\rho c}$$

$F(x) = \int_{-\infty}^{\infty}$

PART - B :-

7(a) Steady State Condition :- $\frac{\partial u}{\partial t} = 0.$ → (1)

One dimensional Wave Equation $\frac{\partial u}{\partial t} = \left(\frac{\partial^2 u}{\partial x^2}\right) c^2$:- → (2)

Since we need 1D Wave Equation at Steady State Condition, (where temperature dont change ~~at~~ with respect to time) Equating (1) & (2) we get

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2}\right) :-$$

$$0 = c^2\left(\frac{\partial^2 u}{\partial x^2}\right)$$

$$\frac{0}{c^2} = \frac{\partial^2 u}{\partial x^2}.$$

$$0 = \frac{\partial^2 u}{\partial x^2} \quad \rightarrow (3)$$

From Equation (3) we get for one dimensional Steady State Condition heat flow Equation :-

(7b). -

1 $[B^2 - 4AC$:-$]$

(i). $A = 1 \quad B = 0 \quad C = 1$:-

$0 - 4(1)(1)$.

$= -4$.

$-4 < 0$ Given Equation is Elliptic Type.

(ii). $B^2 - 4AC$:-

$A = 4 \quad B = 4 \quad C = 1$

$\Rightarrow B^2 - 4AC$.

$\Rightarrow 4^2 - 4(4)(1)$.

$= 16 - 16$

$= 0$ Given Equation is a Parabola.

(8b). Modulation Theorem:-

To Prove

$$F(f(x))\cos ax = \frac{1}{2}[F(s+a) + F(s-a)]$$

We know that $\cos ax = \dfrac{e^{iax} + e^{-iax}}{2}$.

$\Rightarrow$

$$\int_{-\infty}^{\infty} f(x).(\cos ax) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} F(s)(e^{isx})\,dx\left(\frac{e^{iax}+e^{-iax}}{2}\right).$$

$$= \frac{1}{2}\left\{\left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} f(x)(e^{isx}.e^{iax}) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}.e^{-iax}\right]\right\}$$

$$= \frac{1}{2}\left\{\left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} f(x)e^{i(s+a)x} + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{i(s-a)x}\right]\right\}$$

$$= \frac{1}{2}[F(s+a) + F(s-a)] \text{ //.}$$

Thus we showed,

$$F[f(x)(\cos ax)] = \frac{1}{2}[F(s+a) + F(s-a)] :-$$

9(b): Fourier Sine Transform :-

$$F(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) (\sin sx) (dx) :-$$

Fourier Cosine Transform.

$$F(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) (\cos sx) \, dx :-$$

Given e^{-ax} :-

Sine :-

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} (e^{-ax}) (\sin sx) \, dx.$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \text{Si.} \quad 2 \sin ax - \cos ax$$

PART-C :-

($\frac{1}{4}$(3 Sin x - Sin 3x))

(10) (a) :- Given function :-

$y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$

From One dimensional Wave Equation, the Correct Solution :-

$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_3 \cos apt + C_4 \sin apt)$ → ①

~~From~~ The boundary Conditions :-

$y(x,0) = 0 , \quad t \geq 0$

$y_1(x,l) = 0 , \quad t \geq 0$

$\frac{\partial y}{\partial t}(0,t) = 0 , \quad 0 \leq x \leq l.$

$y(0,t) = f(x) = y_0 \sin\left(\frac{\pi x}{l}\right)$

Applying 1st boundary Condition in Eqn ①

$y(x,0) = (C_1 \cos px + C_2 \sin px)(C_3 \cos 0° + C_4 \sin 0°)$

$= (C_1 \cos px + C_2 \sin px)(C_3)$

Where, $C_3 = 0$ & $C_1 \cos px \; C_2 \sin px \neq 0$ which gives trivial Solution.

Sub ($C_3 = 0$) in 1 :-

We get;

$y(x,t) = (C_1 \cos px + C_2 \sin px)(C_4 \sin apt)$ → ②

Applying 2nd boundary Condition in Eqn (2)

we get;

$y(x,0) = (C_1 \cos px + C_2 \sin px)(C_4 \sin apl)$:-

Here $C_4 \sin apl = 0$ which does not gives trivial Solution :-

~~y(x,0)~~ $C_4 \sin apl = 0$.

$\sin apl = 0$

$\sin apl = \sin n\pi$.

$$\boxed{p = \frac{n\pi}{al}}$$:-

Substituting Value of p in Eqn (2):

we get :- Eqn (3) :-

$$y(x,t) = \left(C_1 \cos\left(\frac{n\pi}{al}\right)x + C_2 \sin\left(\frac{n\pi}{al}\right)x\right)\left(C_4 \sin\left(\frac{n\pi}{al}\right)t\right) \quad \rightarrow (3)$$

Substituting 3rd Boundary Condition in Equation (3) :-

Differentiating :-

$$\frac{\partial u}{\partial y}(0,t) = C_1\left(\left(-\sin\frac{n\pi}{al}(x)\left(\frac{n\pi}{al}\right)\right)\right) + C_2 \cos\frac{n\pi}{al}(x)\frac{n\pi}{al}\Big)\left(C_4 \cos\frac{n\pi}{al}(t)\left(\frac{n\pi}{l}\right)\right)$$

$$\frac{\partial u}{\partial y}(0,t) = C_2\left(\frac{n\pi}{al}\right)\left(C_4 \frac{n\pi}{al}(\cos t)\right)$$:-

$\frac{\partial y}{\partial y}(0,t) = C_2 \cdot C_4 \cos\left(\frac{n\pi}{a l}\right)(t)\left(\frac{n\pi}{a l}\right)$:-

$y_p = \frac{1}{4}(3 \cos x - \cos 3x)$:-

$= C_n\left(\frac{1}{4}(3 \cos x - \cos 3x)\right)$:-

$\sin^3 x$ $y_0 = \frac{1}{4}(3 \sin x - \sin 3x)$ //.

$\frac{\partial y}{\partial y}(0,t) =$ C_2

Illy: In Terms of Sine :-

$= C_n \frac{1}{4}(3 \sin x - \sin 3x)$

~~Substitute in Eqn (B)~~.

$c_1 \sin\frac{\pi}{al}x + c_2 \sin\frac{2\pi}{al}x + c_3 \sin\left(\frac{3\pi}{al}\right)x =$

(11). (b) :- $f(x) = 1 - x^2$.

Fourier Transform :-

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx.$$

$$= \frac{1}{\sqrt{2\pi}} (1-x^2)(\cos sx + i\sin sx)\, dx.$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_{-\infty}^{\infty} (1-x^2)(\cos sx)\, dx + \int_{-\infty}^{\infty} (1-x^2)(i\sin sx)\, dx \right.$$

odd

Applying limits :-

$$= \frac{1}{\sqrt{2\pi}} (2) \int_{0}^{1} (1-x^2)(\cos sx)\, dx + 0.$$

$$= \frac{1}{\sqrt{2\pi}} (2) \int_0 (1-x^2)\left(\frac{\sin sx}{s}\right) - (-2x)\left(-\frac{\cos sx}{s^2}\right) + (-2)\left(-\frac{\sin sx}{s^3}\right).$$

$$= \frac{1}{\sqrt{2\pi}} (2) \left[-\frac{s\cos s}{s^2} + \frac{\sin s}{s^3} \right].$$

$$= \frac{2}{\sqrt{2\pi}} \left[\frac{\sin s - s\cos s}{s^3} \right]$$ //

required Fourier Transform :-

~~Applying Inverse We get $\frac{4}{\pi}$~~

(5)

Applying Parseval's Identity :-

$$\int_{-\infty}^{\infty} |F(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds$$

$$F(s) = \int_0^{\infty} \left(\frac{2}{\sqrt{2\pi}}\right)^2 \left(\frac{\sin s - s\cos s}{s^3}\right)^2 = \int_0^1 (1-x^2)$$

$$\frac{4}{2\pi}(2) \int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right)^2 = \int_0^1 (1 + x^4 - 2x^2)$$

$$\frac{16}{\pi} \int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right)^2 = \int_0^1 1 + \frac{x^5}{5} - 2\left(\frac{x^3}{3}\right)$$

$$\frac{16}{\pi} \int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right)^2 = \frac{16}{15}$$

$$\int_0^{\infty} \left(\frac{\sin s - s\cos s}{s^3}\right)^2 = \frac{\pi}{15}$$

Hence Proved.

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SCHOOL OF AERONAUTICAL ENGINEERING

50/50

Name: SENTHIL NATHAN AGS Date 28/12/22

Reg. No. U21AS028 Sem / Year II nd

Examination CLA - III Subject TRANSFORMS AND BOUNDARY VALUE PROBLEMS

Invigilator Sign 28/12/2022

PART-A

ANSWER ALL THE QUESTIONS:

1. Direct Integration:

Given,

$$\frac{\partial^2 z}{\partial x^2} = 0 \rightarrow (1)$$

Partially Integrating equation (1) with respect to x

$$\int \frac{\partial}{\partial x} \frac{\partial z}{\partial x} dx = c$$

Here the constant is function of y 'f(y)'.

$$\frac{\partial z}{\partial x} = f(y)$$

Again partially integrating with respect to 'x' on both sides,

$$\int \frac{\partial z}{\partial x} dx = \int f(y) dx$$

2

$$z = x f(y) + c$$

Here the another constant is function of y 'g(y)'

$$z = x f(y) + g(y)$$

∴ The general solution for the give partial Differential Equation is,

$$z = x f(y) + g(y).$$

2. Eliminating the Arbitrary constants,

Given that,

$$z = ax + by + a^2 + b^2 \rightarrow (1)$$

partially differentiate the equation (1) with respect to x on both sides,

$$\frac{\partial z}{\partial x} = p = a(1) + 0 + 0 + 0$$

$$p = a.$$

partially differentiate the equation (1) with respect to y on both sides,

$$\frac{\partial z}{\partial y} = q = 0 + b(1) + 0 + 0$$

$$q = b.$$

Substitute $p = a$ and $q = b$ in (1), we get

$$z = px + qy + p^2 + q^2$$

which is the required PDE.

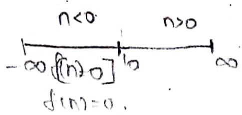
---

3. ONE SIDED Z - TRANSFORM:

★ In the z-Transform when $f(n) = 0$, where $n < 0$, the (Casual Solution). z Transform should be known as one sided z-Transform. The one sided z-Transform is given by,

★ $$z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n} = F(z)$$

which is known as the one sided z-Transform mostly used this z-Transform.



INVERSE OF Z - TRANSFOM:

If the Z- Transform should be

$z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n} = F(z)$. Then the Inverse

Z - Transform is given by,

$$z^{-1}[F(z)] = f(n)$$

which is the inverse Z - Transform,

eg): $z^{-1}\left[\frac{z}{z-a}\right] = a^n$.

---

4. Z - TRANSFORMS OF 1 and $(-1)^n$.

Here,

Z - Transform of 1.

$$z\{1\} = \frac{z}{z-1}$$

$$z\{(-1)^n\} = \frac{z}{z+1}$$

From the conditions of.

$$z\{a^n\} = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{a}{z}\right)^n$$

$$= 1 + \frac{a}{z} + \left(\frac{a}{z}\right)^2 + \left(\frac{a}{z}\right)^3 + \dots$$

WKT $(1-x)^{-1} = 1 + x^2 + x^3 + x^4 + \dots$ here $x = a/z$

$$= \left[1 - \left(a/z\right)\right]^{-1}$$

$$z\{a^n\} = \left[\frac{z-a}{z}\right]^{-1}$$

$$z\{a^n\} = \frac{z}{z-a}$$

Here the value of a is 1

$a = 1 \Rightarrow f(n) = (1)^n = 1.$

$$z\{1\} = \frac{z}{z-1}$$

$a = -1 \Rightarrow f(n) = (-1)^n = (-1)^n$

$$z\{(-1)^n\} = \frac{z}{z-(-1)} = \frac{z}{z+1}.$$

---

5. State and Prove change of Scale Property:

STATEMENT:

If $z\{f(n)\} = F(z) = \sum_{n=0}^{\infty} f(n)\, z^{-n}$ which is the z transform, then, $z\{a^n f(n)\} = F(z/a)$.

which is the change of Scale property.

PROOF:

we know that,

z - Transform,

$$z\{f(n)\} = \sum_{n=0}^{\infty} f(n)\, z^{-n} = F(z).$$

x by a^n with the function. $f(n) = a^n f(n)$

$$z\{a^n f(n)\} = \sum_{n=0}^{\infty} a^n f(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} f(n)\, z^{-n} \left(\frac{1}{a^{-n}}\right)$$

$$z\{a^n f(n)\} = \sum_{n=0}^{\infty} f(n) \left(\frac{z}{a}\right)^{-n}$$

$\therefore\ z\{a^n f(n)\} = F(z/a)$

$\therefore$ Hence the change of scale property is proved

6. z - Transform: here $a = 1$

i) $z(1/n) = \log\left(\frac{z-a}{z}\right)^{-1} = \log\left(\frac{z}{z-a}\right) = \log\left(\frac{z}{z-1}\right)$

ii) $z\left(\frac{1}{n+1}\right) = z\log\left(\frac{z-a}{z}\right)^{-1} = z\log\left(\frac{z}{z-a}\right) = z\log\left(\frac{z}{z-1}\right)$.

i) $z(1/n) = \sum_{n=0}^{\infty} \frac{1}{n} z^{-n}$

$= \sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{1}{z}\right)^n$

$= 0 + 1\left(\frac{1}{z}\right) + \frac{1}{2}\left(\frac{1}{z}\right)^2 + \frac{1}{3}\left(\frac{1}{z}\right)^3 + \dots$

WKT,

$\log(-x+1) = -\left[\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots\right] = \log(1-x)$

here $x = 1/z$

$z(1/n) = -\log(-x+1) = -\log(1-x)$.

$z\left(\frac{1}{n}\right) = \log(-x+1)^{-1}$

$= \log(-1/z + 1)^{-1}$

$= \log\left(\frac{z-1}{z}\right)^{-1}$

i) $\left[z\left(\frac{1}{n}\right) = \log\left(\frac{z}{z-1}\right)\right]$.

lly for

ii) $\left[z\left(\frac{1}{n+1}\right) = z\log\left(\frac{z}{z-1}\right)\right]$

2 good

PART-B.

7 a) Given that,

$\phi(x^2+y^2+z^2, x+y+z)=0$ $\qquad \phi(u,v)=0$

$u=f(v)$.

$x^2+y^2+z^2 = f(x+y+z) \rightarrow ①$

partially differentiate ① with respect to 'x' on both sides

$2x+0+2\frac{\partial z}{\partial x} = f'(x+y+z)\left(1+0+\frac{\partial z}{\partial x}\right)$ $\qquad \therefore \frac{\partial z}{\partial x}=p$

$2x+2p = f'(x+y+z)(1+p) \rightarrow ②$

partially differentiate ① with respect to 'y' on both sides

$0+2y+2\frac{\partial z}{\partial y} = f'(x+y+z)\left(0+1+\frac{\partial z}{\partial y}\right)$ $\qquad \frac{\partial z}{\partial y}=q$

$2y+2q = f'(x+y+z)(1+q). \rightarrow ③$

② ÷ by ③ we get,

$$\frac{2x+2p}{2y+2q} = \frac{f'(x+y+z)(1+p)}{f'(x+y+z)(1+q)}$$

$$\frac{2(x+p)}{2(y+q)} = \frac{(1+p)}{(1+q)}$$

$(x+p)(1+q) = (1+p)(y+q)$

$x+xq+p+pq = y+q+py+pq$

$x-y+xq-q+p-py+pq-pq=0$ $\qquad$ (or) $\quad (1-q)x-(1+p)y+p-q=0$

$x-y+(x-1)q+(1-y)p=0$

which is the required Partial differential Equation.

8 b)

Given that,

$(mz - ny)p + (nx - lz)q = ly - mx$

This is in the form of Clairault's. $(Pp + Qq = R)$.

Here,

$P = mz - ny$, $Q = nx - lz$, $R = ly - mx$

The Subsidary equation is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$$

Taking, x, y, n as one set of Multipliers,

$$\frac{x\,dx + y\,dy + z\,dz}{x(mz - ny) + y(nx - lz) + z(ly - mx)} = \frac{x\,dx + y\,dy + z\,dz}{mxz - nxy + ynx - lyz + zly - mxz}$$

6

$$\frac{x\,dx + y\,dy + z\,dz}{0} = 0.$$

$\therefore x\,dx + y\,dy + z\,dz = 0.$

Integrating the Each Terms, we get,

$\int x\,dx + \int y\,dy + \int z\,dz = \int 0$ $\int 0 = c = a.$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = a.$$

$x^2 + y^2 + z^2 = 2a$ here $2a = b$.

$x^2 + y^2 + z^2 = b$

$\therefore u = x^2 + y^2 + z^2.$

Taking l, m, n as another sets of multipliers.

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz-ny) + m(nx-lz) + n(ly-mx)} = \frac{l\,dx + m\,dy + n\,dz}{mlz - nly + mnx - mlz + nly - mnx}$$

$$\frac{l\,dx + m\,dy + n\,dz}{0} = 0$$

$\therefore l\,dx + m\,dy + n\,dz = 0$

Integrating the each Terms we get,

$$\int l\,dx + \int m\,dy + \int n\,dz = \int 0$$

$$l\int dx + m\int dy + n\int dz = \int 0$$

$$lx + my + nz = c.$$

$$v = lx + my + nz.$$

The general solution is of the form,

$$\phi(u, v) = 0.$$

$$\therefore \phi(x^2+y^2+z^2, lx+my+nz) = 0.$$

which is the required solution.

---

9. a) Given that,

i) $(n+1)(n+2) = f(n) \Rightarrow z\{f(n)\} = z\{(n+1)(n+2)\}$

$$z\{(n+1)(n+2)\} = z\{n^2+3n+2\}$$

$\therefore$ by the linear property of the z-Transform,

$$z\{af(n) + bg(n)\} = a\,z\{f(n)\} + b\,z\{g(n)\}.$$

$\therefore \; Z\{n^2+3n+2\} = Z\{n^2\} + Z\{3n\} + Z\{2\}$

$= Z\{n^2\} + 3Z\{n\} + 2Z\{1\}.$

we know that,

$$Z\{n^2\} = \frac{Z(Z+1)}{(Z-1)^3}$$

$$Z\{n\} = \frac{Z}{(Z-1)^2}$$

$$Z\{1\} = \frac{Z}{Z-1}$$

Substitute in the above Z - Transform, we get,

$$Z\{n^2+3n+2\} = \frac{Z(Z+1)}{(Z-1)^3} + 3\left(\frac{Z}{(Z-1)^2}\right) + 2\left(\frac{Z}{Z-1}\right)$$

$$= \frac{Z(Z+1) + 3Z(Z-1) + 2Z(Z-1)^2}{(Z-1)^3}$$

$$= \frac{Z^2+Z + 3Z^2-3Z + 2Z(Z^2+1-2Z)}{(Z-1)^3}$$

$$= \frac{Z^2+Z+3Z^2-3Z+2Z^3+2Z-4Z^2}{(Z-1)^3}$$

$$= \frac{3Z^2+Z^2-4Z^2+2Z^3-3Z+2Z+Z}{(Z-1)^3}$$

$$Z\{n^2+3n+2\} = \frac{2Z^3}{(Z-1)^3}$$

$$Z\{(n+1)(n+2)\} = 2\left[\frac{Z}{Z-1}\right]^3$$

which is the required Z Transform of function $(n+1)(n+2)$

ii) $(n+1)^2$

$f(n) = (n+1)^2 = n^2 + 2n + 1.$

$\therefore Z\{f(n)\} = Z\{(n+1)^2\} = Z\{n^2 + 2n + 1\}$

by linear property of the Z-Transform

$Z\{n^2+2n+1\} = Z\{n^2\} + Z\{2n\} + Z\{1\}$

$= Z\{n^2\} + 2Z\{n\} + Z\{1\}$

Already we know that,

$Z\{n^2\} = \frac{z(z+1)}{(z-1)^3}$

$Z\{n\} = \frac{z}{(z-1)^2}$

$Z\{1\} = \frac{z}{z-1}$

Substitute in the suitable Z Transform.

6

$= \frac{z(z+1)}{(z-1)^3} + \frac{2z}{(z-1)^2} + \frac{z}{z-1}$

$= \frac{z(z+1) + 2(z-1)z + z(z-1)^2}{(z-1)^3}$

$= \frac{z^2+z + 2z^2 - 2z + z(z^2-2z+1)}{(z-1)^3}$

$= \frac{z^2+2z^2 - 2z^2 + z - 2z + z + z^3}{(z-1)^3}$

$\therefore Z\{(n+1)^2\} = \frac{z^3+z^2}{(z-1)^3} = \frac{z^2(z+1)}{(z-1)^3}$

$\therefore Z\{(n+1)^2\} = Z\{n^2+2n+1\}$

$$= \frac{z^3+z^2}{(z-1)^3}$$

$$Z\{(n+1)^2\} = \frac{z^2(z+1)}{(z-1)^3}$$

which is the required Z - Transform of the function $(n+1)^2$.

PART- C.

$(D^2+2DD'+D'^2)z=0$

m^2+2m+1

b)

Given that,

$$(D^2+2DD'+D'^2)z = x^2y$$

The Arbitrary equation is

$m^2+2m+1 = 0$

replace D by m

D' by 1

remove z.

$m^2+2m+1=0$

$(m+1)(m+1)=0$

$\therefore m = -1, -1$

$\therefore$ The roots are $-1, -1$.

Then the Complementary Function is given by.

$$C.F = f(y-x) + x f(y-x)$$

For the particular Integral P.I

$$P.I = \frac{1}{\phi(D,D')} f(x,y)$$

$$= \frac{1}{(D^2+2DD'+D'^2)} x^2y$$

$(D+D')^2$

$= D^2+D'^2+2DD'$

$$= \frac{1}{(D+D')^2} x^2 y$$

$$= \frac{1}{D^2\left(1+\frac{D'}{D}\right)^2} x^2 y$$

$$= \frac{1}{D^2}\left[1+\frac{D'}{D}\right]^{-2} x^2 y$$

WKT.

$(1+x)^{-2} = 1-2x+3x^2-4x^3+5x^4+\dots$

$$= \frac{1}{D^2}\left[1-2\left(\frac{D'}{D}\right)+3\left(\frac{D'}{D}\right)^2+\dots\right] x^2 y$$

Take only Two other get zero.

$$= \frac{1}{D^2}\left[1-2\left(\frac{D'}{D}\right)\right] x^2 y$$

D' diff wrt to y

D diff wrt to x.

$$= \frac{1}{D^2}\left[x^2 y - \frac{2D'(x^2y)}{D}\right]$$

$$= \frac{1}{D^2}\left[x^2 y - \frac{2x^2(1)}{D}\right]$$

$$= \frac{1}{D^2}\left[x^2 y - 2\int x^2 dx\right]$$

$$= \frac{1}{D^2}\left[x^2 y - 2\frac{x^3}{3}\right]$$

$$= \iint x^2 y \, dx\,dx - \iint 2\frac{x^3}{3} dx\,dx$$

$$= \int \frac{x^3}{3} y\,dx - \int 2\frac{x^4}{12} dx$$

$$= \frac{yx^4}{3\times4} - \frac{x^5}{6\times5}$$

$$\text{P.I} = \frac{yx^4}{12} - \frac{x^5}{30}.$$

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∴ The complete solution of the given function is

$= C.F + P.I$

$= f(y-x) + x f(y-x) + \frac{yx^4}{12} - \frac{x^5}{30}$

which is the required complete solution of the function $(D^2 + 2DD' + D'^2) = x^2y$.

---

11. a) Given that,

$$z^{-1}\left\{\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}\right\}$$

Consider, the, using partial Fraction method.

$$\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)} = \frac{A}{(z-2)} + \frac{B}{(z-3)} + \frac{C}{(z-4)}$$

$$\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)} = \frac{A(z-3)(z-4) + B(z-2)(z-4) + C(z-2)(z-3)}{(z-2)(z-3)(z-4)}$$

$\therefore 3z^2 - 18z + 26 = A(z-3)(z-4) + B(z-2)(z-4) + C(z-2)(z-3)$

Substitute $z = 2$ in the above equation,

$3(2)^2 - 18(2) + 26 = A(2-3)(2-4) + B(0) + C(0)$

$12 - 36 + 26 = A(-1)(-2)$

$12 - 10 = 2A$

$2 = 2A$

$\boxed{A = 1}$

Substitute $z = 3$

$$3(3)^2 - 18(3) + 26 = A(0) + B(3-2)(3-4) + C(0)$$

$$27 - 54 + 26 = B(1)(-1)$$

$$53 - 54 = -B$$

$$+1 = +B$$

$$\boxed{B = 1.}$$

Substitute $z = 4.$

$$3(4)^2 - 18(4) + 26 = A(0) + B(0) + C(4-2)(4-3)$$

$$48 - 72 + 26 = C(2)(1)$$

$$74 - 72 = 2C$$

$$2 = 2C$$

$$\boxed{C = 1.}$$

10

$\therefore$

$$z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right] = \frac{1}{(z-2)} + \frac{1}{(z-3)} + \frac{1}{(z-4)}$$

$\therefore$ by the linear property of z Transform.

$$z(a f(n) + g(n) b) = a z\{f(n)\} + b z\{g(n)\}$$

$$z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right] = z^{-1}\left[\frac{1}{(z-2)}\right] + z^{-1}\left[\frac{1}{z-3}\right] + z^{-1}\left[\frac{1}{z-4}\right]$$

we know that $z^{-1}\left[\frac{1}{z-a}\right] = a^{n-1}.$

$\therefore Z^{-1}\left[\frac{3z^2+26-18z}{(z-2)(z-3)(z-4)}\right] = 2^{n-1} + 3^{n-1} + 4^{n-1}.$

$= 2^n 2^{-1} + 3^n . 3^{-1} + 4^n . 4^{-1}$

$\therefore Z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right] = \left(\frac{1}{2}\right)2^n + \left(\frac{1}{3}\right)3^n + \left(\frac{1}{4}\right)4^n.$

which the required Z inverse of the given function.

good

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16/50

Part - C

11 a)
Ans

Given $z^{-1}\left\{\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right\}$

Let

$$\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)} = \frac{A}{(z-2)} + \frac{B}{(z-3)} + \frac{C}{(z-4)}$$

$$= \frac{A(z-3)(z-4) + B(z-2)(z-4) + C(z-2)(z-3)}{(z-2)(z-3)(z-4)}$$

$$3z^2-18z+26 = A(z-3)(z-4) + B(z-2)(z-4) + C(z-2)(z-3) \rightarrow (1)$$

Here substitute $z=2$ in eqn (1)

From ①

$$3(2)^2 - 18(2) + 26 = A(2-3)(2-4) + 0 + 0$$

$$3(4) - 36 + 26 = A(-1)(-2)$$

$$12 - 36 + 26 = A(2)$$

$$38 - 36 = A(2)$$

$$2 = A(2)$$

$$\boxed{A = 1}$$

Substitute $z = 3$ in eqn ①

$$3(3)^2 - 18(3) + 26 = A(0) + B(3-2)(3-4) + C(0)$$

$$27 - 54 + 26 = 0 + B(1)(-1) + 0$$

$$53 - 54 = -B$$

$$-1 = -B$$

$$\boxed{B = 1}$$

Substitute $z = 4$ in eqn ①

$$3(4)^2 - 18(4) + 26 = A(0) + B(0) + C(4-2)(4-3)$$

$$48 - 72 + 26 = 0 + 0 + C(2)(1)$$

(Margin working: $18 \times 2 = 36$; $26 + 12 = 38$; $18 \times 3 = 54$; $27 + 26 = 53$; $18 \times 4 = 72$; $16 \times 3 = 48$)

$$74 - 72 = C(2)$$

$$2 = C(2)$$

$$\boxed{C = 1}$$

$$\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)} = \frac{1}{(z-2)} + \frac{1}{(z-3)} + \frac{1}{(z-4)}$$

$$z^{-1}\left\{\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}\right\} = z^{-1}\left\{\frac{1}{(z-2)}\right\} + z^{-1}\left\{\frac{1}{(z-3)}\right\} + z^{-1}\left\{\frac{1}{(z-4)}\right\}$$

10

we know that

$$z^{-1}\left\{\frac{1}{z-a}\right\} = a^{n-1}$$

$$z^{-1}\left\{\frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}\right\} = 2^{n-1} + 3^{n-1} + 4^{n-1}$$

$$= \frac{1}{2} \cdot 2^n + \frac{1}{3} \cdot 3^n + \frac{1}{4} \cdot 4^n$$

Part - B

9(b) ii) ~~n.a^n~~ $Z\{n.a^n\}$

W.K.T

$$Z\{a^n.f(x)\} = -z\frac{d}{dz}(zf(x))$$

$$Z\{a^n.n\} = -z\frac{d}{dz}(z\{n\})$$

$$= -z\frac{d}{dz} \frac{1}{x-a}$$

$$= -z\left(\frac{(x-a)1 - x}{(x-a)^2}\right)$$

$$= -z\left(\frac{x-a-x}{(x-a)^2}\right)$$

$$= -z\left(\frac{-a}{(x-a)^2}\right)$$

$$Z\{a^n.n\} = \frac{az}{(x-a)^2}$$

9(b) i) $Z\{n\}$

w.k.T

$$Z\{f(x)\} = \sum_{n=0}^{\infty} f(x)\, z^{-n}$$

$$Z\{n\} = \sum_{n=0}^{\infty} n \cdot z^{-n}$$

3

$$= \sum_{n=0}^{\infty} \frac{n}{z^n}$$

$$= \sum_{n=1}^{\infty} \frac{n}{z^n}$$

$$= \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + \frac{4}{z^4} + \cdots\cdots$$

w.K.T

$$(1-x)^{-2} = (1 + 2x^2 + 3x^3 + 4x^4 + \cdots)$$

Here $x = \frac{1}{z})$

$$= \left(1 - \frac{1}{z}\right)^{-2}$$

$$= \left(\frac{z-1}{z}\right)^{-2}$$

$$= \frac{1}{\frac{(z-1)^2}{z^2}}$$

$$Z\{n\} = \left(\frac{z}{z-1}\right)^2$$

Part-A

6) $z\{\frac{1}{n}\}$

W.K.T

$$z\{n\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$z\{\frac{1}{n}\} = \sum_{n=0}^{\infty} \frac{1}{n} \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{nz^n}\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{nz^n}$$

$$= \frac{1}{1 \cdot z} + \frac{1}{2 \cdot z^2} + \frac{1}{3 \cdot z^3} + \frac{1}{4 \cdot z^4} + \cdots$$

$$= -\log($$

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Examination CLA-3 Subject U20MABT03 Transforms and Boundary value Problems.
Invigilator Sign. ...

25/50

Part-C

11. (b) Given,

To Find the z-transform of $\cos n\theta$ and $r^n \cos n\theta$.

For $z\{\cos n\theta\}$

take, $z\{a^n\} = \frac{z}{z-a}$

Let $a = e^{i\theta}$ $\quad a^n = e^{i\theta n}$

$$z\{e^{i\theta n}\} = \frac{z}{z-e^{i\theta}}$$

$$\left[\because e^{i\theta} = \cos\theta + i\sin\theta, \quad e^{-i\theta} = \cos\theta - i\sin\theta\right]$$

$$= \frac{z}{z-(\cos\theta + i\sin\theta)}$$

$$= \frac{z}{(z-\cos\theta) - i\sin\theta} \times \frac{(z-\cos\theta)+i\sin\theta}{(z-\cos\theta)+i\sin\theta}$$

$$= \frac{z\left[(z-\cos\theta)+i\sin\theta\right]}{(z-\cos\theta)^2 - (i^2)\sin^2\theta} \quad \left[\because i = \sqrt{-1}, \quad i^2 = -1\right]$$

$$= \frac{z(z-\cos\theta) + iz\sin\theta}{z^2 - 2z\cos\theta + (\cos^2\theta + \sin^2\theta)} \quad [\because \cos^2\theta + \sin^2\theta = 1]$$

$$= \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1} + i\frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

$\therefore$

$$Z\{\cos n\theta\} + iZ\{\sin n\theta\} = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1} + i\frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

by comparing both sides.

$$Z\{\cos n\theta\} = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$$

$$Z\{\sin n\theta\} = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$$

Now, To find $Z\{r^n \cos n\theta\}$

$$Z\{a^n f(n)\} = F\left(\frac{z}{a}\right)$$

$$Z\{f(n)\} = F(z)$$

$$F(z) = Z\{\cos n\theta\} = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$$

$$F\left(\frac{z}{r}\right) = \frac{\frac{z}{r}\left(\frac{z}{r}-\cos\theta\right)}{\left(\frac{z}{r}\right)^2 - 2\frac{z}{r}\cos\theta + 1}$$

$$= \frac{\frac{z}{r}\left(\frac{z-r\cos\theta}{r}\right)}{\frac{z^2}{r^2} - 2\frac{z}{r}\cos\theta + \frac{1}{1}}$$

$$= \frac{\frac{z(z-r\cos\theta)}{r^2}}{\frac{z^2-2zr\cos\theta+r^2}{r^2}}$$

$$= \frac{z(z-r\cos\theta)}{z^2-2zr\cos\theta+r^2}$$

$\therefore F\left(\frac{z}{r}\right) = Z\{r^n \cos n\theta\}$

$$\therefore Z\{r^n \cos n\theta\} = \frac{z(z-r\cos\theta)}{z^2-2zr\cos\theta+r^2}$$

Part- A

2. Given that,

$z = ax+by+a^2+b^2$ — ①

Now, Partially Derivating equation ① with respect to 'x'

$\frac{\partial z}{\partial x} = a(1) = p \Rightarrow \boxed{p=a}$ — ②

Now, Partially Derivating equation ① with respect to 'y'

$\frac{\partial z}{\partial y} = q = 0 + b(1) + 0 + 0$

$\Rightarrow \boxed{q=b}$ — ③

Substitute eq ② and ③ in eq ①

we get, $\boxed{z = px+qy+p^2+q^2}$

3. Z-transform:

In one sided z-transform only one side of the equation is solved. where

$$z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$z\{f(n)\} = F(z)$$

One-sided inverse z-transform:

same as one sided z-transform only a side is solved. where

$$\text{inverse } z\text{-transform} = \frac{1}{z\text{-transform}}$$

Ex:- $z^{-1}\left\{\frac{z}{z-a}\right\} = z\{a^n\}$

4. $z\{1\} = \sum_{n=0}^{\infty} (1)(z)^{-n}$

$$z\{1\} = \frac{z}{z-1}$$

$$z\{(-1)^n\} = \frac{z}{z+1}$$

6. $z\left\{\frac{1}{n}\right\} = \sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{1}{z}\right)^n$

$$\Rightarrow z\left\{\frac{1}{n}\right\} = \log\frac{z}{z-1}$$

$$z\left\{\frac{1}{n+1}\right\} = \sum_{n=0}^{\infty} \frac{1}{n+1}\left(\frac{1}{z}\right)^n$$

$$\Rightarrow z\left\{\frac{1}{n+1}\right\} = z\log\frac{z}{z-1}$$

(2)

U21AS010

Part - B

9.
(b) To find z-transform of

i) $z\{n\}$

we know that, $z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$

$z\{n\} = \sum_{n=0}^{\infty} n\left(\frac{1}{z}\right)^n$

$= 0 + (1)\left(\frac{1}{z}\right)^1 + 2\left(\frac{1}{z}\right)^2 + 3\left(\frac{1}{z}\right)^3 + \cdots$

⇒ It is in the form of

$(1-x)^{-1} = x + 2x^2 + 3x^3 + \cdots$

here $x = \left(\frac{1}{z}\right)$

$z\{n\} = \left(1-\frac{1}{z}\right)^{-1} = \left(\frac{z-1}{z}\right)^{-1} = \frac{z}{z-1}$

$= \frac{1}{z}\left[1 + 2\left(\frac{1}{z}\right)^1 + 3\left(\frac{1}{z}\right)^2 + 4\left(\frac{1}{z}\right)^3 + \cdots\right]$

$= \left[(1-x)^{-2}\right] = 1 + 2x + 3x^2 + 4x^3 + \cdots$

4

⇒ $x = \frac{1}{z}$ $z\{n\} = \left(1-\frac{1}{z}\right)^{-2}\frac{1}{z} = \left(\frac{z-1}{z}\right)^{-2}\frac{1}{z} = \left(\frac{z}{z-1}\right)^2 \cdot \frac{1}{z}$

$z\{n\} = \frac{z^2}{(z-1)^2} \times \frac{1}{z}$

$z\{n\} = \frac{z}{(z-1)^2}$

ii) $z\{n a^n\}$

we know that $z\{f(n)\} = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$z\{na^n\} = \sum_{n=0}^{\infty} n \frac{a^n}{z^n} = \sum_{n=0}^{\infty} n\left(\frac{a}{z}\right)^n$$

$$= \left(\frac{a}{z}\right)^1 + 2\left(\frac{a}{z}\right)^2 + 3\left(\frac{a}{8}\right)^3 + 4\left(\frac{a}{z}\right)^4 + \cdots$$

$$= \frac{a}{z}\left(1 + 2\left(\frac{a}{z}\right) + 3\left(\frac{a}{z}\right)^2 + 4\left(\frac{a}{z}\right)^3 + \cdots\right]$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

here $x = \frac{a}{z}$

$$= \frac{a}{z}\left(1 - \frac{a}{z}\right)^{-2} = \frac{a}{z}\left(\frac{z-a}{z}\right)^{-2}$$

$$= \frac{a}{z}\left[\frac{z}{z-a}\right]^2 = \frac{a}{z}\left(\frac{z^2}{(z-a)^2}\right)$$

$$z\{na^n\} = \frac{a z^2}{z(z-a)^2}$$

~~$z\{n f(n)\} = -z\frac{\partial}{\partial z} f(n) = -z\frac{\partial}{\partial z} a^n = -z\, n a^{n-1}$~~

rough

$z\{n\} \times z\{a^n\}$

$= \frac{z}{(z-1)^2} \times \frac{z}{(z-a)}$

$= \frac{z^2}{(z-1)^2(z-a)}$

$= \frac{z^2}{z^2 - 2z + 1\,(z-a)}$

$z^3 - 2z^2 + z - az^2 - 2az + a$

z^2

$= -z\frac{\partial}{\partial z} f(n)$

Part - A

1.

$$\frac{\partial^2 z}{\partial x^2} = 0$$

$$\frac{\partial z}{\partial x} = f(y)$$

Now, f(y) be the constant

again integrate,

$$\partial x = f(y)\, dx$$

$$z = x f(y) + g(y)$$

This is the required general equation.

Part-C

10.
(b) Given that,

$$\left(D^2+2DD'+D'^2\right)z = x^2y$$

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=x^2y$$

$\Rightarrow$

8(a) = Given that,

$$px+qy=z$$

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